# Exercise walk-through

## Conservation of Angular Momentum ▪ 6.4

eXercise

# You must be able to

- state that **angular momentum is conserved** 角动量守恒 when no net external torque acts
- apply $I_i\omega_i = I_f\omega_f$
- explain the skater effect (pulling arms in speeds the spin)
- note that kinetic energy need not be conserved

## Method — Conservation condition

If the net external torque is zero,

$$I_i \omega_i = I_f \omega_f.$$

# Method — The skater effect

A skater pulls her arms in, lowering $I$. Since $I\omega$ is fixed, $\omega$ rises:

$$I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \frac{I_i\omega_i}{I_f}.$$

If $I$ halves, $\omega$ doubles.

## Method — Energy is not conserved here

The skater does work pulling her arms in, so the rotational kinetic energy actually **increases** even though $L$ stays constant.

## Practice 2.1 ▪ 1 mark

State the condition for angular momentum to be conserved.

# Solution 2.1

**Worked steps**

When the net external torque is zero B1.

## Practice 2.2 ▪ 2 marks

A skater at $I_i = 6$ kg m$^2$ spins at $2$ rad s$^{-1}$, then pulls in to $I_f = 2$ kg m$^2$. Find the new $\omega$.

# Solution 2.2

Method: The skater effect

**Worked steps**

$6(2) = 2\omega_f \Rightarrow \omega_f = 6$ rad s$^{-1}$ M1 A1.

## Practice 2.3 ▪ 1 mark

If a spinning body's rotational inertia halves (no external torque), what happens to $\omega$?

# Solution 2.3

Method: Conservation condition

**Worked steps**

It doubles B1.

## Practice 2.4 ▪ 1 mark

Is rotational kinetic energy always conserved when angular momentum is? Answer yes or no.

# Solution 2.4

Method: Energy is not conserved here

**Worked steps**

No B1.

## Exam-style 3.1 ▪ 1 mark

Angular momentum is conserved when the net external torque is

**A** maximum

**B** zero

**C** increasing

**D** equal to $I\alpha$

# Solution 3.1

Method: Conservation condition

**A** maximum

**B** zero ✓

**C** increasing

**D** equal to $I\alpha$

## Worked steps

**B** — zero net external torque.

## Exam-style 3.2 ▪ 1 mark

A star collapses to one third of its rotational inertia. Its spin rate becomes

**A** one third

**B** unchanged

**C** three times faster

**D** nine times faster

# Solution 3.2

**A** one third

**B** unchanged

**C** three times faster ✓

**D** nine times faster

**Worked steps**

**C** — $I \to I/3$ means $\omega \to 3\omega$.

## Exam-style 3.3 ▪ 2 marks

A turntable ($I = 4$ kg m$^2$) spins at $3$ rad s$^{-1}$. A blob of clay is dropped on, raising $I$ to $6$ kg m$^2$.

(a) Find the new angular speed.

(b) State whether angular momentum is conserved and why.

# Solution 3.3

**Worked steps**

(a) $4(3) = 6\omega_f \Rightarrow \omega_f = 2$ rad s$^{-1}$ M1 A1. (b) Yes — no external torque acts on the turntable-plus-clay system B1.

## Exam-style 3.4 ▪ 2 marks

A diver tucks to spin faster during a somersault.
(a) Explain, using $I\omega$, why tucking speeds the spin.
(b) State what provides no external torque here.

# Solution 3.4

Method: Conservation condition

**Worked steps**

(a) Tucking lowers $I$; since $I\omega$ is fixed, $\omega$ rises M1 A1. (b) Gravity acts through the center of mass, giving no torque about it B1.