

Past-paper walk-through

Angular Momentum and Angular Impulse ■ 6.3

eXercise

Questions in this set

- 2018 Q3
- 2016 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- **//** separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 3

2018 ■ Q3

A triangular rod, shown above, has length L , mass M , and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

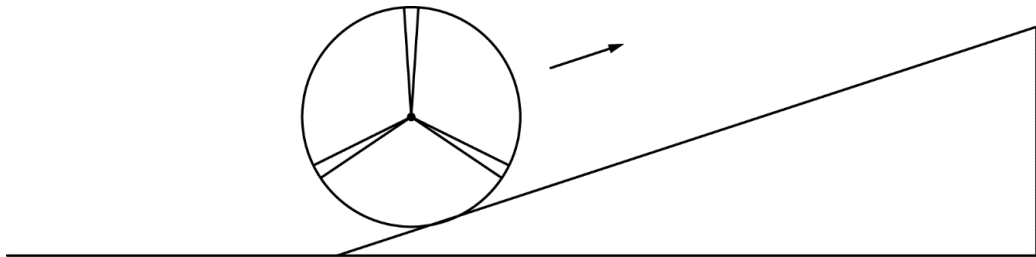
[Figure 1: A plain circle (thin hoop), unlabeled interior.]

[Figure 2: A circle (thin hoop) with three identical rods fastened inside it, each rod running from the center of the hoop out to the rim, spaced so the three rods form a symmetric tripod/Y-like pattern (three line segments from the center to three points on the circle).]

Question 3

The thin hoop shown above in Figure 1 has a mass M , radius L , and a rotational inertia around its center of ML^2 . Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

Question 3



Question 3

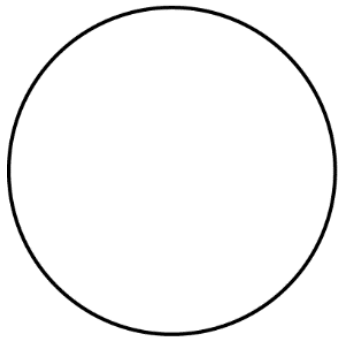


Figure 1

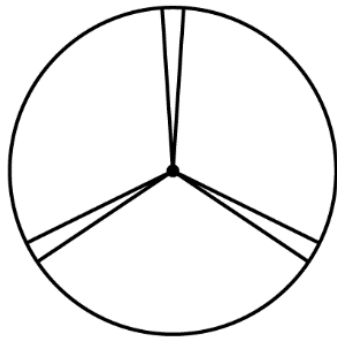
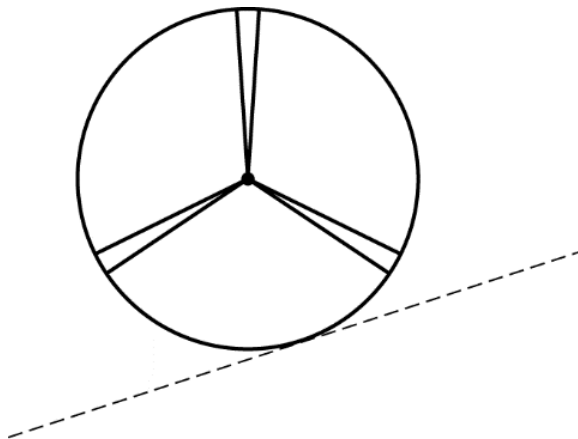


Figure 2

Question 3



Question 3

2018 ■ Q3

A triangular rod, shown above, has length L , mass M , and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M , L , and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt .

Question 3

2018 ■ Q3

A triangular rod, shown above, has length L , mass M , and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(c) Derive an expression for the final angular speed ω of the hoop-rods system.

Express your answer in terms of M , L , F , Δt , and physical constants, as appropriate.

[Diagram: The hoop-rods system (circle with three internal rods forming a Y-pattern) sitting at the base of an inclined ramp, with a curved arrow above/left of the hoop indicating a force F applied tangent to the hoop, pushing it up the ramp. The ramp rises from left to right, and the hoop sits at the bottom, touching the horizontal surface to the left and the incline surface.]

Question 3

The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

Question 3

2018 ■ Q3

A triangular rod, shown above, has length L , mass M , and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(d)(i) On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.

[Diagram: The hoop-rods system (circle with three internal rods in a Y-pattern) resting on a dashed line representing an inclined surface, for the student to draw force vectors on.]

(d)(ii) Justify your choice for the direction of each of the forces drawn in part (d)i.

Question 3

2018 ■ Q3

A triangular rod, shown above, has length L , mass M , and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M , L , I_{tot} , ω , and physical constants, as appropriate.

Solution 3

(a) Mark scheme

■ $I = \int r^2 dm = \int x^2 dm$. Since $\lambda = \frac{2M}{L^2}x$, $dm = \lambda dx = \frac{2M}{L^2}x dx$. So

$$I = \int_0^L \frac{2M}{L^2} x^3 dx = \left[\frac{2M}{L^2} \cdot \frac{x^4}{4} \right]_0^L = \frac{2M}{L^2} \cdot \frac{L^4}{4} = \frac{ML^2}{2},$$
 as required. 1 point for relating x to r properly in the integral; 1 point for correctly substituting the linear mass density to get dm ; 1 point for integrating with correct limits (or constant of integration).

Solution 3

(b) Mark scheme

- The hoop-rods system's inertia about the hoop's center is the sum of the hoop's own inertia (ML^2) and the three rods' inertia about that same center (each rod's inertia about its end, from part (a), is $ML^2/2$):

$I_{tot} = 3I_{rod} + I_{hoop} = 3\left(\frac{ML^2}{2}\right) + ML^2 = \frac{5}{2}ML^2$. 1 point for setting the total rotational inertia equal to the sum of the hoop's and the three rods' inertias; 1 point for the correct final answer $I_{tot} = \frac{5}{2}ML^2$.

Solution 3

(c) Mark scheme

- Angular impulse-momentum theorem: $\tau\Delta t = I(\omega_2 - \omega_1)$, and starting from rest:
 $Fr\Delta t = I\omega \Rightarrow \omega = \frac{Fr\Delta t}{I}$. The lever arm equals the hoop's radius L (force applied tangent to the hoop): $\omega = \frac{FL\Delta t}{I}$. Substituting $I_{tot} = \frac{5}{2}ML^2$ from part (b): $\omega = \frac{FL\Delta t}{\frac{5}{2}ML^2} = \frac{2F\Delta t}{5ML}$. 1 point for using an appropriate equation for the final angular speed; 1 point for relating the lever arm to the length L ; 1 point for correct substitution of I_{tot} from part (b).

Solution 3

(d)(i) Mark scheme

- Three forces act on the rolling hoop-rods system: the weight F_W , drawn as a distinct arrow starting at the center of the hoop and pointing straight down; the normal force F_N , drawn as an arrow starting at the contact point with the incline and pointing perpendicular to the incline surface (up and to the left); and friction f , drawn as an arrow starting at the contact point and pointing along the incline surface, up the ramp (opposing the tendency to slip as the hoop rolls up). 1 point each for the weight, the normal force, and the friction drawn in the correct direction and at the correct point of application; a maximum of two of the three points can be earned if any extraneous vectors are drawn or if any vector has an incorrect point of exertion.

Solution 3

(d)(ii)

Mark scheme

- The normal force is always perpendicular to the surface, so it is directed up and to the left (away from the incline). The gravitational force is always vertically downward. The friction force is opposite the direction of (relative) rotation/slipping at the contact point, so it is directed up the incline.

Solution 3

(e) Mark scheme

- Energy conservation from the bottom of the ramp to maximum height:

$$K_i = U_{g2} \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH, \text{ so } H = \frac{\frac{1}{2}mv^2 + \frac{1}{2}I_{tot}\omega^2}{mg}. \text{ Using the}$$

rolling-without-slipping condition $v = L\omega$:

$$H = \frac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg} = \frac{\frac{1}{2}(mL^2 + I_{tot})\omega^2}{mg}. \text{ The total rolling mass is } m = 4M$$

(the hoop of mass M plus the three rods of mass M each):

$$H = \frac{\frac{1}{2}((3M + M)L^2 + I_{tot})\omega^2}{(3M + M)g} = \frac{(4ML^2 + I_{tot})\omega^2}{8Mg}. \text{ 1 point for including both}$$

translational and rotational kinetic energy in the energy-conservation equation for

Solution 3

the final height; 1 point for correctly substituting $v = L\omega$; 1 point for correctly substituting the total inertias/masses into the energy equation.

Question 3

2016 ■ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.]

[Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in

Mass	Rotational Inertia	Block	m	Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$

(about the end of the rod) | | Platform | $m_P = 5m$ | $\frac{m_P R^2}{2}$ (about the central axis) |

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

Question 3

(a) Derive an expression for the spring constant of the spring.

Question 3

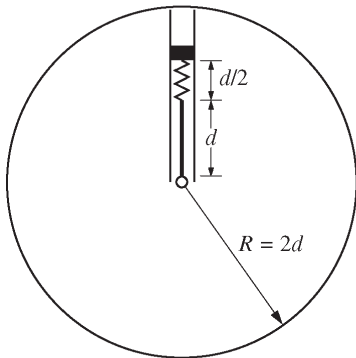


Figure 1

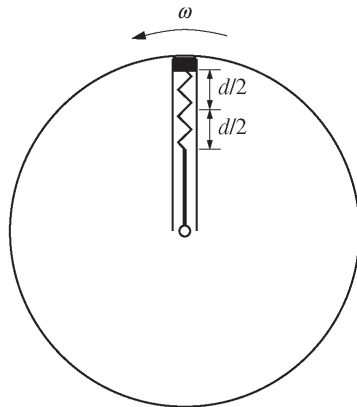


Figure 2

Question 3

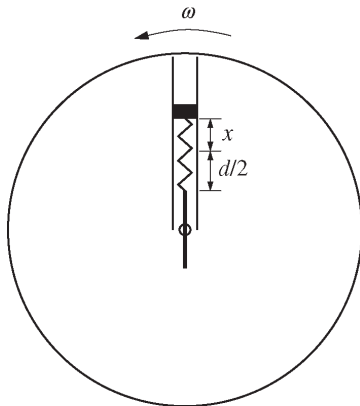


Figure 3

Question 3

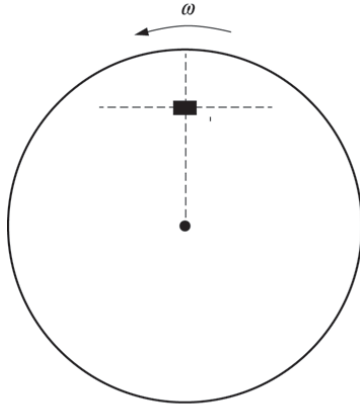


Figure 4

Question 3

2016 ■ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Question 3

length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

Mass	Rotational Inertia	Block	m	Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)	Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)
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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

(b)(i) Determine an expression for the rotational inertia of the block around the axis of the platform.

Question 3

(b)(ii) Derive an expression for the rotational inertia of the entire system about the axis of the platform.

Question 3

2016 ■ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

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Question 3

length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

Mass	Rotational Inertia	Block	m	Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
		Platform	$m_P = 5m$		$\frac{m_P R^2}{2}$ (about the central axis)	

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

Question 3

[Figure 3: The same circular platform, rotating counterclockwise at ω . The rod's block position is shown partway along the rod, with the spring stretched by amount x and a remaining segment labeled $d/2$ below it.]

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

Question 3

2016 ■ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Question 3

length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

Mass	Rotational Inertia	Block	m	Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
		Platform	$m_P = 5m$		$\frac{m_P R^2}{2}$ (about the central axis)	

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω . Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

Question 3

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

Question 3

2016 ■ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Question 3

	Mass		Rotational Inertia		— — —		Block		m		Rod		$m_R = 3m$		$\frac{m_R d^2}{3}$	(about the
end of the rod)																
	Platform		$m_P = 5m$		$\frac{m_P R^2}{2}$	(about the central axis)										

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Question 3

Justify your answer.

Question 3

2016 ■ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Mass	Rotational Inertia	Block	m	Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)	Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)
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Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

(f) In order to keep the system rotating with constant angular speed ω , is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Question 3

Justify your answer.

Question 3

2016 ■ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

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Question 3

			Mass		Rotational Inertia		— — —		Block		m			Rod		$m_R = 3m$		$\frac{m_R d^2}{3}$	(about the
end of the rod)			Platform		$m_P = 5m$		$\frac{m_P R^2}{2}$	(about the central axis)											

Answer the following questions for the platform rotating at constant angular speed ω . Express all algebraic answers in terms of m , d , ω , and physical constants, as appropriate.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

Question 3

[Figure 4: The circular platform rotating counterclockwise at ω , with the block shown at the center on the axis (a small dark square marker), a dashed horizontal line through it, and a dot below the platform on the vertical rod line, for the student to draw an acceleration vector on the block.]

Solution 3

(a)

Mark scheme

- Radial form of Newton's second law: spring force equals centripetal force, $F_S = F_C = ma_C$, i.e. $kx = mr\omega^2$. With spring stretch $x = d/2$ and orbit radius $r = 2d$ (per the given geometry): $k(\frac{d}{2}) = m(2d)\omega^2$. Solving: $k = 4m\omega^2$. 1 pt for indicating $F_S = F_C = ma_C$; 1 pt for a correct substitution for x ; 1 pt for a correct substitution for r that lets you solve for k . Correct answer: $k = 4m\omega^2$.

Solution 3

(b)(i)

Mark scheme

- Treating the block as a point mass at radius $r = 2d$:
 $I = \sum mr^2 = m(2d)^2 = 4md^2$. 1 pt for a correct answer, or an answer consistent with the radius used in part (a).

Solution 3

(b)(ii) Mark scheme

- Total rotational inertia is the sum of the platform's, rod's, and block's:

$I = I_P + I_R + I_O = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + m r^2$. Substituting $m_P = 5m$, $R = 2d$ (platform as a disk), $m_R = 3m$ (rod), and $m r^2 = 4m d^2$ from (b)i:

$I = \frac{5m(2d)^2}{2} + \frac{3m d^2}{3} + 4m d^2 = 10m d^2 + m d^2 + 4m d^2 = 15m d^2$. 1 pt for indicating the system's rotational inertia must include the platform, rod, and object; 1 pt for correctly substituting each piece consistent with (b)i. Correct answer: $I = 15m d^2$.

Solution 3

(c)

Mark scheme

- $L = I\omega$. Using $I = 15md^2$ from part (b)ii: $L = 15md^2\omega$. 1 pt for an answer consistent with the answer from part (b)ii.

Solution 3

(d)

Mark scheme

- Same radial equation as part (a): $F_S = F_C = ma_C \Rightarrow kx = m\omega^2 r$. Using $k = 4m\omega^2$ (from part (a), or the student's own consistent value) and the new radius $r = d + x$ (rod's center now at the axis, spring stretched by x):
 $(4m\omega^2)x = m(d + x)\omega^2 \Rightarrow 4x = d + x \Rightarrow 3x = d \Rightarrow x = d/3$. 1 pt for indicating the spring force equals mass times centripetal acceleration; 1 pt for a correct substitution for k (or one consistent with part (a)); 1 pt for a correct substitution for r that lets you solve for x . Correct answer: $x = d/3$.

Solution 3

(e)

Mark scheme

- Answer: Decreasing. As the block moves inward toward the axis, the system's rotational inertia I decreases while ω is held constant; since $L = I\omega$, a decreasing I with constant ω means L decreases. 1 pt for selecting 'Decreasing' (no credit for the justification if the wrong selection is made); 1 pt for a correct justification (I decreases while ω stays the same, so $L = I\omega$ decreases).

Solution 3

(f)

Mark scheme

- Answer: Negative. With ω held constant while I decreases, the rotational kinetic energy $K = \frac{1}{2}I\omega^2$ decreases; by the work-energy theorem the net work on the rotating system is negative, so the motor is doing negative work (removing energy to keep ω constant as I shrinks). 1 pt for selecting 'Negative' (no credit for the justification if the wrong selection is made); 1 pt for a correct justification (ω is constant, I has decreased, so rotational KE has decreased, so the work done must be negative).

Solution 3

(g)

Mark scheme

- The scoring guidelines show the correct acceleration direction as a single arrow starting on the block and pointing down and to the right — i.e. combining a centripetal (inward) component with a tangential component consistent with the block's inward radial motion while the platform rotates counterclockwise. 1 pt for an arrow starting on the box and pointing down and to the right.