Exercise walk-through

Angular Momentum and Angular Impulse ▪ 6.3

eXercise

You must be able to

- use **angular momentum** 角动量: $L = I\omega$ (and $L = mvr$ for a point mass)
- use **angular impulse** 角冲量: $\tau\,\Delta t = \Delta L$
- recognise the analogy with $p = mv$ and $F\Delta t = \Delta p$
- state the unit of angular momentum (kg m^2 s^{-1})

Method — Angular momentum

$$L = I\omega.$$

A wheel with $I = 3$ kg m^2 at $\omega = 2$ rad s^{-1} has $L = 6$ kg m^2 s^{-1}. For a point mass, $L = mvr$.

Method — Angular impulse

A torque acting over a time changes the angular momentum:

$$\tau \, \Delta t = \Delta L.$$

A 4 N m torque for 3 s adds $\Delta L = 12$ kg m^2 s^{-1}.

Method — The rate form

Equivalently $\tau = \dfrac{dL}{dt}$ – torque is the rate of change of angular momentum.

Practice 2.1 ▪ 2 marks

A flywheel with $I = 5$ kg m^2 spins at 4 rad s^{-1}. Find its angular momentum.

Solution 2.1

Worked steps

$L = I\omega = 5 \times 4 = 20$ kg m^2 s^{-1} (M1)(A1).

Practice 2.2 ▪ 1 mark

State the unit of angular momentum.

Solution 2.2

Worked steps

kg m^2 s^{-1} B1.

Practice 2.3 ▪ 2 marks

A 3 N m torque acts for 5 s. Find the change in angular momentum.

Solution 2.3

Worked steps

$\Delta L = \tau\,\Delta t = 3 \times 5 = 15$ kg m^2 s^{-1} M1 A1.

Practice 2.4 ▪ 2 marks

A 2 kg ball moves at 3 m s^{-1} in a circle of radius 4 m. Find its angular momentum.

Solution 2.4

Worked steps

$L = mvr = 2 \times 3 \times 4 = 24$ kg m^2 s^{-1} M1 A1.

Exam-style 3.1 ▪ 1 mark

Angular momentum is the rotational analogue of

A force

B momentum

C energy

D acceleration

Solution 3.1

A force

B momentum ✓

C energy

D acceleration

Worked steps

B — angular momentum is the rotational analogue of linear momentum.

Exam-style 3.2 ▪ 1 mark

For a rigid body, L equals

A $I\alpha$

B $I\omega$

C $\frac{1}{2}I\omega^2$

D $\tau\theta$

Solution 3.2

A $I\alpha$

B $I\omega$ ✓

C $\frac{1}{2}I\omega^2$

D $\tau\theta$

Worked steps

B — $L = I\omega$.

Exam-style 3.3 ▪ 1 mark

A wheel with $I = 2$ kg m^2 at $\omega = 3$ rad s^{-1} receives a steady 2 N m torque for 4 s.
(a) Find its initial angular momentum.
(b) Find its angular momentum after the torque acts.

Solution 3.3

Method: Angular momentum

Worked steps

(a) $L_i = 2 \times 3 = 6$ kg m^2 s^{-1} B1. (b) $\Delta L = 2 \times 4 = 8$; $L_f = 6 + 8 = 14$ kg m^2 s^{-1} M1 A1.

Exam-style 3.4 ▪ 1 mark

A skater has $I = 4$ kg m^2 spinning at 2 rad s^{-1}.
(a) Find her angular momentum.
(b) State what quantity torque is the rate of change of.

Solution 3.4

Method: The rate form

Worked steps

(a) $L = 4 \times 2 = 8$ kg m^2 s^{-1} B1. (b) Angular momentum ($\tau = dL/dt$) B1.