Past-paper walk-through

Rotational Kinetic Energy ▪ 6.1

eXercise

Questions in this set

- 2026 Q4
- 2023 Set 1 Q3
- 2023 Set 2 Q3
- 2021 Set 1 Q3
- 2021 Set 2 Q2
- 2019 Set 1 Q3
- 2018 Q3
- 2017 Q3
- 2015 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2026 ▪ Q4

A unicycle is placed on a stand such that the wheel does not touch the ground. The rotational inertia of the wheel about the axle is I_W. One end of a crank arm can be attached to the wheel, as shown in the side view.
[Figure: A side-view illustration of a unicycle (seat, seat post, frame, wheel with spokes, and stand legs) standing on the ground with the wheel raised off the ground, labeled "Unicycle." A dashed-line callout box magnifies the wheel region, labeled "Side View," showing the wheel's outer rim labeled "Wheel," the central hub labeled "Axle," and a straight bar extending from the axle outward labeled "Crank Arm."]
In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each

Question 4

crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

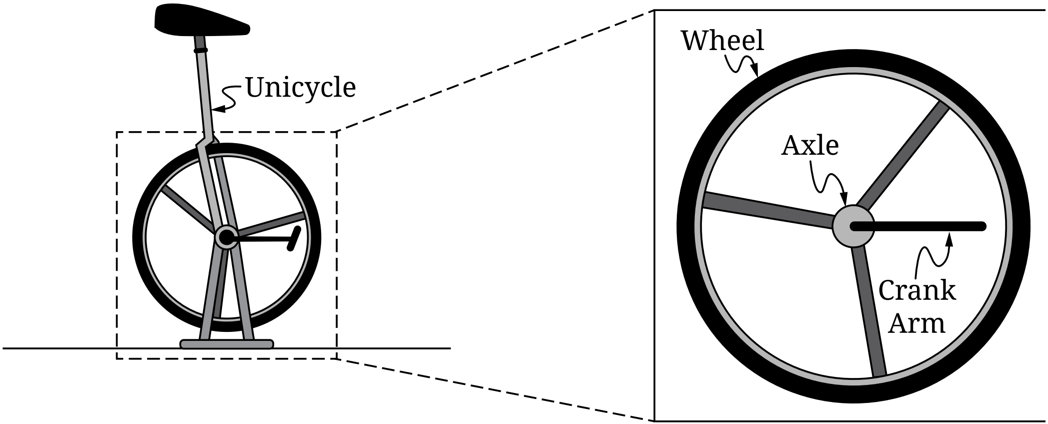
(a) Indicate whether ω_2 is greater than, less than, or equal to ω_1 by writing one of the following.

- $\omega_2 > \omega_1$
- $\omega_2 < \omega_1$
- $\omega_2 = \omega_1$

Question 4

Justify your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

Question 4



Question 4

2026 ▪ Q4

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- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

(b) Derive an expression for the angular speed ω_1 of the wheel in Scenario 1 immediately after the crank arm has been turned through one rotation. Express your answer in terms of I_W, F, ℓ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

2026 ▪ Q4

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In Scenarios 1 and 2, crank arms of different lengths are connected to the wheel. In each scenario, the wheel is initially at rest. A force of magnitude F is then exerted at the end of each crank arm. The direction of the force remains perpendicular to each crank arm at all times. The mass of each crank arm is negligible. The scenarios differ as described.

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- Scenario 1: The length of the crank arm is ℓ_1. The angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_1.
- Scenario 2: The length of the crank arm is ℓ_2, where $\ell_2 > \ell_1$. The angular speed of the wheel immediately after the longer crank arm has been turned through one rotation is ω_2.

(c) In Scenario 3, the unicycle is placed on the ground such that the unicycle can move forward without the wheel slipping on the ground. A force of the same magnitude F is exerted at the end of the crank arm of length ℓ_1, as in Scenario 1. The direction of the force remains perpendicular to the crank arm at all times. The resulting angular speed of the wheel immediately after the crank arm has been turned through one rotation is ω_3.

Indicate whether ω_3 is greater than, less than, or equal to ω_1 by writing one of the following.

Question 4

- $\omega_3 > \omega_1$
- $\omega_3 < \omega_1$
- $\omega_3 = \omega_1$

Briefly **justify** your answer. Your justification may reference equations but must include conceptual reasoning beyond algebraic solutions.

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total

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rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

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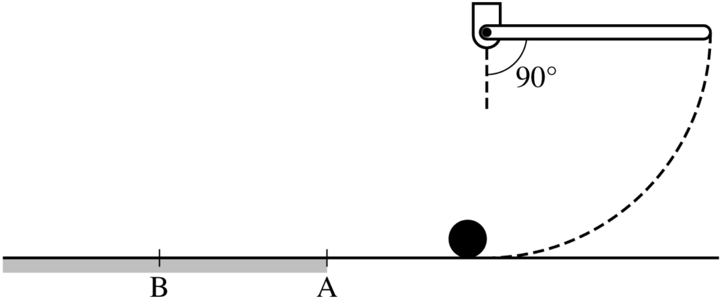

Note: Figure not drawn to scale.

Figure 1

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2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate. After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

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2023 Set 1 ▪ Q3

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.
[A circle is shown representing the sphere, for the student to draw and label force vectors starting on the circle.]

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2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(d)(i) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

The linear velocity v of the center of mass of the sphere as a function of time t

(d)(ii) The angular velocity ω of the sphere as a function of time t

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2023 Set 1 ▪ Q3

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(e)(i) Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

(e)(ii) Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.
[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]
[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

Question 3

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{cm} = \dfrac{1}{18}ML^2$.

Question 3

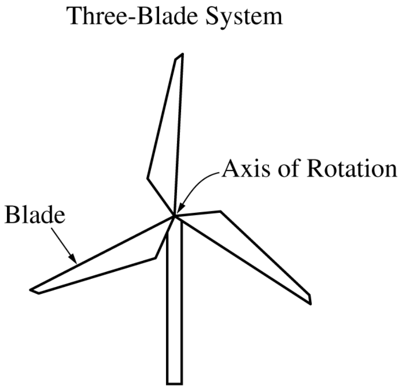

Figure 1

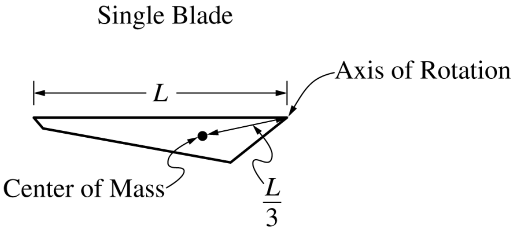

Figure 2

Question 3

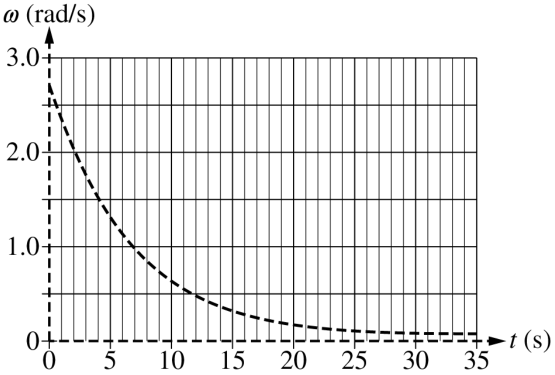

Figure 4

Question 3

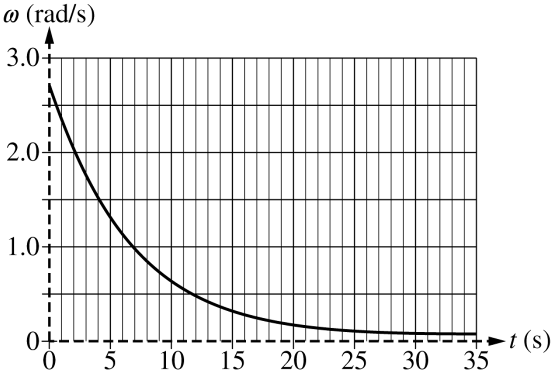

Figure 3

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

Question 3

2023 Set 2 ▪ Q3

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[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(c)(i) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

Question 3

[Figure 3: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; curve starts at $(0, 2.6)$ and decays exponentially toward 0 as t increases, with a dashed vertical line at $t = 0$ marking the initial value.]
Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.
(c)(ii) Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.
(c)(iii) Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.
The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the

Question 3

equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.
[Figure 4: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; a dashed curve shows the original $\omega_0 e^{-\beta_0 t}$ decay from $(0, 2.6)$ toward 0.]

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

Question 3

2021 Set 1 ▪ Q3

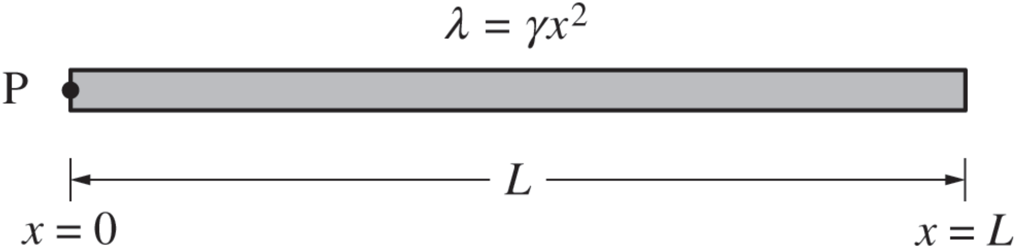
[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

Question 3

Question 3



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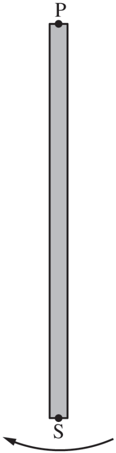

Question 3

2021 Set 1 ▪ Q3

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A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_______ Greater than _______ Less than _______ Equal to

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

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A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(d) The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.

[Two blank axes: left axis labeled τ (vertical) vs. t (horizontal); right axis labeled ω (vertical) vs. t (horizontal). Both currently blank, to be sketched by the student.]

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

______ Increasing ______ Decreasing ______ Not changing

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

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A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(f) [Figure: The rod hangs vertically, pivoted at point P at the top, with point S marked at the bottom end. A curved arrow beneath S indicates the rod swinging.] The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

Solution 3

(a) Mark scheme

- Set up the rotational-inertia integral about point P: $I = \int r^2\, dm$, with the given linear mass density $\lambda = \gamma x^2$ so $dm = \lambda\, dx = \gamma x^2\, dx$ (1 point). Substitute and integrate from $x = 0$ to $x = L$, using $\gamma = 3M/L^3$ (from normalizing the total mass $M = \int_0^L \gamma x^2\, dx = \gamma L^3/3$) (1 point):
$$I = \int_0^L x^2(\gamma x^2\, dx) = \gamma \left[\frac{x^5}{5}\right]_0^L = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5}\right) = \frac{3}{5}ML^2.$$

Solution 3

(b) Mark scheme

- Set up the center-of-mass integral: $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\int x\, dm}{\int dm}$ (1 point). Substitute $dm = \gamma x^2\, dx$ into the numerator (1 point), and either substitute M directly for the denominator or evaluate $\int dm$ to get the rod's mass (1 point): $X_{CM} = \frac{\int_0^L x(\gamma x^2)\, dx}{M} = \frac{[\gamma x^4/4]_0^L}{M} = \frac{(3M/L^3)(L^4/4)}{M} = \frac{3}{4}L$. The center of mass is located $\frac{3}{4}L$ from point P.

Solution 3

(c)

Mark scheme

- Answer: Greater than (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) more of the rod's mass is concentrated at the end opposite P (farther from the rotation axis), so the rotational inertia about P exceeds that about the center of mass; or (ii) by the parallel-axis theorem $I = I_{cm} + md^2$, an axis displaced from the center of mass ($d > 0$) always gives a larger rotational inertia than the axis through the center of mass.

Solution 3

(d) Mark scheme

- Sketch two graphs vs. time t, from release (rod horizontal) to when the center of mass reaches its lowest point (rod vertical). Torque τ vs. t (1 point): starts at its maximum value at $t = 0$ and decreases along a concave-down curve to zero (torque vanishes as the rod becomes vertical, when the weight acts along the line to the pivot). Angular speed ω vs. t (1 point): starts at 0 and increases along a concave-down curve that flattens, approaching a horizontal asymptote (constant ω) as t increases (since $\alpha = \tau/I \to 0$ as $\tau \to 0$). Consistency (1 point): ω must keep rising as long as $\tau > 0$ and level off exactly as $\tau \to 0$, matching $d\omega/dt = \tau/I$.

Solution 3

(e)

Mark scheme

- Answer: Decreases (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) from $\tau = rF\sin\theta$, as the rod swings downward the angle θ between the weight and the lever arm decreases, so the torque decreases; or (ii) the horizontal lever arm between P and the rod's center of mass continuously shrinks as the rod rotates downward, so the torque decreases. Since I is constant and $\alpha = \tau/I$, angular acceleration decreases correspondingly.

Solution 3

(f) Mark scheme

- Use conservation of energy for the rod swinging from horizontal (at rest) to vertical: $U_i + K_i = U_f + K_f \Rightarrow U_i = K_f$ (1 point). Substitute the gravitational PE (using the center-of-mass drop $h_i = \frac{3}{4}L$ found in part (b)) and the rotational KE about P (1 point): $Mgh_i = \frac{1}{2}I\omega_f^2 \Rightarrow Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\omega_f^2$. Using $I = \frac{3}{5}ML^2$ from part (a) and $\omega_f = v/L$, where v is the linear speed of point S (the free end, distance L from P) (1 point): $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \Rightarrow v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}(9.8\text{ m/s}^2)(1.0\text{ m})} = 4.9\text{ m/s}$. (Given $M = 3.0$ kg, $L = 1.0$ m; note M cancels out of the final expression for v.)

Question 2

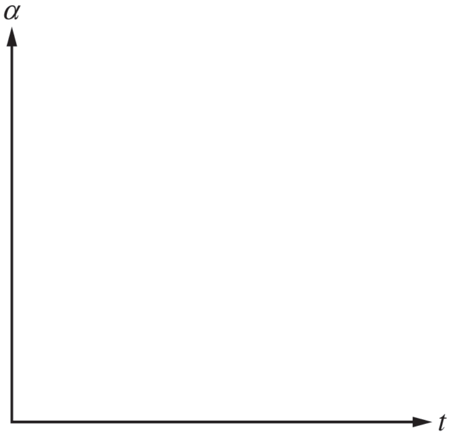
2021 Set 2 ▪ Q2

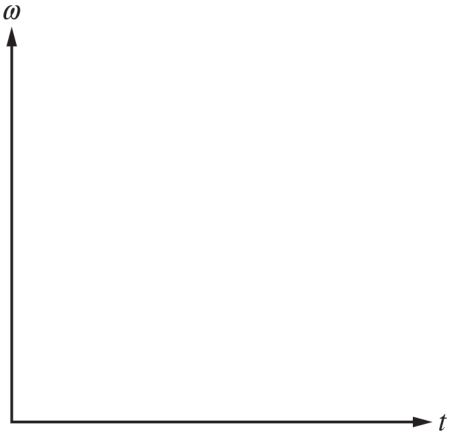
[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]
Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.
(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.
[Figure: "Object B" — an L-shaped (right-angle) object formed of two rods, shown on an x-y coordinate system with origin O at the bottom-left. A pivot is marked at the top-left corner (labelled "Pivot", on the y-axis). A horizontal rod of length L extends

Question 2

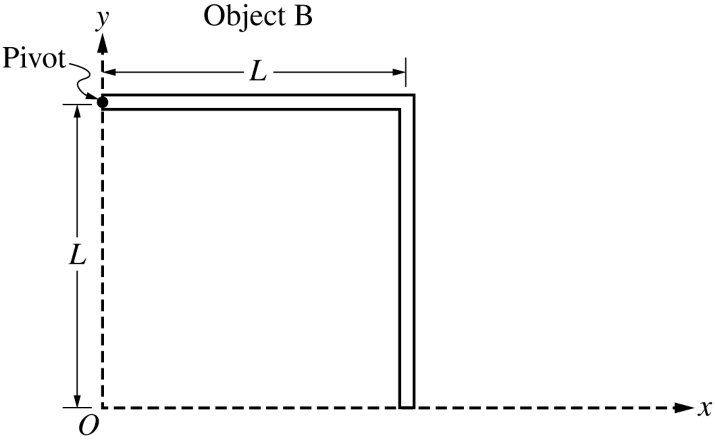
right from the pivot along the top (labelled "L"), and a vertical rod of length L extends down from the pivot to O along the y-axis (labelled "L"). The x-axis points right from O, the y-axis points up from O through the pivot.]
Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above.
Express your answers in part (b) in terms of L.

Question 2

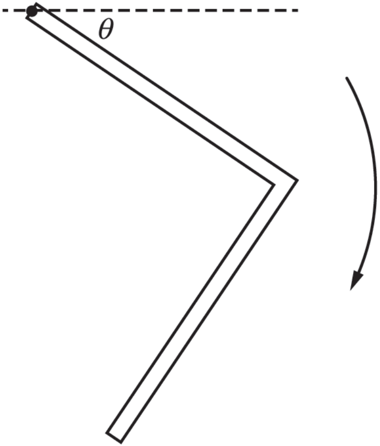




Question 2



Question 2



Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(b)(i) Determine the following for the given coordinate system shown in the figure.

The x-coordinate of the center of mass of object B

(b)(ii) The y-coordinate of the center of mass of object B

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(c) Object B has a rotational inertia of I_B about its pivot.

Is the value of I_B greater than, less than, or equal to I_A?

_______Greater than _______Less than _______Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.

[Two blank sets of axes are provided: left axis labelled α (vertical) vs. t (horizontal); right axis labelled ω (vertical) vs. t (horizontal). Both axes are otherwise unmarked (no gridlines or scale), for the student to sketch curves.]

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(e) [Figure: The L-shaped object B is shown rotated partway from its initial horizontal/vertical position, tilted through an angle θ (marked at the top between a dashed horizontal reference line and the now-tilted top rod), with a curved arrow indicating the direction of rotation (downward/clockwise) at the lower end of the shape.]

Question 2

While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

Increasing Decreasing Notchanging

Justify your answer.

[Figure: Object B is shown in a new orientation — the vertical rod of length L now lies along the bottom (horizontal), and the horizontal rod of length L extends straight up from its right end (vertical), forming an upside-down L / right-angle shape resting on the ground.]

Object B rotates through the position shown above.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

Solution 2

(a) Mark scheme

- Using integral calculus with λ the linear mass density of the rod (length $2L$, mass M): $I = \int_{r=0}^{r=2L} \lambda r^2 \, dr = \lambda \left[\frac{r^3}{3} \right]_0^{2L} = \frac{\lambda}{3}(2L)^3$ (1 pt, for correct limits/constant of integration). Since $\lambda = m/\ell = M/(2L)$: $I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)(8L^3) = \frac{4}{3}ML^2$ (1 pt, for correctly relating λ to M and L). This confirms $I_A = \frac{4}{3}ML^2$.

Solution 2

(b)(i) Mark scheme

- $X_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} = \dfrac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)}{\frac{M}{2} + \frac{M}{2}} = \dfrac{\frac{ML}{4} + \frac{ML}{2}}{M} = \dfrac{3}{4}L$. Final answer: $X_{CM} = \dfrac{3}{4}L$.

Solution 2

(b)(ii) Mark scheme

- $Y_{CM} = \dfrac{\sum m_i y_i}{\sum m_i} = \dfrac{\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)}{\frac{M}{2} + \frac{M}{2}} = \dfrac{\frac{ML}{2} + \frac{ML}{4}}{M} = \dfrac{3}{4}L$. Final answer: $Y_{CM} = \dfrac{3}{4}L$.

Solution 2

(c)

Mark scheme

- Select 'Less than' (1 pt for selection + attempted relevant justification) with a correct justification (1 pt). Example: 'Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.' I.e. $I_B < I_A$.

Solution 2

(d)

Mark scheme

- Sketch two graphs covering the interval from release to the moment the center of mass reaches its lowest point: the angular acceleration α vs. *t* graph must be concave down and begin horizontally (start at a nonzero maximum value with zero slope) (1 pt); the angular speed ω vs. *t* graph must be concave down and end horizontally (rise from zero to a plateau with zero final slope) (1 pt); the two graphs must be consistent with each other — ω increasing exactly while $\alpha > 0$, and both approaching their limiting behavior together as $\alpha \to 0$ near the lowest point (1 pt).

Solution 2

(e)

Mark scheme

- Select 'Decreasing' (1 pt for selection + attempted relevant justification) with a justification that identifies the lever arm for the torque as decreasing (1 pt). Example: 'Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.'

Solution 2

(f) Mark scheme

- Conservation of energy: $U_{g1} = K_2$ (1 pt). Relate the change in rotational KE to the change in gravitational PE: $mgh = \frac{1}{2} I\omega^2$ (1 pt). Substitute $h = L/2$ (the drop in height of the center of mass): $Mg\left(\frac{L}{2}\right) = \frac{1}{2} I_B\omega^2$ (1 pt). Solve for ω in terms of the allowed symbols M, L, I_B: $\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}} = \sqrt{\frac{MgL}{I_B}}$ (1 pt).

  Final answer: $\omega = \sqrt{\frac{MgL}{I_B}}$.

Question 3

2019 Set 1 ▪ Q3

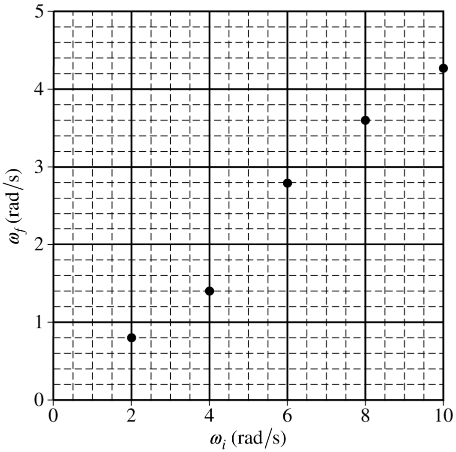
[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

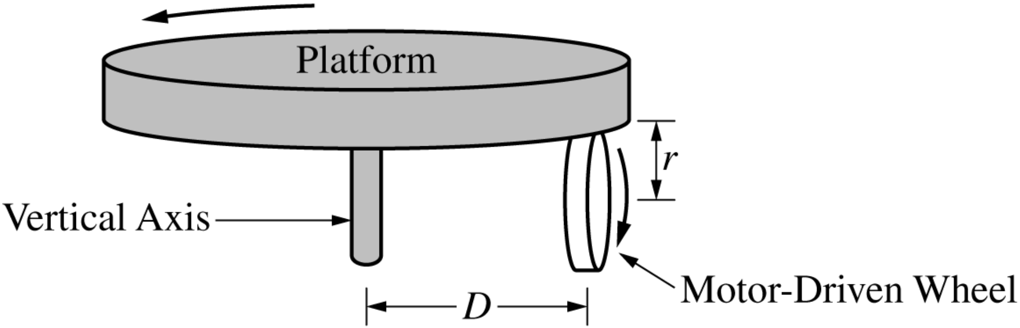
Question 3

(a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

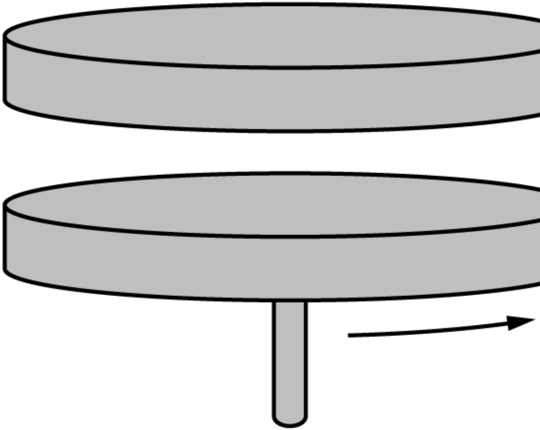
Question 3



Question 3



Question 3



Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(c) Derive an expression for the angular speed ω_W of the wheel when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(d) [Figure: the platform shown alone from a 3D perspective, spinning (arrow indicating rotation direction), with the motor-driven wheel removed.]

When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f. Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(e)(i) A student now uses the rotating platform ($I_P = 3.1$ kg·m^2) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.
[Graph: ω_f (rad/s) vs. ω_i (rad/s); y-axis from 0 to 5 in increments of 1, x-axis from 0 to 10 in increments of 2. Data points roughly follow a straight line through the origin, e.g. near (2, 0.9), (4, 1.4), (6, 2.6), (8, 3.6), (10, 4.2).]
On the graph on the previous page, draw a best-fit line for the data.
(e)(ii) Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(f) The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

______ $K_f < K_i$ ______ $K_f = K_i$ ______ $K_f > K_i$

Justify your answer.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

______ Greater than   ______ Less than   ______ Equal to

Justify your answer.

Solution 3

(a)

Mark scheme

- Substitute into the rotational form of Newton's second law (1 pt): $\tau = I\alpha \Rightarrow FD = I_P\alpha \Rightarrow \alpha = \dfrac{FD}{I_P}$. Substitute into a rotational kinematic equation to find the angular speed (1 pt): $\omega_2 = \omega_1 + \alpha\Delta t = 0 + \dfrac{FD}{I_P}\Delta t$, giving $\omega_P = \dfrac{FD\Delta t}{I_P}$.

Solution 3

(b)

Mark scheme

- Use the rotational kinetic energy equation (1 pt): $K = \frac{1}{2}I\omega_P^2 = \frac{1}{2}(I)\left(\frac{FD\Delta t}{I}\right)^2$. Give an answer consistent with part (a) (1 pt): $K = \dfrac{(FD\Delta t)^2}{2I}$.

Solution 3

(c)

Mark scheme

- Indicate that the linear speed of the platform's edge equals the linear speed of the wheel's edge at the point of contact (1 pt): $v_P = v_W$ or $(r\omega)_P = (r\omega)_W$. Correctly relate the linear speeds to the angular speeds (1 pt):
$$D\omega_P = r\omega_W \Rightarrow \omega_W = \frac{D\omega_P}{r}.$$

Solution 3

(d)

Mark scheme

- Use conservation of angular momentum for the disk landing on the platform (1 pt): $L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2$. Correctly substitute (the disk has the same rotational inertia I_P as the platform, so the combined inertia doubles) (1 pt): $I_P\omega_P = (2I_P)\omega_f \Rightarrow \omega_f = \frac{1}{2}\omega_P$.

Solution 3

(e)(i)

Mark scheme

- Draw a reasonable straight best-fit line through the plotted ω_f vs. ω_i data points on the given graph (1 pt for an appropriate best-fit line).

Solution 3

(e)(ii)

Mark scheme

- Use conservation of angular momentum to derive an expression containing I_U (1 pt): $L_i = L_f \Rightarrow I_P\omega_i = (I_P + I_U)\omega_f \Rightarrow I_U = I_P\left(\frac{\omega_i}{\omega_f}\right) - I_P$. Substitute two points read from the best-fit line (1 pt): $I_U = (3.1\ \mathrm{kg}\cdot\mathrm{m}^2)\left(\frac{4.0 - 1.0\ \mathrm{rad/s}}{9.3 - 2.4\ \mathrm{rad/s}}\right) - (3.1\ \mathrm{kg}\cdot\mathrm{m}^2) = 4.1\ \mathrm{kg}\cdot\mathrm{m}^2$ (the point (0,0) may be used implicitly if the best-fit line passes through the origin).

Solution 3

(f)

Mark scheme

- Select $K_f < K_i$ with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the two disks end up rotating together at the same final angular speed, this is an inelastic collision, so kinetic energy is lost during the collision — therefore $K_f < K_i$.

Solution 3

(g)

Mark scheme

- Select 'Greater than' with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the object's center of mass is off the platform's axis, the parallel-axis theorem $I = I_{CM} + Mh^2$ applies, so the true (actual) rotational inertia about the axis through the center of mass is smaller than what was measured — the experimental value obtained in part (e) is therefore greater than the actual value, increased by Mh^2.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

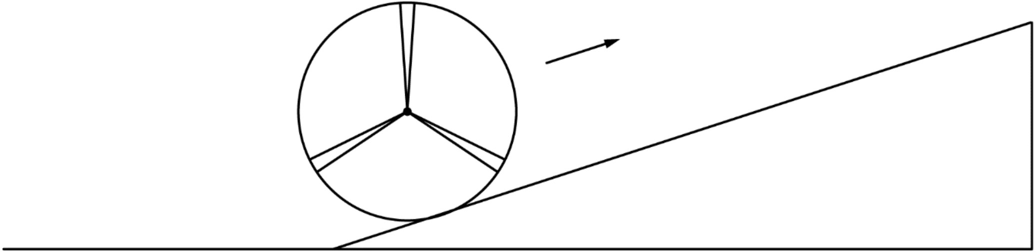
[Figure 1: A plain circle (thin hoop), unlabeled interior.]

[Figure 2: A circle (thin hoop) with three identical rods fastened inside it, each rod running from the center of the hoop out to the rim, spaced so the three rods form a symmetric tripod/Y-like pattern (three line segments from the center to three points on the circle).]

Question 3

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

Question 3



Question 3

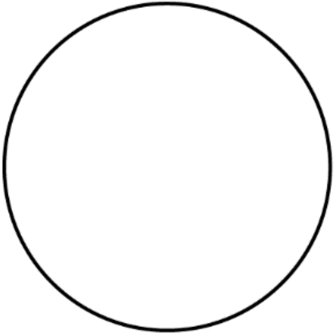


Figure 1

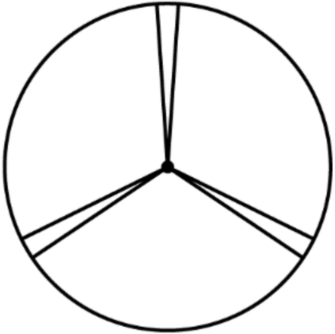
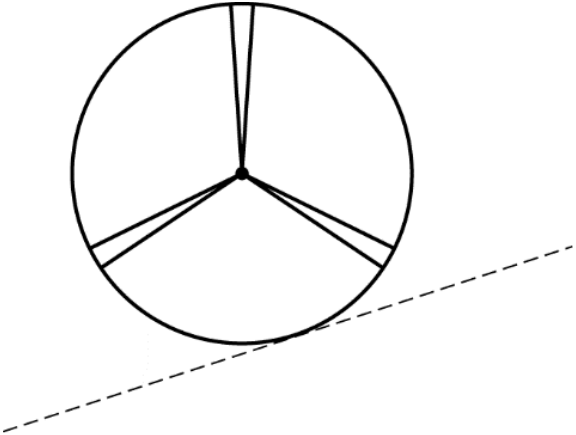


Figure 2

Question 3



Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \dfrac{2M}{L^2}x$, where x is the distance from one end of the rod.

(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate. [Diagram: The hoop-rods system (circle with three internal rods forming a Y-pattern) sitting at the base of an inclined ramp, with a curved arrow above/left of the hoop indicating a force F applied tangent to the hoop, pushing it up the ramp. The ramp rises from left to right, and the hoop sits at the bottom, touching the horizontal surface to the left and the incline surface.]

Question 3

The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(d)(i) On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.

[Diagram: The hoop-rods system (circle with three internal rods in a Y-pattern) resting on a dashed line representing an inclined surface, for the student to draw force vectors on.]

(d)(ii) Justify your choice for the direction of each of the forces drawn in part (d)i.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

Solution 3

(a) Mark scheme

- $I = \int r^2\, dm = \int x^2\, dm$. Since $\lambda = \dfrac{2M}{L^2}x$, $dm = \lambda\, dx = \dfrac{2M}{L^2}x\, dx$. So $I = \displaystyle\int_0^L \frac{2M}{L^2}x^3\, dx = \left[\frac{2M}{L^2}\cdot\frac{x^4}{4}\right]_0^L = \frac{2M}{L^2}\cdot\frac{L^4}{4} = \frac{ML^2}{2}$, as required. 1 point for relating x to r properly in the integral; 1 point for correctly substituting the linear mass density to get dm; 1 point for integrating with correct limits (or constant of integration).

Solution 3

(b) Mark scheme

- The hoop-rods system's inertia about the hoop's center is the sum of the hoop's own inertia (ML^2) and the three rods' inertia about that same center (each rod's inertia about its end, from part (a), is $ML^2/2$):
  $I_{tot} = 3I_{rod} + I_{hoop} = 3\left(\frac{ML^2}{2}\right) + ML^2 = \frac{5}{2}ML^2$. 1 point for setting the total rotational inertia equal to the sum of the hoop's and the three rods' inertias; 1 point for the correct final answer $I_{tot} = \frac{5}{2}ML^2$.

Solution 3

(c) Mark scheme

- Angular impulse-momentum theorem: $\tau\Delta t = I(\omega_2 - \omega_1)$, and starting from rest: $Fr\Delta t = I\omega \;\Rightarrow\; \omega = \dfrac{Fr\Delta t}{I}$. The lever arm equals the hoop's radius L (force applied tangent to the hoop): $\omega = \dfrac{FL\Delta t}{I}$. Substituting $I_{tot} = \frac{5}{2}ML^2$ from part (b): $\omega = \dfrac{FL\Delta t}{\frac{5}{2}ML^2} = \dfrac{2F\Delta t}{5ML}$. 1 point for using an appropriate equation for the final angular speed; 1 point for relating the lever arm to the length L; 1 point for correct substitution of I_{tot} from part (b).

Solution 3

(d)(i) Mark scheme

- Three forces act on the rolling hoop-rods system: the weight F_W, drawn as a distinct arrow starting at the center of the hoop and pointing straight down; the normal force F_N, drawn as an arrow starting at the contact point with the incline and pointing perpendicular to the incline surface (up and to the left); and friction f, drawn as an arrow starting at the contact point and pointing along the incline surface, up the ramp (opposing the tendency to slip as the hoop rolls up). 1 point each for the weight, the normal force, and the friction drawn in the correct direction and at the correct point of application; a maximum of two of the three points can be earned if any extraneous vectors are drawn or if any vector has an incorrect point of exertion.

Solution 3

(d)(ii)

Mark scheme

- The normal force is always perpendicular to the surface, so it is directed up and to the left (away from the incline). The gravitational force is always vertically downward. The friction force is opposite the direction of (relative) rotation/slipping at the contact point, so it is directed up the incline.

Solution 3

(e) Mark scheme

- Energy conservation from the bottom of the ramp to maximum height: $K_i = U_{g2} \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$, so $H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$. Using the rolling-without-slipping condition $v = L\omega$: $H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg} = \dfrac{\frac{1}{2}(mL^2 + I_{tot})\omega^2}{mg}$. The total rolling mass is $m = 4M$ (the hoop of mass M plus the three rods of mass M each): $H = \dfrac{\frac{1}{2}((3M + M)L^2 + I_{tot})\omega^2}{(3M + M)g} = \dfrac{(4ML^2 + I_{tot})\omega^2}{8Mg}$. 1 point for including both translational and rotational kinetic energy in the energy-conservation equation for

Solution 3

the final height; 1 point for correctly substituting $v = L\omega$; 1 point for correctly substituting the total inertias/masses into the energy equation.

Question 3

2017 ▪ Q3

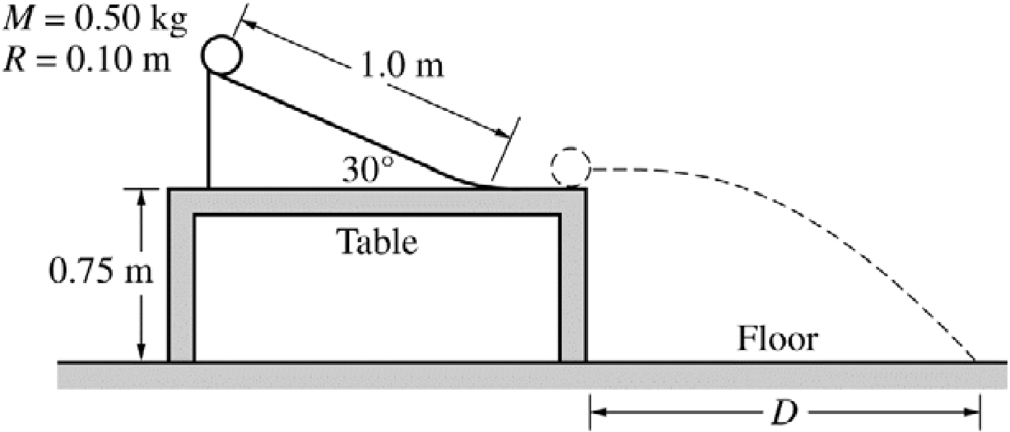
[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition

Question 3

from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.
(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

Question 3



Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

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Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

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Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

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Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the

Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(e)(i) Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

(e)(ii) Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Question 3

(e)(iii) Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Solution 3

(a)

Mark scheme

- Using conservation of energy from the top of the incline to the bottom (table level): $U_{g,top} = K_{table}$, i.e. $mgh = K_{table}$, where $h = L\sin\theta = (1.0\ \mathrm{m})\sin 30^\circ = 0.50\ \mathrm{m}$. So $K_{table} = mgh = (0.50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(1.0\ \mathrm{m})(\sin 30^\circ) = 2.45\ \mathrm{J}$ (or $2.5\ \mathrm{J}$ using $g = 10\ \mathrm{m/s^2}$). 1 point for correctly applying conservation of energy, 1 point for the correct numeric answer with units.

Solution 3

(b) Mark scheme

- Total KE is translational plus rotational: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. For a cylinder, $I = \frac{1}{2}MR^2$ and $v = R\omega$, so $K = \frac{1}{2}M(R\omega)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}M(R\omega)^2 + \frac{1}{4}M(R\omega)^2 = \frac{3}{4}M(R\omega)^2$. Solving for ω: $\omega = \sqrt{\frac{4K}{3MR^2}} = \sqrt{\frac{4(2.45\ \mathrm{J})}{3(0.50\ \mathrm{kg})(0.10\ \mathrm{m})^2}} = 25.6\ \mathrm{rad/s}$ (or $26.0\ \mathrm{rad/s}$ using $g = 10\ \mathrm{m/s^2}$ and $K = 2.5\ \mathrm{J}$). 1 point for correctly writing K as translational plus

Solution 3

rotational KE, 1 point for correctly substituting $v = R\omega$ and $I = \frac{1}{2}MR^2$, 1 point for the correct numeric answer.

Solution 3

(c) Mark scheme

- No torque acts on the cylinder during its fall off the table (only gravity, through the center of mass), so the rotational KE at the floor equals the rotational KE at the table edge: $K_{rot} = \frac{1}{4}M(R\omega)^2$. The total KE at the floor equals the total KE at the table ($K_{table} = \frac{3}{4}M(R\omega)^2$) plus the additional KE gained by falling the table height $h_{table} = 0.75\ \mathrm{m}$: $K_{tot} = \frac{3}{4}M(R\omega)^2 + Mgh_{table}$. So $\frac{K_{rot}}{K_{tot}} = \frac{(R\omega)^2}{3(R\omega)^2 + 4gh_{table}} = \frac{[(0.10\ \mathrm{m})(25.6\ \mathrm{rad/s})]^2}{3[(0.10)(25.6)]^2 + 4(9.81\ \mathrm{m/s^2})(0.75\ \mathrm{m})} = 0.133$ (or 0.135 using $g = 10\ \mathrm{m/s^2}$). [Equivalent alternate approach given in the scoring

Solution 3

guideline: $K_{rot}/U_{total} = \frac{1}{4}(R\omega)^2/[g(h+y)]$, using the total incline-plus-table height drop, gives the same ≈ 0.13.] 1 point for a correct ratio expression, 1 point for the correct substituted numeric answer.

Solution 3

(d) Mark scheme

- Vertical motion (free fall from the table, height $y = 0.75\ \mathrm{m}$): $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(0.75\ \mathrm{m})}{9.81\ \mathrm{m/s^2}}} = 0.39\ \mathrm{s}$. Horizontal motion (constant velocity $v = R\omega$, no horizontal force during the fall): $D = (R\omega)t = (0.10\ \mathrm{m})(25.6\ \mathrm{rad/s})(0.39\ \mathrm{s}) = 1.0\ \mathrm{m}$. 1 point for correctly finding the fall time from vertical kinematics, 1 point for correctly using $D = R\omega t$ to get $D = 1.0\ \mathrm{m}$.

Solution 3

(e)(i)

Mark scheme

- Correct answer: Equal to. By conservation of energy, the total KE at the floor equals the total gravitational PE lost, $Mg(h+y)$. Since the sphere has the same mass and falls through the same total height as the cylinder, its total KE at the floor is the same as the cylinder's. 1 point for selecting 'Equal to' with an attempted relevant justification, 1 point for a fully correct justification.

Solution 3

(e)(ii) Mark scheme

- Correct answer: Less than. Because the sphere's rotational inertia is less than the cylinder's, the sphere rotates faster and, since $v = R\omega$ for rolling without slipping, moves with a greater linear speed. Because the mass is the same and the linear speed is greater, the sphere has greater linear (translational) kinetic energy. Since the total kinetic energies of the sphere and cylinder at the floor are equal (part i), the sphere must therefore have LESS rotational kinetic energy than the cylinder. 1 point for selecting 'Less than' with an attempted relevant justification, 1 point for a fully correct justification.

Solution 3

(e)(iii)

Mark scheme

- Correct answer: Greater than. Since the sphere has a greater linear speed than the cylinder as it leaves the table (from part ii), and both fall through the same height in the same time, the sphere travels a greater horizontal distance $D = vt$ before reaching the floor than the cylinder does. 1 point for selecting 'Greater than' with an attempted relevant justification, 1 point for a fully correct justification.

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.
[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]
(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.
[Diagram: A ceiling mount at the top left holding a horizontal rod via a pivot, with the rod extending to the right and labeled A at its free end. A curved arrow sweeps clockwise from the horizontal rod position down to a vertical dashed line, showing the

Question 3

rod falling from horizontal to vertical. The vertical dashed position is labeled B at its bottom end.]

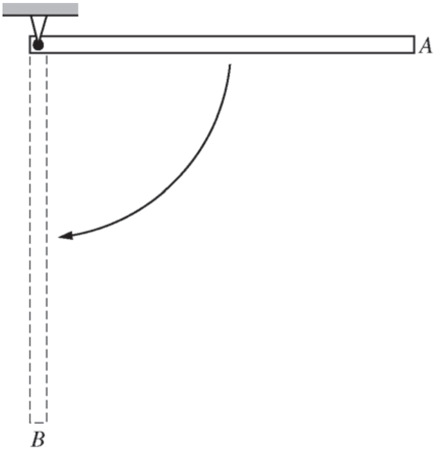
Question 3

Question 3

horizontal position A and uses photogates to measure the velocity of the free end at position B. T
recorded below.

Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3

Question 3



Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(b) The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B.

Derive an expression for the velocity of the free end of the rod at position B. Express your answer in terms of M, L, and physical constants, as appropriate.

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.
[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(c) An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position A and uses photogates to measure the velocity of the free end at position B. The data are recorded below.

Question 3

Length (m)	**0.25**	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3

[Two additional blank rows below the data table, with the same six columns, for the student to fill in derived quantities.]

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

Horizontal axis: ______________________

Question 3

Vertical axis: ____________________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data. [A blank grid divided into a 5×6 array of larger cells, each subdivided into finer dashed gridlines, for plotting data points and drawing a best-fit line. No axes are labeled and no data is plotted.]

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(e)(i) Using your straight line, determine an experimental value for g.

(e)(ii) Describe two ways in which the effects of air resistance could be reduced.