Past-paper walk-through

Newton's Second Law in Rotational Form ▪ 5.6

eXercise

Questions in this set

- 2025 Q4
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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 4

2025 ▪ Q4

A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.
The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.
The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.
[Figure 1: A horizontal surface with a Disk (solid gray circle, radius R labeled) on the left and a Ring (gray annulus/hoop) to its right, both moving to the right (motion lines shown behind each) toward a ramp inclined at angle θ.]

Question 4

[Figure 2: The disk and ring shown together partway up the ramp inclined at angle θ, moving up-slope (motion lines behind them), with the ring positioned slightly behind/around the disk.]

(a) While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_D and f_R, respectively.

Indicate whether f_D is greater than, less than, or equal to f_R by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

Justify your answer using qualitative reasoning beyond referencing equations.

Question 4

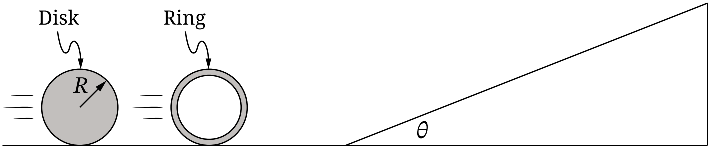
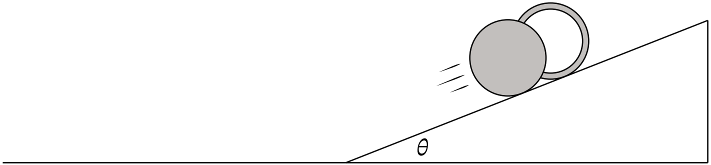


Figure 1



Question 4

2025 ▪ Q4

A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

[Figure 1: A horizontal surface with a Disk (solid gray circle, radius R labeled) on the left and a Ring (gray annulus/hoop) to its right, both moving to the right (motion lines shown behind each) toward a ramp inclined at angle θ.]

Question 4

[Figure 2: The disk and ring shown together partway up the ramp inclined at angle θ, moving up-slope (motion lines behind them), with the ring positioned slightly behind/around the disk.]

(b) A cylinder has mass M, radius R, and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.

Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M, R, I, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 4

2025 ▪ Q4

A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

[Figure 1: A horizontal surface with a Disk (solid gray circle, radius R labeled) on the left and a Ring (gray annulus/hoop) to its right, both moving to the right (motion lines shown behind each) toward a ramp inclined at angle θ.]

Question 4

[Figure 2: The disk and ring shown together partway up the ramp inclined at angle θ, moving up-slope (motion lines behind them), with the ring positioned slightly behind/around the disk.]

(c) In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp. **Indicate** whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping. Briefly **justify** your answer.

Solution 4

(a) Mark scheme

- Compare the magnitudes of static friction f_D (on the disk) and f_R (on the ring) as both roll without slipping on the ramp (same mass, radius, initial conditions). Answer: $f_D < f_R$ [Point A1]. Justification (model, from the scoring guidelines): the ring has a greater rotational inertia because it has more mass distributed toward the edge, so the ring travels farther and has less acceleration down the ramp; because the ring and disk have the same mass, the gravitational force components on them are the same, so from Newton's second law the ring must have less net force down the ramp and more friction (up the ramp) than the disk. This justification compares the disk's and ring's rotational inertias (the ring's is larger, since its mass sits farther from the axis) [Point A2: a justification

Solution 4

comparing translational kinematics, OR rotational kinematics, OR rotational inertias of the disk and ring – any ONE suffices], and it reasons via Newton's second law in translational form (equal gravity component, so the shape with the smaller net accelerating force down the ramp – the ring – must have the larger friction force) [Point A3: justification using N2L in translational form, OR rotational form, OR conservation of energy – any ONE suffices]. Reasoning must go beyond simply citing equations.

Solution 4

(b) Mark scheme

- Cylinder of mass M, radius R, rotational inertia I about its central axis, rolling without slipping UP a ramp inclined at angle θ. Derive an expression for the magnitude of the static friction force f from the ramp, in terms of M, R, I, θ. Rotational form of Newton's second law about the center of mass (friction f is the only force producing a torque about the axis, torque arm R): $\alpha_{sys} = \dfrac{\sum \tau}{I_{sys}} \Rightarrow \dfrac{a}{R} = \dfrac{fR}{I} \Rightarrow a = \dfrac{fR^2}{I}$ (uses the rolling-without-slipping constraint $a = R\alpha$) [Point B3: uses $a = r\alpha$ in an attempt to solve the system of equations]. Translational form of Newton's second law along the incline (the cylinder decelerates as it rolls up, so net force points down the incline):

Solution 4

$\vec{a}_{sys} = \dfrac{\sum \vec{F}}{m_{sys}} \Rightarrow f - Mg\sin\theta = -Ma$ – friction (up the incline) and the gravity component (down the incline) enter with opposite signs [Point B2: opposite signs for the gravitational and frictional force terms]. Combining Newton's second law in BOTH translational and rotational forms is the required multistep derivation [Point B1]. Substituting:
$f - Mg\sin\theta = -M\left(\dfrac{fR^2}{I}\right) \Rightarrow f + M\dfrac{fR^2}{I} = Mg\sin\theta \Rightarrow f\left(1 + \dfrac{MR^2}{I}\right) = Mg\sin\theta$,
giving $f = \dfrac{Mg\sin\theta}{1 + \dfrac{MR^2}{I}}$. (A response using conservation of energy instead may earn full credit per the scoring note.)

Solution 4

(c) Mark scheme

- New scenario: the disk and the ring (same initial center-of-mass speed v as before) roll up a DIFFERENT ramp on which they initially SLIP while rolling. Compare the magnitude of the kinetic friction force on the disk to that on the ring while both are slipping. Answer: 'Equal to' [Point C1]. Justification (model, from the scoring guidelines): the friction here is kinetic (since the objects are slipping), and kinetic friction depends only on the coefficient of kinetic friction between the surfaces in contact and on the normal force ($f_k = \mu_k N$), not on rotational inertia or shape. Since the disk and ring have the same mass and are on the same ramp, their normal forces are equal, and the coefficient of kinetic friction is the same for both (same pair of surfaces), so the kinetic friction forces on the disk and the ring

Solution 4

are equal in magnitude [Point C2: justification includes that kinetic friction depends only on the surfaces/coefficient of kinetic friction and the normal force].

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]
Note: Figure not drawn to scale.
A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal

Question 3

string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(a) On the following representation of the clay-disk system, draw and label the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.

[Figure: A shaded disk viewed face-on, with a dot at the center marking the axle, and point A marked on the upper-right edge of the disk (blank, for the student to draw force arrows on).]

Question 3

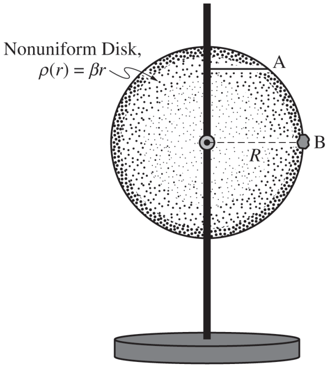


Figure 3

<u>Note</u>: Figure not drawn to scale.

Question 3

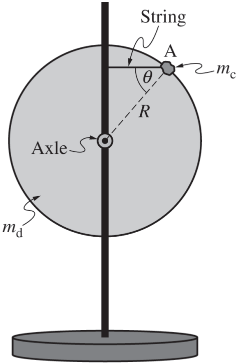


Figure 1

Note: Figure not drawn to scale.

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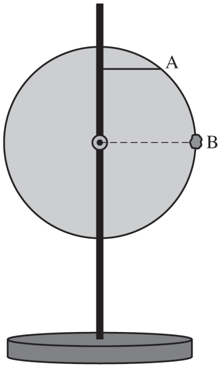

Figure 2

Note: Figure not drawn to scale.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

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A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(b) Derive an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

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2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

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Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(c) [Figure 2: The same disk-axle-pole setup, now with point A marked on the upper edge of the disk and point B marked at the right edge of the disk, horizontally in line with the axle; a dashed horizontal line connects the axle to point B. Note: Figure not drawn to scale.]

The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,new}$ compare with tension F_T from part (b)? Justify your reasoning.

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2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(d)(i) [Figure 3: A nonuniform disk (shown with radial shading/stippling denoting varying density) mounted on the same axle-pole setup, labeled "Nonuniform Disk, $\rho(r) = \beta r$". Point A is marked on the upper edge of the disk, and point B is marked at the right edge of the disk, horizontally in line with the axle (dashed line from axle to B). The disk radius is labeled R. Note: Figure not drawn to scale.]

A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60$ kg and the disk has a radius $R = 0.30$ m. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0\ \text{kg/m}^3$. Calculate the rotational inertia of the disk about the axle.

Question 3

(d)(ii) The string connecting the disk to the pole is cut. Calculate the magnitude of the initial angular acceleration of the clay-disk system.

Solution 3

(a) Mark scheme

- Force diagram on the clay-disk system with three distinct, correctly placed/labeled external forces (each arrow starting on, and pointing away from, its point of application): (1) a downward gravitational force on the disk, $m_d g$ (or a single downward gravitational force on the clay-disk system as a whole), drawn at the disk's center / appropriate location; (2) a downward gravitational force on the lump of clay, $m_c g$, drawn at Point A; (3) a leftward tension force F_T exerted by the string, drawn at Point A; and (4) a force directed up-and-to-the-right exerted by the axle, F_{axle}, drawn at the axle – with no other (extraneous) forces shown.

Solution 3

(b) Mark scheme

- Net torque about the axle is zero (the disk is in rotational equilibrium): $\Sigma\tau = 0$, i.e. $\tau_{clay} - \tau_{string} = 0$, where only the weight of the clay and the string tension exert torque about the axle. Torque from the clay's weight: $\tau_{clay} = Rm_c g\cos\theta$. Torque from the string tension: $\tau_{string} = RF_T\sin\theta$. Setting them equal: $Rm_c g\cos\theta = RF_T\sin\theta \Rightarrow F_T = \dfrac{m_c g\cos\theta}{\sin\theta} = m_c g\cot\theta$. Points: 1 for a multi-step derivation indicating the net torque is zero, 1 for indicating only the clay's weight and the string tension exert torque on the system about the axle, 1 for a correct torque expression from the clay's weight ($\tau_{clay} = Rm_c g\cos\theta$), 1 for a correct torque expression from the string tension ($\tau_{string} = RF_T\sin\theta$). (A maximum of 3

Solution 3

of these 4 points can be earned if sin and cos are consistently reversed in both torque expressions.)

Solution 3

(c)

Mark scheme

- $F_{T,new} > F_T$ (the tension is greater when the clay is at Point B). Justification: there is a direct relationship between the torque exerted by the clay's weight and the string tension (from part (b), $F_T \propto \tau_{clay}$). At Point B (horizontally in line with the axle) the component of the clay's weight perpendicular to the radial direction – equivalently the lever arm for the weight – is larger than at Point A, so the torque from the clay's weight is greater at B. Since the net torque must still be zero, the tension needed to balance it, $F_{T,new}$, must also be greater than F_T.

Solution 3

(d)(i) Mark scheme

- Rotational inertia of the nonuniform disk ($\rho(r) = \beta r$, radius $R = 0.3$ m) found by integration, treating a thin ring of radius r and thickness dr as $dm = \rho(2\pi r)dr = \beta r(2\pi r)dr = 2\pi\beta r^2\, dr$:

$$I = \int r^2\, dm = \int_0^{0.3\text{ m}} r^2 \rho(2\pi r)\, dr = \int_0^{0.3\text{ m}} 2\pi\beta r^4\, dr = \frac{2\pi\beta R^5}{5}.$$

  With $\beta = 4.0\ \text{kg/m}^3$ and $R = 0.3$ m: $I = \dfrac{2\pi(4.0\ \text{kg/m}^3)(0.3\ \text{m})^5}{5} = 0.012\ \text{kg}{\cdot}\text{m}^2$.
  Points: 1 for setting up integration to calculate rotational inertia, 1 for either

Solution 3

substituting $\rho(2\pi r)\,dr$ for dm or indicating the correct integration limits, 1 for the correct final answer $I = 0.012$ kg·m^2, including units.

Solution 3

(d)(ii) Mark scheme

- After the string is cut, apply the rotational form of Newton's second law, $\Sigma\tau = I\alpha$, to the clay-disk system: the only remaining torque about the axle is from the clay's weight, $\tau_{net} = Rm_c g$ (clay still at Point A). The system's rotational inertia is the sum of the disk's and the clay's rotational inertia, $I_{system} = I_{disk} + I_{clay}$, where $I_{clay} = m_c R^2$ (point mass at radius R) and $I_{disk} = 0.012$ kg·m^2 from part (d)(i). So

$$\alpha = \frac{\tau_{net}}{I_{system}} = \frac{Rm_c g}{I_{disk} + I_{clay}} = \frac{(0.3\ \text{m})(0.60\ \text{kg})(9.8\ \text{m/s}^2)}{(0.012\ \text{kg}\cdot\text{m}^2) + (0.60\ \text{kg})(0.3\ \text{m})^2} = 26.7\ \text{rad/s}^2.$$

Solution 3

Points: 1 for using the rotational form of Newton's second law, 1 for a correct expression for the net torque exerted on the clay-disk system, 1 for indicating that the system's rotational inertia is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay.

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

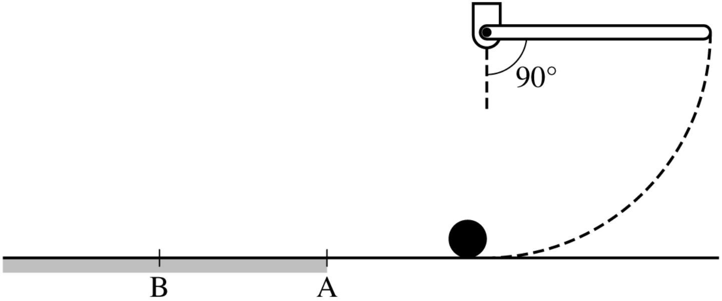
A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total

Question 3

rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

Question 3



Note: Figure not drawn to scale.

Figure 1

Question 3

2023 Set 1 ▪ Q3

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A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate. After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.

[A circle is shown representing the sphere, for the student to draw and label force vectors starting on the circle.]

Question 3

2023 Set 1 ▪ Q3

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(d)(i) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

The linear velocity v of the center of mass of the sphere as a function of time t

(d)(ii) The angular velocity ω of the sphere as a function of time t

Question 3

2023 Set 1 ▪ Q3

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Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(e)(i) Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

(e)(ii) Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

Question 3

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{cm} = \frac{1}{18}ML^2$.

Question 3

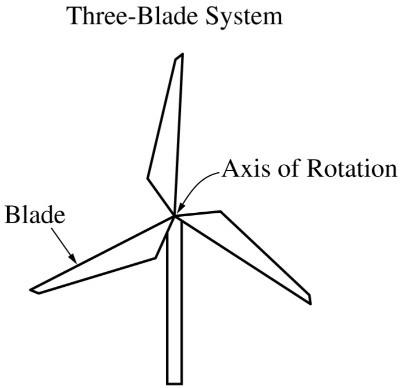


Figure 1

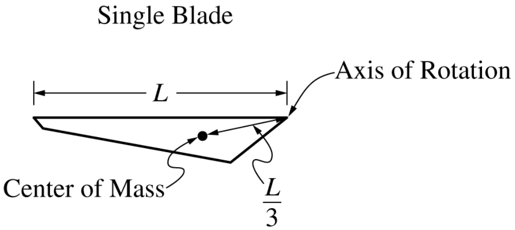


Figure 2

Question 3

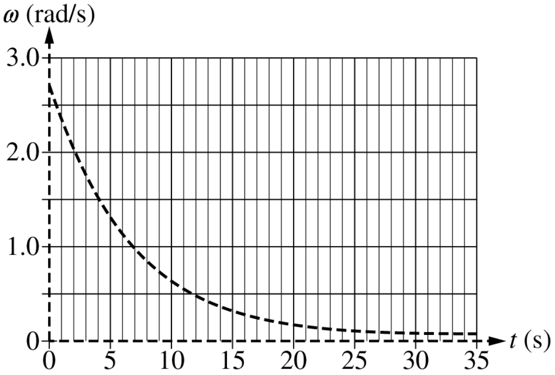


Figure 4

Question 3

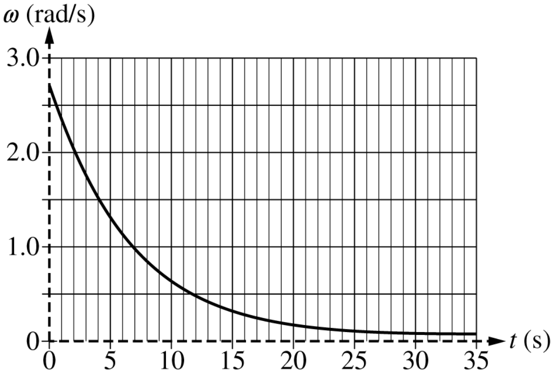


Figure 3

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(c)(i) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

Question 3

[Figure 3: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; curve starts at $(0, 2.6)$ and decays exponentially toward 0 as t increases, with a dashed vertical line at $t = 0$ marking the initial value.]

Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

(c)(ii) Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

(c)(iii) Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the

Question 3

equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.
[Figure 4: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; a dashed curve shows the original $\omega_0 e^{-\beta_0 t}$ decay from $(0, 2.6)$ toward 0.]

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
[Diagram, Figure 2: The same disk-and-stand setup, now rotated clockwise through a small angle θ (marked at the axle). The string on the right side of the disk now goes over the top and down to a hanging "Block" on the right side. The spring on the left side is now shown stretched, labeled "Stretched Spring", connecting down to the ground.]
Note: Figures not drawn to scale.

Question 3

A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_d R^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

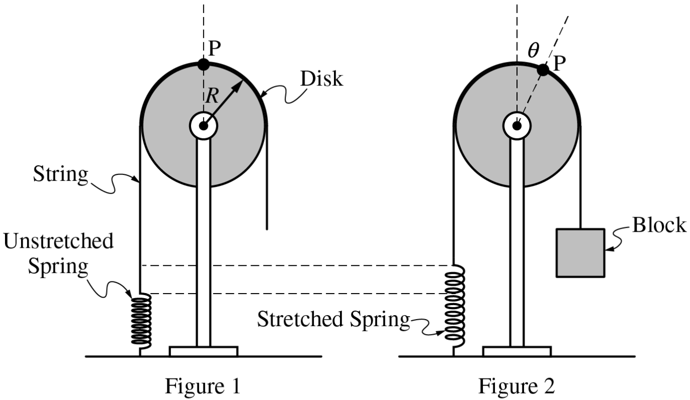
A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

Question 3

(a) Derive an expression for the mass m_B of the block.

Question 3



Note: Figures not drawn to scale.

Question 3

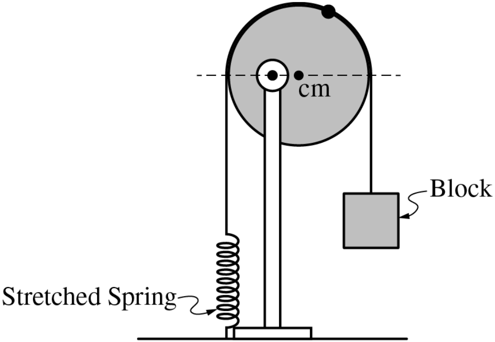
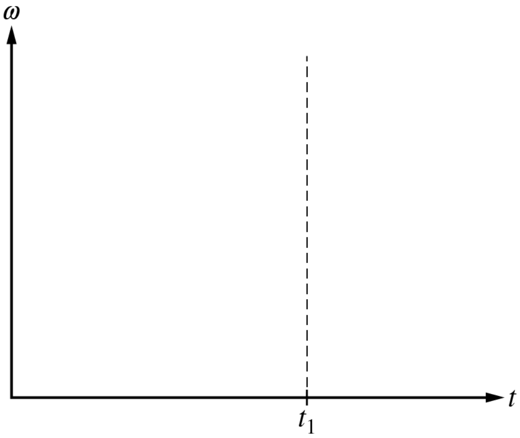


Figure 3

Note: Figure not drawn to scale.

Question 3



Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
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Question 3

string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

Question 3

[Diagram: A circle representing the disk, with a dot at its center marking the axle, and a diagonal dashed line through the center, on which forces should be drawn.]

Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
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Question 3

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A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
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Question 3

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A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.

[Graph: axes labeled "ω" (vertical) vs. "t" (horizontal); a dashed vertical gridline is marked at t_1 on the time axis; blank axes for the student to sketch on.]

Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]

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Note: Figures not drawn to scale.

A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_dR^2$. A

Question 3

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A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(e) [Diagram, Figure 3: The disk-and-stand setup, now mounted so the axle (marked with a dot, off-center within the disk, labeled "axle") does not pass through the center of the disk (marked "com" for center of mass). A string goes over the top of the disk down to a hanging "Block" on the right side. A "Stretched Spring" connects from the left side down to the ground.]

Note: Figure not drawn to scale.

Question 3

The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

Solution 3

(a)

Mark scheme

- Derivation of m_B. Indicate the disk is in rotational (or translational) equilibrium before the string is cut: $\Sigma\tau_{\text{on disk}} = 0 \Rightarrow \tau_g = \tau_s$ (or equivalently $\Sigma F = 0 \Rightarrow F_g = F_s$) (1 point). Substitute the force expressions (both forces act at the same radius R, gravity via the block's weight through the string, spring force $F_s = k\Delta x$): $F_g R = F_s R \Rightarrow m_B g R = k\Delta x R \Rightarrow m_B g = k\Delta x$ (1 point). Substitute $\Delta x = R\theta$ (arc length for the disk's rotation angle θ): $m_B g = kR\theta \Rightarrow m_B = \dfrac{kR\theta}{g}$ (1 point).

Solution 3

(b) Mark scheme

- Free-body diagram of the disk immediately after the string on the right is cut (block falling, only the left string/spring remains attached): draw and label (1) the tension/spring force F_s (or F_{spring}, T_{spring}) tangent to the disk, anywhere from point P to the left edge of the disk inclusive (1 point); (2) the force of gravity F_g located at the center, pointing straight down (1 point); (3) the force from the axle F_{axle} (or F_N, F_n) located at the center, directed so the disk remains in translational equilibrium, i.e. $\Sigma F = 0$ — typically straight up with magnitude balancing F_g and the vertical component of F_s (1 point). A response with extraneous forces/vectors caps at a maximum of 2 of the 3 points.

Solution 3

(c) Mark scheme

- Derivation of angular acceleration α. Indicate the net torque on the disk is due only to the tension/spring force, via the rotational form of Newton's second law: $\tau_s = I_d\alpha$ (1 point). Express that torque as the spring force times its lever arm R: $F_sR = I_d\alpha$ (1 point). Substitute $F_s = -k\Delta x$: $-k\Delta xR = I_d\alpha$ (1 point). Substitute the disk's moment of inertia $I_d = \frac{1}{2}M_dR^2$ and $\Delta x = R\theta$ (consistent with part (a)): $-k(R\theta)R = \frac{1}{2}M_dR^2\alpha \Rightarrow \alpha = -\dfrac{2k\theta}{M_d}$ (1 point). The negative sign is not required to earn this point.

Solution 3

(d)

Mark scheme

- Sketch of ω vs. t from $t = 0$ to $t = t_1$ (point P returns directly above the axle at t_1): the curve must start at zero and increase monotonically over the whole interval (1 point), and be concave down throughout — i.e. increasing but at a decreasing rate (1 point). Only the portion of the graph from 0 to t_1 is scored.

Solution 3

(e) Mark scheme

- Setup: the disk is remounted so its axle no longer passes through its center of mass; a stretched spring pulls down-left and a string over the top holds a hanging block on the right. As the configuration is adjusted (disk rotated / axle offset from center of mass), the response must indicate three torque changes, each earning 1 point: (1) the torque due to gravity ON THE DISK increased, because the disk's weight now acts with a nonzero lever arm about the (now off-center) axle, where previously it had none; (2) the torque due to the tension from the hanging block (transmitted via the string on the right) increased, because that force's lever arm about the axle grew longer; (3) the torque due to the spring's tension (on the left) increased, because it is the only force that can supply the

Solution 3

extra counterclockwise torque needed to counteract the increased clockwise torques from (1) and (2) and keep the disk in equilibrium. Scoring note: a response that also states the torque from the axle force stays the same can still earn all 3 points; a response that says the axle-force torque changes, or invokes additional torques, caps at a maximum of 2 of the 3 points.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.
[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]
[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow away from the board, and should represent the location at which that force acts.
[A blank horizontal rectangle representing the board, with a dashed vertical centerline and dashed horizontal centerline, for the student to draw and label force vectors acting on the board.]

Question 3

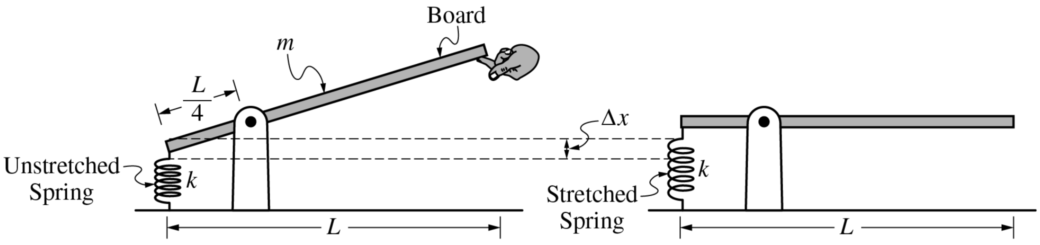
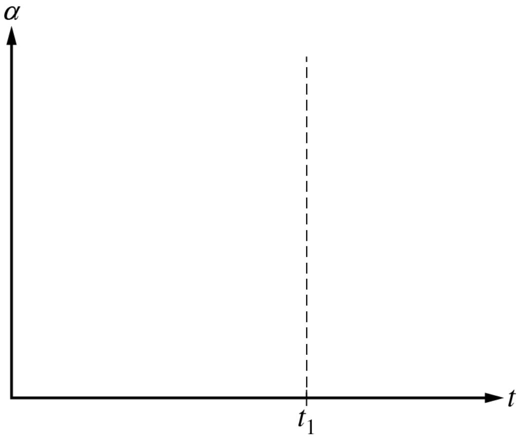


Figure 1

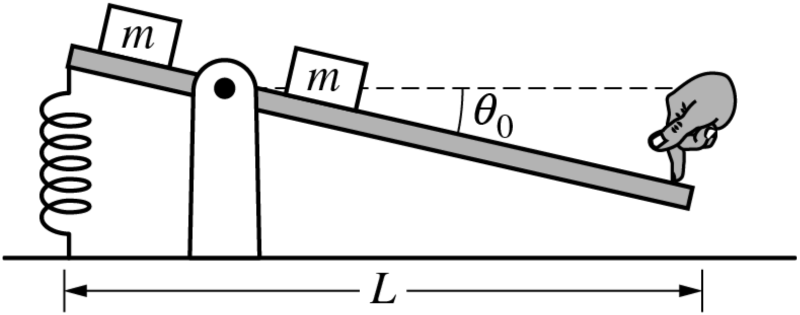
Figure 2

Note: Figures not drawn to scale.

Question 3



Question 3



Note: Figure not drawn to scale.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(c)(i) [Figure: A board pushed down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest. Note: Figure not drawn to scale.]
A student pushes down the board on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.
At time $t = 0$, the board is released. At time t_1, the board first crosses the horizontal.
Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.
[A blank graph with vertical axis α and horizontal axis t, with a marked point t_1 on the time axis, for the student to sketch angular acceleration vs. time.]

Question 3

(c)(ii) Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(d) [Figure: The same board-spring-pivot setup, with two blocks of equal mass *m* attached to the board equal distances from the pivot point, as shown. Note: Figure not drawn to scale.]
Two blocks of equal mass *m* are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. The new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?
$\alpha' > \alpha_0$ $\alpha' < \alpha_0$ $\alpha' = \alpha_0$
Justify your answer.

Solution 3

(a) Mark scheme

- Free-body diagram on the board rectangle, 1 point each: (1) Spring force F_{spring} (or F_S, "spring force") drawn as an arrow pointing downward at the location where the spring attaches (the right end of the board, $L/4$ from the pivot). (2) Force of gravity F_g (or mg, F_G, W, "grav force" — not "G"/"g" alone) drawn pointing downward at the center of the board (its center of mass). (3) Pivot force F_{pivot} (or F_N, F_n, N, "normal force") drawn pointing upward at the pivot location (the fixed support at the board's center). Each force must be correctly located and labeled; extraneous forces cap the score at 2 points.

Solution 3

(b) Mark scheme

- Indicate the sum of torques about the pivot equals zero (equilibrium) — 1 point: $\Sigma\tau = 0$. Substitute mg for the gravitational force and $k\Delta x$ for the spring force into the torque equation, using the given lever arms ($L/4$ each) — 1 point: $\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$. Solve for a correct expression for Δx — 1 point: $\Delta x = \frac{mg}{k}$ (earned regardless of an overall sign).

Solution 3

(c)(i)

Mark scheme

- Sketch of α vs. t from $t = 0$ to $t = t_1$, 1 point each: (1) The curve begins at a maximum (positive) value at $t = 0$ and monotonically decreases to a minimum of zero at $t = t_1$ (when the board reaches horizontal/equilibrium, so the net torque and hence α is zero there). (2) The curve is concave down over the entire interval $0 < t < t_1$ (not a straight line or concave-up decrease). Behavior of the sketch beyond t_1 is not scored.

Solution 3

(c)(ii) Mark scheme

- Write the rotational form of Newton's second law with the spring and gravitational torques in opposite directions — 1 point: $\tau_{spring} - \tau_g = I\alpha$ (i.e. $I_B\alpha_0$). Include $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque (the correct perpendicular lever-arm factor for the small angle θ_0) — 1 point: $F_{spring}d_{spring}(\cos\theta_0) - F_g d_g(\cos\theta_0) = I_B\alpha_0$. Substitute mg for the force of gravity and $k\Delta x_2$ for the spring force — 1 point: $d_{spring}k\Delta x_2(\cos\theta_0) - d_g\, mg(\cos\theta_0) = I_B\alpha_0$. Substitute the correct lever arms $d_{spring} = d_g = L/4$ — 1 point: $\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$, giving $\alpha_0 = \frac{L}{4I_B}\left(k\Delta x_2\cos\theta_0 - mg\cos\theta_0\right)$.

Solution 3

Solution 3

(d) Mark scheme

- Answer: $\alpha' < \alpha_0$ — select this choice with an attempt at a relevant justification (1 point). Justification must indicate the net torque on the system does not change — 1 point (the spring and gravity on the board still exert the same torques at the same lever arms; the two added equal masses sit at equal distances on opposite sides of the pivot, so their gravitational torques cancel and add no net torque). Justification must also indicate the rotational inertia of the system increases — 1 point (the two added point masses increase the total moment of inertia I about the pivot). Example full justification: The net torque on the system is unchanged (the spring and the board's own weight exert the same torques as before, and the additional torques from the two added masses cancel

Solution 3

since they sit at equal distances on opposite sides of the pivot), but the rotational inertia of the system is now larger. Since $\alpha = \tau_{net}/I$, with τ_{net} unchanged and I larger, the angular acceleration is now smaller: $\alpha' < \alpha_0$.

Question 3

2021 Set 1 ▪ Q3

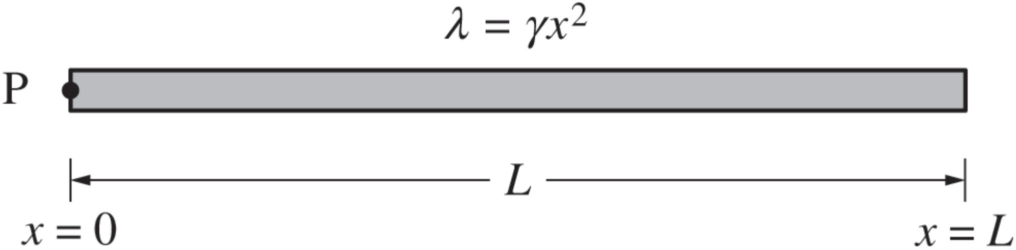
[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

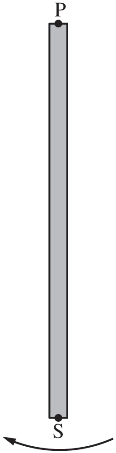
(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

Question 3

Question 3



Question 3



Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_______ Greater than _______ Less than _______ Equal to

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(d) The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.

[Two blank axes: left axis labeled τ (vertical) vs. t (horizontal); right axis labeled ω (vertical) vs. t (horizontal). Both currently blank, to be sketched by the student.]

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_______ Increasing _______ Decreasing _______ Not changing

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(f) [Figure: The rod hangs vertically, pivoted at point P at the top, with point S marked at the bottom end. A curved arrow beneath S indicates the rod swinging.] The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

Solution 3

(a) Mark scheme

- Set up the rotational-inertia integral about point P: $I = \int r^2\, dm$, with the given linear mass density $\lambda = \gamma x^2$ so $dm = \lambda\, dx = \gamma x^2\, dx$ (1 point). Substitute and integrate from $x = 0$ to $x = L$, using $\gamma = 3M/L^3$ (from normalizing the total mass $M = \int_0^L \gamma x^2\, dx = \gamma L^3/3$) (1 point):
$I = \int_0^L x^2(\gamma x^2\, dx) = \gamma \left[\frac{x^5}{5}\right]_0^L = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5}\right) = \frac{3}{5}ML^2.$

Solution 3

(b) Mark scheme

- Set up the center-of-mass integral: $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\int x\,dm}{\int dm}$ (1 point). Substitute $dm = \gamma x^2\,dx$ into the numerator (1 point), and either substitute M directly for the denominator or evaluate $\int dm$ to get the rod's mass (1 point): $X_{CM} = \frac{\int_0^L x(\gamma x^2)\,dx}{M} = \frac{[\gamma x^4/4]_0^L}{M} = \frac{(3M/L^3)(L^4/4)}{M} = \frac{3}{4}L$. The center of mass is located $\frac{3}{4}L$ from point P.

Solution 3

(c)

Mark scheme

- Answer: Greater than (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) more of the rod's mass is concentrated at the end opposite P (farther from the rotation axis), so the rotational inertia about P exceeds that about the center of mass; or (ii) by the parallel-axis theorem $I = I_{cm} + md^2$, an axis displaced from the center of mass ($d > 0$) always gives a larger rotational inertia than the axis through the center of mass.

Solution 3

(d) Mark scheme

- Sketch two graphs vs. time t, from release (rod horizontal) to when the center of mass reaches its lowest point (rod vertical). Torque τ vs. t (1 point): starts at its maximum value at $t = 0$ and decreases along a concave-down curve to zero (torque vanishes as the rod becomes vertical, when the weight acts along the line to the pivot). Angular speed ω vs. t (1 point): starts at 0 and increases along a concave-down curve that flattens, approaching a horizontal asymptote (constant ω) as t increases (since $\alpha = \tau/I \to 0$ as $\tau \to 0$). Consistency (1 point): ω must keep rising as long as $\tau > 0$ and level off exactly as $\tau \to 0$, matching $d\omega/dt = \tau/I$.

Solution 3

(e)

Mark scheme

- Answer: Decreases (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) from $\tau = rF\sin\theta$, as the rod swings downward the angle θ between the weight and the lever arm decreases, so the torque decreases; or (ii) the horizontal lever arm between P and the rod's center of mass continuously shrinks as the rod rotates downward, so the torque decreases. Since I is constant and $\alpha = \tau/I$, angular acceleration decreases correspondingly.

Solution 3

(f) Mark scheme

- Use conservation of energy for the rod swinging from horizontal (at rest) to vertical: $U_i + K_i = U_f + K_f \Rightarrow U_i = K_f$ (1 point). Substitute the gravitational PE (using the center-of-mass drop $h_i = \frac{3}{4}L$ found in part (b)) and the rotational KE about P (1 point): $Mgh_i = \frac{1}{2}I\omega_f^2 \Rightarrow Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\omega_f^2$. Using $I = \frac{3}{5}ML^2$ from part (a) and $\omega_f = v/L$, where v is the linear speed of point S (the free end, distance L from P) (1 point):
  $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \Rightarrow v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}(9.8\ \text{m/s}^2)(1.0\ \text{m})} = 4.9\ \text{m/s}$. (Given $M = 3.0$ kg, $L = 1.0$ m; note M cancels out of the final expression for v.)

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]
Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.
(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.
[Figure: "Object B" — an L-shaped (right-angle) object formed of two rods, shown on an x-y coordinate system with origin O at the bottom-left. A pivot is marked at the top-left corner (labelled "Pivot", on the y-axis). A horizontal rod of length L extends

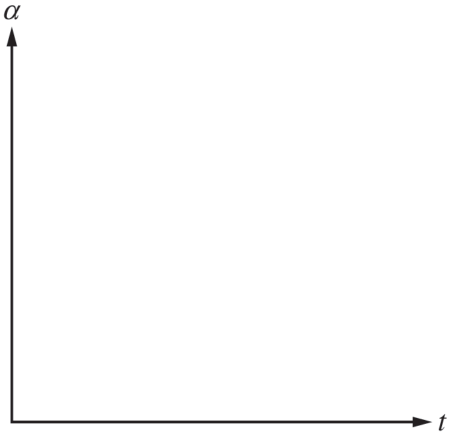
Question 2

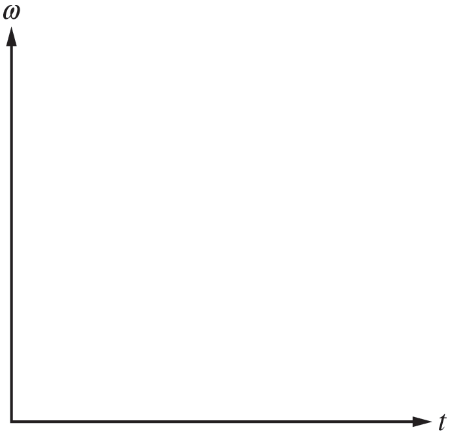
right from the pivot along the top (labelled "L"), and a vertical rod of length L extends down from the pivot to O along the y-axis (labelled "L"). The x-axis points right from O, the y-axis points up from O through the pivot.]

Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above.

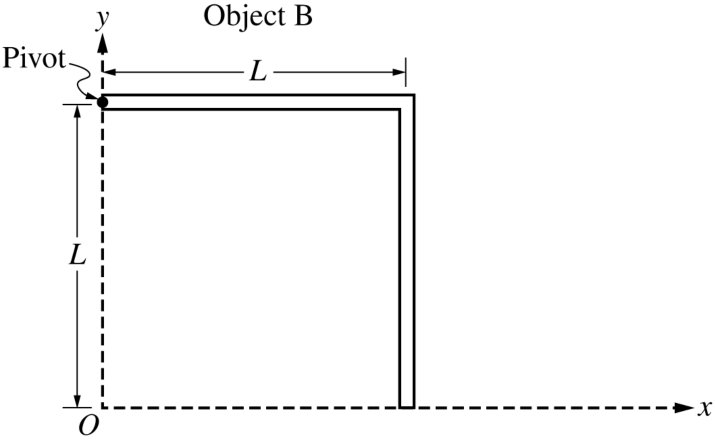
Express your answers in part (b) in terms of L.

Question 2

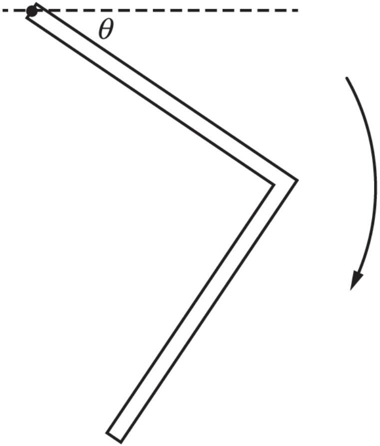




Question 2



Question 2



Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(b)(i) Determine the following for the given coordinate system shown in the figure. The x-coordinate of the center of mass of object B

(b)(ii) The y-coordinate of the center of mass of object B

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(c) Object B has a rotational inertia of I_B about its pivot.

Is the value of I_B greater than, less than, or equal to I_A?

_______Greater than _______Less than _______Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.

[Two blank sets of axes are provided: left axis labelled α (vertical) vs. t (horizontal); right axis labelled ω (vertical) vs. t (horizontal). Both axes are otherwise unmarked (no gridlines or scale), for the student to sketch curves.]

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(e) [Figure: The L-shaped object B is shown rotated partway from its initial horizontal/vertical position, tilted through an angle θ (marked at the top between a dashed horizontal reference line and the now-tilted top rod), with a curved arrow indicating the direction of rotation (downward/clockwise) at the lower end of the shape.]

Question 2

While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

$I^{ncreasing}{}_{D^{ecreasing}{}_{N^{otchanging}}}$

Justify your answer.

[Figure: Object B is shown in a new orientation — the vertical rod of length L now lies along the bottom (horizontal), and the horizontal rod of length L extends straight up from its right end (vertical), forming an upside-down L / right-angle shape resting on the ground.]

Object B rotates through the position shown above.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

Solution 2

(a) Mark scheme

- Using integral calculus with λ the linear mass density of the rod (length $2L$, mass M): $I = \int_{r=0}^{r=2L} \lambda r^2\, dr = \lambda \left[\frac{r^3}{3}\right]_0^{2L} = \frac{\lambda}{3}(2L)^3$ (1 pt, for correct limits/constant of integration). Since $\lambda = m/\ell = M/(2L)$: $I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)(8L^3) = \frac{4}{3}ML^2$ (1 pt, for correctly relating λ to M and L). This confirms $I_A = \frac{4}{3}ML^2$.

Solution 2

(b)(i) Mark scheme

- $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)}{\frac{M}{2} + \frac{M}{2}} = \frac{\frac{ML}{4} + \frac{ML}{2}}{M} = \frac{3}{4}L$. Final answer: $X_{CM} = \frac{3}{4}L$.

Solution 2

(b)(ii) Mark scheme

- $Y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)}{\frac{M}{2} + \frac{M}{2}} = \frac{\frac{ML}{2} + \frac{ML}{4}}{M} = \frac{3}{4}L$. Final answer: $Y_{CM} = \frac{3}{4}L$.

Solution 2

(c)

Mark scheme

- Select 'Less than' (1 pt for selection + attempted relevant justification) with a correct justification (1 pt). Example: 'Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.' I.e. $I_B < I_A$.

Solution 2

(d)

Mark scheme

- Sketch two graphs covering the interval from release to the moment the center of mass reaches its lowest point: the angular acceleration α vs. t graph must be concave down and begin horizontally (start at a nonzero maximum value with zero slope) (1 pt); the angular speed ω vs. t graph must be concave down and end horizontally (rise from zero to a plateau with zero final slope) (1 pt); the two graphs must be consistent with each other — ω increasing exactly while $\alpha > 0$, and both approaching their limiting behavior together as $\alpha \to 0$ near the lowest point (1 pt).

Solution 2

(e)

Mark scheme

- Select ‘Decreasing’ (1 pt for selection + attempted relevant justification) with a justification that identifies the lever arm for the torque as decreasing (1 pt). Example: ‘Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.’

Solution 2

(f) Mark scheme

- Conservation of energy: $U_{g1} = K_2$ (1 pt). Relate the change in rotational KE to the change in gravitational PE: $mgh = \frac{1}{2}I\omega^2$ (1 pt). Substitute $h = L/2$ (the drop in height of the center of mass): $Mg\left(\frac{L}{2}\right) = \frac{1}{2}I_B\omega^2$ (1 pt). Solve for ω in terms of the allowed symbols M, L, I_B: $\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}} = \sqrt{\frac{MgL}{I_B}}$ (1 pt).

  Final answer: $\omega = \sqrt{\frac{MgL}{I_B}}$.

Question 3

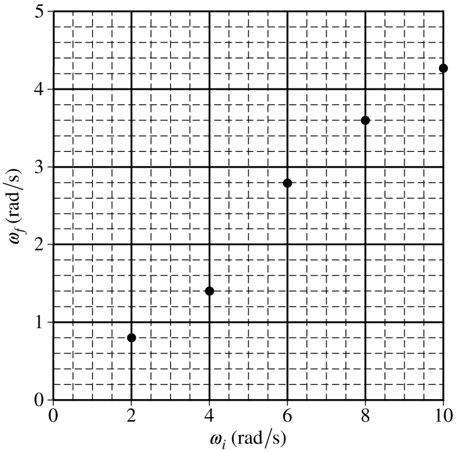
2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]
A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

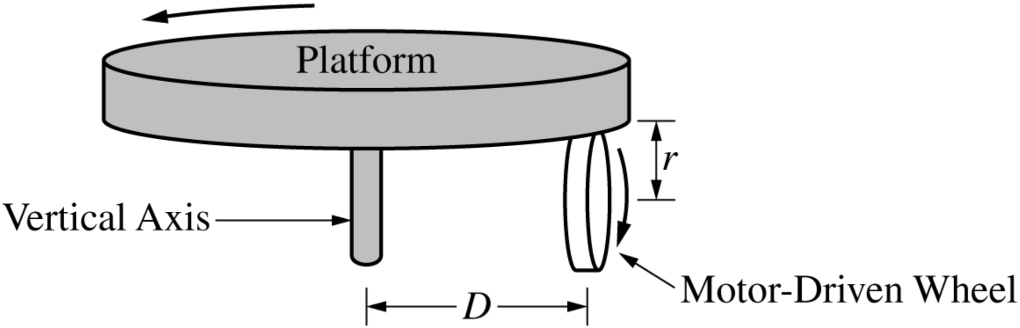
Question 3

(a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

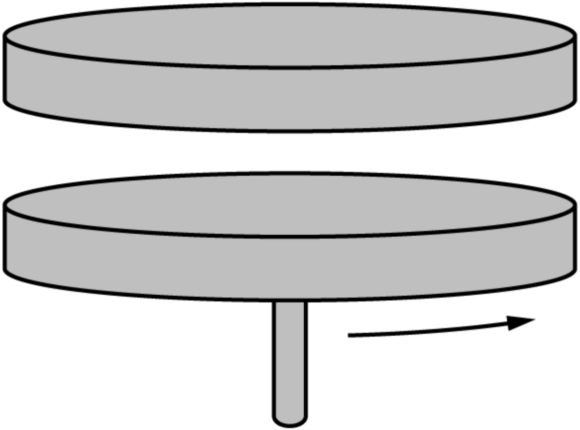
Question 3



Question 3



Question 3



Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(c) Derive an expression for the angular speed ω_W of the wheel when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(d) [Figure: the platform shown alone from a 3D perspective, spinning (arrow indicating rotation direction), with the motor-driven wheel removed.]
When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.
Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(e)(i) A student now uses the rotating platform ($I_P = 3.1$ kg·m^2) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.
[Graph: ω_f (rad/s) vs. ω_i (rad/s); y-axis from 0 to 5 in increments of 1, x-axis from 0 to 10 in increments of 2. Data points roughly follow a straight line through the origin, e.g. near (2, 0.9), (4, 1.4), (6, 2.6), (8, 3.6), (10, 4.2).]
On the graph on the previous page, draw a best-fit line for the data.
(e)(ii) Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(f) The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

______ $K_f < K_i$ ______ $K_f = K_i$ ______ $K_f > K_i$

Justify your answer.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

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Question 3

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

______ Greater than   ______ Less than   ______ Equal to

Justify your answer.

Solution 3

(a)

Mark scheme

- Substitute into the rotational form of Newton's second law (1 pt): $\tau = I\alpha \Rightarrow FD = I_P\alpha \Rightarrow \alpha = \dfrac{FD}{I_P}$. Substitute into a rotational kinematic equation to find the angular speed (1 pt): $\omega_2 = \omega_1 + \alpha\Delta t = 0 + \dfrac{FD}{I_P}\Delta t$, giving $\omega_P = \dfrac{FD\Delta t}{I_P}$.

Solution 3

(b)

Mark scheme

- Use the rotational kinetic energy equation (1 pt): $K = \frac{1}{2} I\omega_P^2 = \frac{1}{2}(I)\left(\frac{FD\Delta t}{I}\right)^2$. Give an answer consistent with part (a) (1 pt): $K = \frac{(FD\Delta t)^2}{2I}$.

Solution 3

(c)

Mark scheme

- Indicate that the linear speed of the platform's edge equals the linear speed of the wheel's edge at the point of contact (1 pt): $v_P = v_W$ or $(r\omega)_P = (r\omega)_W$. Correctly relate the linear speeds to the angular speeds (1 pt): $D\omega_P = r\omega_W \Rightarrow \omega_W = \dfrac{D\omega_P}{r}$.

Solution 3

(d)

Mark scheme

- Use conservation of angular momentum for the disk landing on the platform (1 pt): $L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2$. Correctly substitute (the disk has the same rotational inertia I_P as the platform, so the combined inertia doubles) (1 pt): $I_P\omega_P = (2I_P)\omega_f \Rightarrow \omega_f = \frac{1}{2}\omega_P$.

Solution 3

(e)(i)

Mark scheme

- Draw a reasonable straight best-fit line through the plotted ω_f vs. ω_i data points on the given graph (1 pt for an appropriate best-fit line).

Solution 3

(e)(ii)

Mark scheme

- Use conservation of angular momentum to derive an expression containing I_U (1 pt): $L_i = L_f \Rightarrow I_P\omega_i = (I_P + I_U)\omega_f \Rightarrow I_U = I_P\left(\frac{\omega_i}{\omega_f}\right) - I_P$. Substitute two points read from the best-fit line (1 pt): $I_U = (3.1\ \mathsf{kg} \cdot \mathsf{m}^2)\left(\frac{4.0 - 1.0\ \mathsf{rad/s}}{9.3 - 2.4\ \mathsf{rad/s}}\right) - (3.1\ \mathsf{kg} \cdot \mathsf{m}^2) = 4.1\ \mathsf{kg} \cdot \mathsf{m}^2$ (the point (0,0) may be used implicitly if the best-fit line passes through the origin).

Solution 3

(f)

Mark scheme

- Select $K_f < K_i$ with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the two disks end up rotating together at the same final angular speed, this is an inelastic collision, so kinetic energy is lost during the collision — therefore $K_f < K_i$.

Solution 3

(g)

Mark scheme

- Select 'Greater than' with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the object's center of mass is off the platform's axis, the parallel-axis theorem $I = I_{CM} + Mh^2$ applies, so the true (actual) rotational inertia about the axis through the center of mass is smaller than what was measured — the experimental value obtained in part (e) is therefore greater than the actual value, increased by Mh^2.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \dfrac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

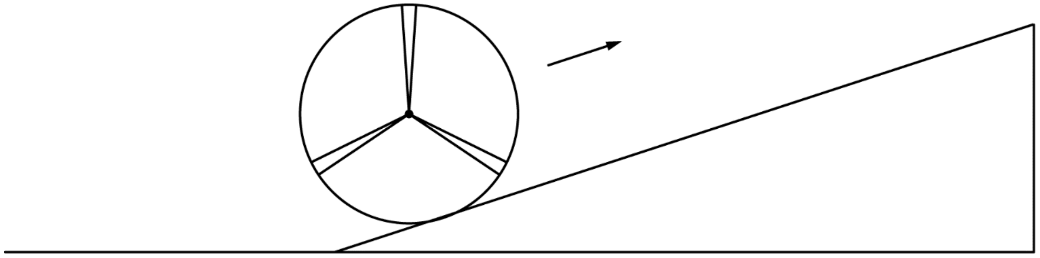
[Figure 1: A plain circle (thin hoop), unlabeled interior.]

[Figure 2: A circle (thin hoop) with three identical rods fastened inside it, each rod running from the center of the hoop out to the rim, spaced so the three rods form a symmetric tripod/Y-like pattern (three line segments from the center to three points on the circle).]

Question 3

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

Question 3



Question 3

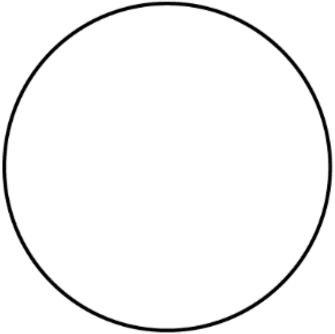


Figure 1

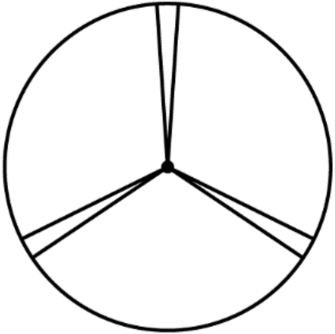
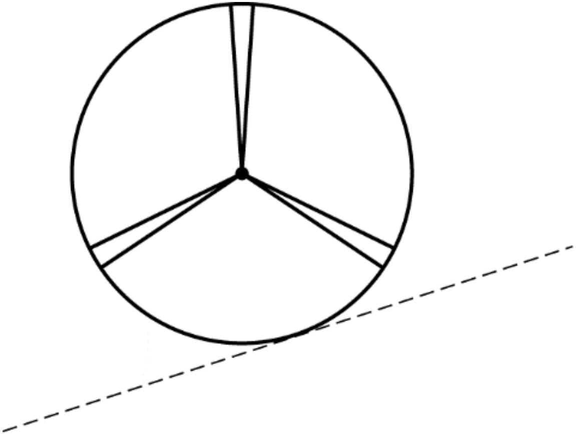


Figure 2

Question 3



Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \dfrac{2M}{L^2}x$, where x is the distance from one end of the rod.

(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate. [Diagram: The hoop-rods system (circle with three internal rods forming a Y-pattern) sitting at the base of an inclined ramp, with a curved arrow above/left of the hoop indicating a force F applied tangent to the hoop, pushing it up the ramp. The ramp rises from left to right, and the hoop sits at the bottom, touching the horizontal surface to the left and the incline surface.]

Question 3

The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(d)(i) On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.

[Diagram: The hoop-rods system (circle with three internal rods in a Y-pattern) resting on a dashed line representing an inclined surface, for the student to draw force vectors on.]

(d)(ii) Justify your choice for the direction of each of the forces drawn in part (d)i.

Question 3

2018 ▪ Q3

A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \dfrac{2M}{L^2}x$, where x is the distance from one end of the rod.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

Solution 3

(a) Mark scheme

- $I = \int r^2\, dm = \int x^2\, dm$. Since $\lambda = \frac{2M}{L^2}x$, $dm = \lambda\, dx = \frac{2M}{L^2}x\, dx$. So $I = \int_0^L \frac{2M}{L^2}x^3\, dx = \left[\frac{2M}{L^2} \cdot \frac{x^4}{4}\right]_0^L = \frac{2M}{L^2} \cdot \frac{L^4}{4} = \frac{ML^2}{2}$, as required. 1 point for relating x to r properly in the integral; 1 point for correctly substituting the linear mass density to get dm; 1 point for integrating with correct limits (or constant of integration).

Solution 3

(b) Mark scheme

- The hoop-rods system's inertia about the hoop's center is the sum of the hoop's own inertia (ML^2) and the three rods' inertia about that same center (each rod's inertia about its end, from part (a), is $ML^2/2$):
  $I_{tot} = 3I_{rod} + I_{hoop} = 3\left(\dfrac{ML^2}{2}\right) + ML^2 = \dfrac{5}{2}ML^2$. 1 point for setting the total rotational inertia equal to the sum of the hoop's and the three rods' inertias; 1 point for the correct final answer $I_{tot} = \frac{5}{2}ML^2$.

Solution 3

(c) Mark scheme

- Angular impulse-momentum theorem: $\tau\Delta t = I(\omega_2 - \omega_1)$, and starting from rest: $Fr\Delta t = I\omega \;\Rightarrow\; \omega = \dfrac{Fr\Delta t}{I}$. The lever arm equals the hoop's radius L (force applied tangent to the hoop): $\omega = \dfrac{FL\Delta t}{I}$. Substituting $I_{tot} = \frac{5}{2}ML^2$ from part (b): $\omega = \dfrac{FL\Delta t}{\frac{5}{2}ML^2} = \dfrac{2F\Delta t}{5ML}$. 1 point for using an appropriate equation for the final angular speed; 1 point for relating the lever arm to the length L; 1 point for correct substitution of I_{tot} from part (b).

Solution 3

(d)(i) Mark scheme

- Three forces act on the rolling hoop-rods system: the weight F_W, drawn as a distinct arrow starting at the center of the hoop and pointing straight down; the normal force F_N, drawn as an arrow starting at the contact point with the incline and pointing perpendicular to the incline surface (up and to the left); and friction f, drawn as an arrow starting at the contact point and pointing along the incline surface, up the ramp (opposing the tendency to slip as the hoop rolls up). 1 point each for the weight, the normal force, and the friction drawn in the correct direction and at the correct point of application; a maximum of two of the three points can be earned if any extraneous vectors are drawn or if any vector has an incorrect point of exertion.

Solution 3

(d)(ii)

Mark scheme

- The normal force is always perpendicular to the surface, so it is directed up and to the left (away from the incline). The gravitational force is always vertically downward. The friction force is opposite the direction of (relative) rotation/slipping at the contact point, so it is directed up the incline.

Solution 3

(e) Mark scheme

- Energy conservation from the bottom of the ramp to maximum height: $K_i = U_{g2} \;\Rightarrow\; \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgH$, so $H = \dfrac{\frac{1}{2}mv^2 + \frac{1}{2}I_{tot}\omega^2}{mg}$. Using the rolling-without-slipping condition $v = L\omega$: $H = \dfrac{\frac{1}{2}m(L\omega)^2 + \frac{1}{2}I_{tot}\omega^2}{mg} = \dfrac{\frac{1}{2}(mL^2 + I_{tot})\omega^2}{mg}$. The total rolling mass is $m = 4M$ (the hoop of mass M plus the three rods of mass M each): $H = \dfrac{\frac{1}{2}((3M + M)L^2 + I_{tot})\omega^2}{(3M + M)g} = \dfrac{(4ML^2 + I_{tot})\omega^2}{8Mg}$. 1 point for including both translational and rotational kinetic energy in the energy-conservation equation for

Solution 3

the final height; 1 point for correctly substituting $v = L\omega$; 1 point for correctly substituting the total inertias/masses into the energy equation.