Past-paper walk-through

Rotational Equilibrium and Newton's First Law in Rotational Form ▪ 5.5

eXercise

Questions in this set

- 2024 Set 1 Q3
- 2024 Set 2 Q3
- 2022 Set 1 Q3
- 2022 Set 2 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

Question 3

(a) [Figure: A schematic representation of the rod alone (no pole, no string, no block), extending diagonally with a pivot circle at the lower-left end. Points marked along the rod: P, C, and Q (in that order from the pivot outward).]
On the following representation of the rod, draw and label the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.

Question 3

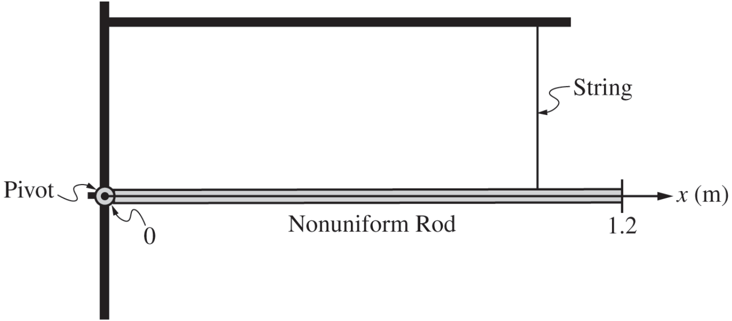

Figure 3

Note: Figure not drawn to scale.

Question 3

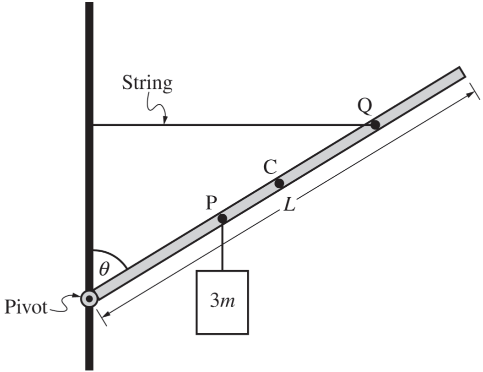

Figure 1

Note: Figure not drawn to scale.

Question 3

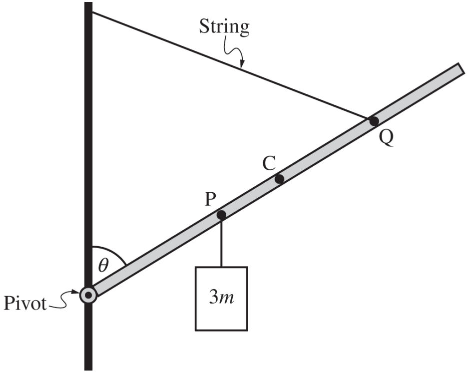

Figure 2

Note: Figure not drawn to scale.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot.

Question 3

Derive an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(c) [Figure 2: Same setup as Figure 1, but the string now connects from Point Q on the rod to a higher point on the vertical pole (the string attachment point is above the

Question 3

top of the pole shown in Figure 1), making a steeper angle with the pole. The rod still makes angle θ with the pole, with Points P, C, Q marked along the rod, and the block of mass $3m$ hanging from Point P. Pivot labeled "Pivot-S" at the base. Note: Figure not drawn to scale.]

The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\ new}$ compare with the original tension F_T from part (b)? Justify your reasoning.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(d)(i) [Figure 3: A nonuniform rod attached horizontally to a pivot on a vertical pole, extending to the right along a horizontal x (m) axis from 0 at the pivot to 1.2 at the

Question 3

far end labeled "Nonuniform Rod". A string labeled "String" connects from a point partway along the rod up to the top of the vertical pole, forming a right-triangle-like support. Pivot labeled "Pivot-S" at the base of the pole. Note: Figure not drawn to scale.]

A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

Calculate the mass of the rod.

(d)(ii) Calculate the rotational inertia of the rod about the pivot.

Solution 3

(a) Mark scheme

- Draw and label: a downward force $F_{T,block}$ (or $3mg$) at point P (tension from the string holding the hanging block); a downward gravitational force mg at point C (the rod's own weight, acting at its center of mass); a leftward force $F_{T,string}$ at point Q (tension from the string to the pole); and a force directed up and to the right at the pivot, F_{pivot} (the pivot's reaction force), with no other (extraneous) forces drawn. Points: (1) correctly drawn/labeled downward forces at P and C; (2) correctly drawn/labeled leftward force at Q; (3) correctly drawn/labeled up-and-right force at the pivot, with no extraneous forces.

Solution 3

(b) Mark scheme

- Balance torques about the pivot ($\Sigma\tau = 0$), with the hanging mass $3m$ at P ($\frac{3L}{8}$ from pivot), the rod's weight mg at C ($\frac{4L}{8} = \frac{L}{2}$ from pivot), and the string tension F_T at Q ($\frac{6L}{8}$ from pivot, acting with perpendicular-component $F_T \cos\theta$ since the string is horizontal and the rod makes angle θ with the vertical pole): $3mg \sin\theta \left(\frac{3L}{8}\right) + mg \sin\theta \left(\frac{4L}{8}\right) - F_T \cos\theta \left(\frac{6L}{8}\right) = 0$, so $\left(\frac{9}{8}\right) mg \sin\theta + \left(\frac{4}{8}\right) mg \sin\theta = \left(\frac{6}{8}\right) F_T \cos\theta$, giving $13mg \sin\theta = 6F_T \cos\theta \Rightarrow F_T = \dfrac{13}{6} mg \tan\theta$. (Scoring note: a maximum of 3 of the 4 points may be earned if sin and cos are reversed throughout for all three torque terms.) Points: (1) net torque on rod equals zero; (2) correct torque

Solution 3

expression for the hanging mass; (3) correct torque expression for the rod's own gravitational force; (4) correct torque expression for the string tension.

Solution 3

(c) Mark scheme

- Because the torque that the string must supply on the rod is always the same (it must still balance the unchanged torques from the rod's weight and the hanging mass), moving the string's attachment point higher on the pole increases the angle between the string and the rod, which increases the string's perpendicular moment arm. To produce the same torque with a larger moment arm, the tension $F_{T,new}$ must be smaller. So as the angle between the string and rod increases, the force exerted on the rod by the string decreases. Points: (1) indicating the torque exerted on the rod by the string is always the same; (2) stating that as the angle between the string and rod increases, the tension force decreases.

Solution 3

(d)(i) Mark scheme

- The nonuniform rod has linear mass density $\lambda(x) = A + Bx$ (with $A = 6.0$ kg/m, $B = 10.0$ kg/m^2) over its 1.2 m length. Total mass: $M = \int dm = \int_0^{1.2} \lambda\, dx = \int_0^{1.2} (6 + 10x)\, dx = \left(6x + \frac{10x^2}{2}\right)\Big|_0^{1.2} = 6(1.2) + 5(1.2)^2 = 7.2 + 7.2 = 14.4$ kg. Points: (1) indicating the total mass is the sum (integral) of differential masses along the rod, $M = \int dm$; (2) correctly writing dm in terms of x (i.e. $dm = \lambda\, dx = (6 + 10x)\, dx$); (3) a correct numeric answer with correct units, $M = 14.4$ kg.

Solution 3

(d)(ii) Mark scheme

- Rotational inertia about the pivot: $I = \int r^2\, dm$ with $dm = \lambda\, dx$ and $r = x$: $I = \int_0^{1.2} (A + Bx)x^2\, dx = \int_0^{1.2} (6 + 10x)x^2\, dx = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Big|_0^{1.2} = \frac{6(1.2)^3}{3} + \frac{10(1.2)^4}{4} = 3.456 + 5.184 = 8.64$ kg $\cdot$ m^2. Points: (1) correct substitution of λ into an integral expression for rotational inertia; (2) correct integration; (3) a correct numeric answer with correct units, $I = 8.64$ kg $\cdot$ m^2.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]
Note: Figure not drawn to scale.
A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal

Question 3

string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(a) On the following representation of the clay-disk system, draw and label the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.

[Figure: A shaded disk viewed face-on, with a dot at the center marking the axle, and point A marked on the upper-right edge of the disk (blank, for the student to draw force arrows on).]

Question 3

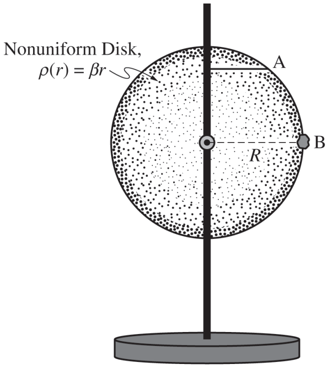

Figure 3

<u>Note</u>: Figure not drawn to scale.

Question 3

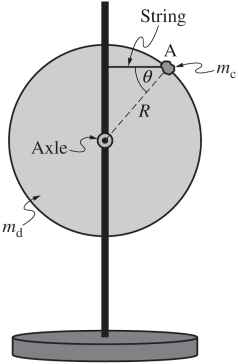

Figure 1

Note: Figure not drawn to scale.

Question 3

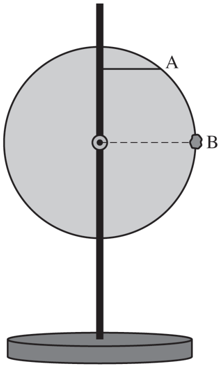

Figure 2

Note: Figure not drawn to scale.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(b) Derive an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

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A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(c) [Figure 2: The same disk-axle-pole setup, now with point A marked on the upper edge of the disk and point B marked at the right edge of the disk, horizontally in line with the axle; a dashed horizontal line connects the axle to point B. Note: Figure not drawn to scale.]

The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,new}$ compare with tension F_T from part (b)? Justify your reasoning.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(d)(i) [Figure 3: A nonuniform disk (shown with radial shading/stippling denoting varying density) mounted on the same axle-pole setup, labeled "Nonuniform Disk, $\rho(r) = \beta r$". Point A is marked on the upper edge of the disk, and point B is marked at the right edge of the disk, horizontally in line with the axle (dashed line from axle to B). The disk radius is labeled R. Note: Figure not drawn to scale.]

A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60$ kg and the disk has a radius $R = 0.30$ m. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0$ kg/m^3. Calculate the rotational inertia of the disk about the axle.

Question 3

(d)(ii) The string connecting the disk to the pole is cut. Calculate the magnitude of the initial angular acceleration of the clay-disk system.

Solution 3

(a) Mark scheme

- Force diagram on the clay-disk system with three distinct, correctly placed/labeled external forces (each arrow starting on, and pointing away from, its point of application): (1) a downward gravitational force on the disk, $m_d g$ (or a single downward gravitational force on the clay-disk system as a whole), drawn at the disk's center / appropriate location; (2) a downward gravitational force on the lump of clay, $m_c g$, drawn at Point A; (3) a leftward tension force F_T exerted by the string, drawn at Point A; and (4) a force directed up-and-to-the-right exerted by the axle, F_{axle}, drawn at the axle – with no other (extraneous) forces shown.

Solution 3

(b) Mark scheme

- Net torque about the axle is zero (the disk is in rotational equilibrium): $\Sigma\tau = 0$, i.e. $\tau_{clay} - \tau_{string} = 0$, where only the weight of the clay and the string tension exert torque about the axle. Torque from the clay's weight: $\tau_{clay} = Rm_c g\cos\theta$. Torque from the string tension: $\tau_{string} = RF_T\sin\theta$. Setting them equal: $Rm_c g\cos\theta = RF_T\sin\theta \Rightarrow F_T = \dfrac{m_c g\cos\theta}{\sin\theta} = m_c g\cot\theta$. Points: 1 for a multi-step derivation indicating the net torque is zero, 1 for indicating only the clay's weight and the string tension exert torque on the system about the axle, 1 for a correct torque expression from the clay's weight ($\tau_{clay} = Rm_c g\cos\theta$), 1 for a correct torque expression from the string tension ($\tau_{string} = RF_T\sin\theta$). (A maximum of 3

Solution 3

of these 4 points can be earned if sin and cos are consistently reversed in both torque expressions.)

Solution 3

(c)

Mark scheme

- $F_{T,new} > F_T$ (the tension is greater when the clay is at Point B). Justification: there is a direct relationship between the torque exerted by the clay's weight and the string tension (from part (b), $F_T \propto \tau_{clay}$). At Point B (horizontally in line with the axle) the component of the clay's weight perpendicular to the radial direction – equivalently the lever arm for the weight – is larger than at Point A, so the torque from the clay's weight is greater at B. Since the net torque must still be zero, the tension needed to balance it, $F_{T,new}$, must also be greater than F_T.

Solution 3

(d)(i) Mark scheme

- Rotational inertia of the nonuniform disk ($\rho(r) = \beta r$, radius $R = 0.3$ m) found by integration, treating a thin ring of radius r and thickness dr as $dm = \rho(2\pi r)dr = \beta r(2\pi r)dr = 2\pi\beta r^2\,dr$:

$$I = \int r^2\,dm = \int_0^{0.3\text{ m}} r^2\rho(2\pi r)\,dr = \int_0^{0.3\text{ m}} 2\pi\beta r^4\,dr = \frac{2\pi\beta R^5}{5}.$$

With $\beta = 4.0$ kg/m^3 and $R = 0.3$ m: $I = \dfrac{2\pi(4.0\text{ kg/m}^3)(0.3\text{ m})^5}{5} = 0.012$ kg$\cdot$m^2.
Points: 1 for setting up integration to calculate rotational inertia, 1 for either

Solution 3

substituting $\rho(2\pi r)\, dr$ for dm or indicating the correct integration limits, 1 for the correct final answer $I = 0.012$ kg·m^2, including units.

Solution 3

(d)(ii) Mark scheme

- After the string is cut, apply the rotational form of Newton's second law, $\Sigma\tau = I\alpha$, to the clay-disk system: the only remaining torque about the axle is from the clay's weight, $\tau_{net} = Rm_cg$ (clay still at Point A). The system's rotational inertia is the sum of the disk's and the clay's rotational inertia, $I_{system} = I_{disk} + I_{clay}$, where $I_{clay} = m_cR^2$ (point mass at radius R) and $I_{disk} = 0.012$ kg·m^2 from part (d)(i). So

$$\alpha = \frac{\tau_{net}}{I_{system}} = \frac{Rm_cg}{I_{disk} + I_{clay}} = \frac{(0.3\text{ m})(0.60\text{ kg})(9.8\text{ m/s}^2)}{(0.012\text{ kg}\cdot\text{m}^2) + (0.60\text{ kg})(0.3\text{ m})^2} = 26.7\text{ rad/s}^2.$$

Solution 3

Points: 1 for using the rotational form of Newton's second law, 1 for a correct expression for the net torque exerted on the clay-disk system, 1 for indicating that the system's rotational inertia is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay.

Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
[Diagram, Figure 2: The same disk-and-stand setup, now rotated clockwise through a small angle θ (marked at the axle). The string on the right side of the disk now goes over the top and down to a hanging "Block" on the right side. The spring on the left side is now shown stretched, labeled "Stretched Spring", connecting down to the ground.]
Note: Figures not drawn to scale.

Question 3

A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_dR^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.
A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.
Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

Question 3

(a) Derive an expression for the mass m_B of the block.

Question 3

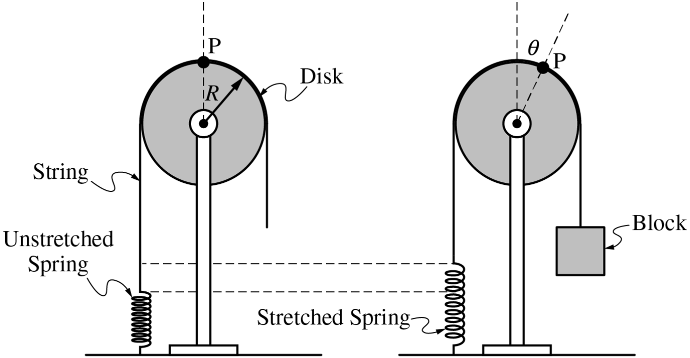


Figure 1

Figure 2

<u>Note</u>: Figures not drawn to scale.

Question 3

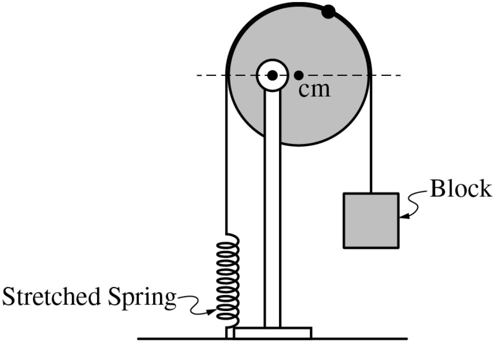
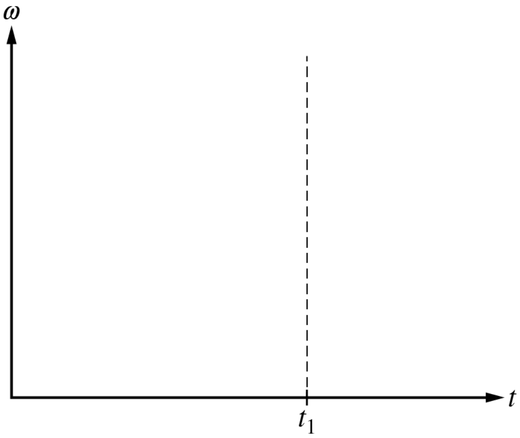


Figure 3

Note: Figure not drawn to scale.

Question 3



Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
[Diagram, Figure 2: The same disk-and-stand setup, now rotated clockwise through a small angle θ (marked at the axle). The string on the right side of the disk now goes over the top and down to a hanging "Block" on the right side. The spring on the left side is now shown stretched, labeled "Stretched Spring", connecting down to the ground.]
Note: Figures not drawn to scale.
A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \dfrac{1}{2}M_dR^2$. A

Question 3

string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

Question 3

[Diagram: A circle representing the disk, with a dot at its center marking the axle, and a diagonal dashed line through the center, on which forces should be drawn.]

Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
[Diagram, Figure 2: The same disk-and-stand setup, now rotated clockwise through a small angle θ (marked at the axle). The string on the right side of the disk now goes over the top and down to a hanging "Block" on the right side. The spring on the left side is now shown stretched, labeled "Stretched Spring", connecting down to the ground.]
Note: Figures not drawn to scale.
A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_dR^2$. A

Question 3

string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.

A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

Question 3

2022 Set 1 ▪ Q3

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Question 3

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Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.

[Graph: axes labeled "ω" (vertical) vs. "t" (horizontal); a dashed vertical gridline is marked at t_1 on the time axis; blank axes for the student to sketch on.]

Question 3

2022 Set 1 ▪ Q3

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Question 3

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A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(e) [Diagram, Figure 3: The disk-and-stand setup, now mounted so the axle (marked with a dot, off-center within the disk, labeled "axle") does not pass through the center of the disk (marked "com" for center of mass). A string goes over the top of the disk down to a hanging "Block" on the right side. A "Stretched Spring" connects from the left side down to the ground.]

Note: Figure not drawn to scale.

Question 3

The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

Solution 3

(a)

Mark scheme

- Derivation of m_B. Indicate the disk is in rotational (or translational) equilibrium before the string is cut: $\Sigma\tau_{\text{on disk}} = 0 \Rightarrow \tau_g = \tau_s$ (or equivalently $\Sigma F = 0 \Rightarrow F_g = F_s$) (1 point). Substitute the force expressions (both forces act at the same radius R, gravity via the block's weight through the string, spring force $F_s = k\Delta x$): $F_g R = F_s R \Rightarrow m_B g R = k\Delta x R \Rightarrow m_B g = k\Delta x$ (1 point). Substitute $\Delta x = R\theta$ (arc length for the disk's rotation angle θ): $m_B g = kR\theta \Rightarrow m_B = \dfrac{kR\theta}{g}$ (1 point).

Solution 3

(b) Mark scheme

- Free-body diagram of the disk immediately after the string on the right is cut (block falling, only the left string/spring remains attached): draw and label (1) the tension/spring force F_s (or F_{spring}, T_{spring}) tangent to the disk, anywhere from point P to the left edge of the disk inclusive (1 point); (2) the force of gravity F_g located at the center, pointing straight down (1 point); (3) the force from the axle F_{axle} (or F_N, F_n) located at the center, directed so the disk remains in translational equilibrium, i.e. $\Sigma F = 0$ — typically straight up with magnitude balancing F_g and the vertical component of F_s (1 point). A response with extraneous forces/vectors caps at a maximum of 2 of the 3 points.

Solution 3

(c) Mark scheme

- Derivation of angular acceleration α. Indicate the net torque on the disk is due only to the tension/spring force, via the rotational form of Newton's second law: $\tau_s = I_d\alpha$ (1 point). Express that torque as the spring force times its lever arm R: $F_sR = I_d\alpha$ (1 point). Substitute $F_s = -k\Delta x$: $-k\Delta xR = I_d\alpha$ (1 point). Substitute the disk's moment of inertia $I_d = \frac{1}{2}M_dR^2$ and $\Delta x = R\theta$ (consistent with part (a)): $-k(R\theta)R = \frac{1}{2}M_dR^2\alpha \Rightarrow \alpha = -\dfrac{2k\theta}{M_d}$ (1 point). The negative sign is not required to earn this point.

Solution 3

(d)

Mark scheme

- Sketch of ω vs. t from $t = 0$ to $t = t_1$ (point P returns directly above the axle at t_1): the curve must start at zero and increase monotonically over the whole interval (1 point), and be concave down throughout — i.e. increasing but at a decreasing rate (1 point). Only the portion of the graph from 0 to t_1 is scored.

Solution 3

(e) Mark scheme

- Setup: the disk is remounted so its axle no longer passes through its center of mass; a stretched spring pulls down-left and a string over the top holds a hanging block on the right. As the configuration is adjusted (disk rotated / axle offset from center of mass), the response must indicate three torque changes, each earning 1 point: (1) the torque due to gravity ON THE DISK increased, because the disk's weight now acts with a nonzero lever arm about the (now off-center) axle, where previously it had none; (2) the torque due to the tension from the hanging block (transmitted via the string on the right) increased, because that force's lever arm about the axle grew longer; (3) the torque due to the spring's tension (on the left) increased, because it is the only force that can supply the

Solution 3

extra counterclockwise torque needed to counteract the increased clockwise torques from (1) and (2) and keep the disk in equilibrium. Scoring note: a response that also states the torque from the axle force stays the same can still earn all 3 points; a response that says the axle-force torque changes, or invokes additional torques, caps at a maximum of 2 of the 3 points.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.
[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]
[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow away from the board, and should represent the location at which that force acts.
[A blank horizontal rectangle representing the board, with a dashed vertical centerline and dashed horizontal centerline, for the student to draw and label force vectors acting on the board.]

Question 3

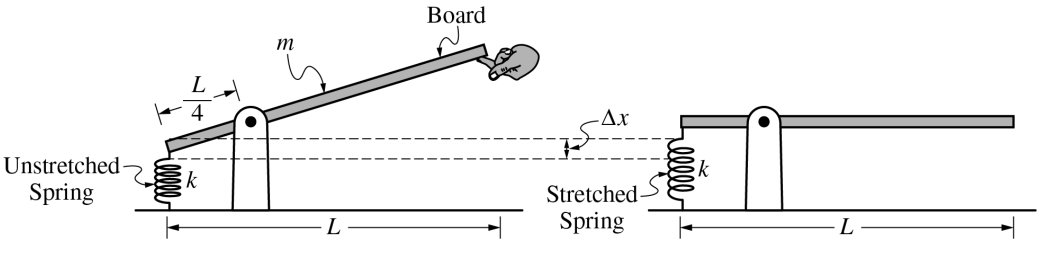
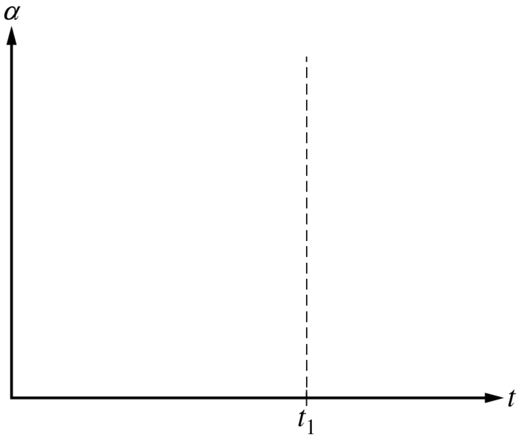


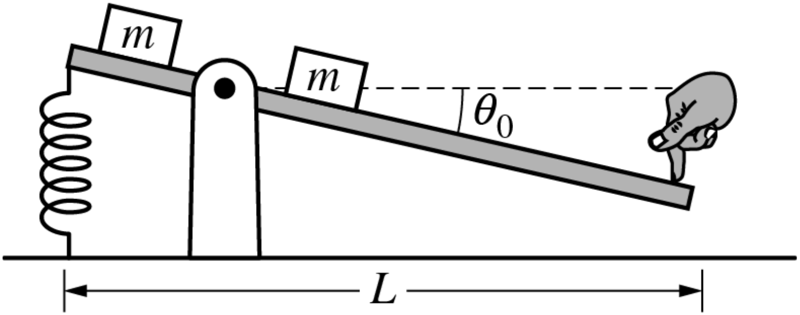
Figure 1 Figure 2

Note: Figures not drawn to scale.

Question 3



Question 3



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Question 3

2022 Set 2 ▪ Q3

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[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

Question 3

2022 Set 2 ▪ Q3

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[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(c)(i) [Figure: A board pushed down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest. Note: Figure not drawn to scale.]
A student pushes down the board on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.
At time $t = 0$, the board is released. At time t_1, the board first crosses the horizontal.
Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.
[A blank graph with vertical axis α and horizontal axis t, with a marked point t_1 on the time axis, for the student to sketch angular acceleration vs. time.]

Question 3

(c)(ii) Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(d) [Figure: The same board-spring-pivot setup, with two blocks of equal mass *m* attached to the board equal distances from the pivot point, as shown. Note: Figure not drawn to scale.]

Two blocks of equal mass *m* are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. The new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

$\alpha' > \alpha_0$ $\alpha' < \alpha_0$ $\alpha' = \alpha_0$

Justify your answer.

Solution 3

(a) Mark scheme

- Free-body diagram on the board rectangle, 1 point each: (1) Spring force F_{spring} (or F_S, "spring force") drawn as an arrow pointing downward at the location where the spring attaches (the right end of the board, $L/4$ from the pivot). (2) Force of gravity F_g (or mg, F_G, W, "grav force" — not "G"/"g" alone) drawn pointing downward at the center of the board (its center of mass). (3) Pivot force F_{pivot} (or F_N, F_n, N, "normal force") drawn pointing upward at the pivot location (the fixed support at the board's center). Each force must be correctly located and labeled; extraneous forces cap the score at 2 points.

Solution 3

(b) Mark scheme

- Indicate the sum of torques about the pivot equals zero (equilibrium) — 1 point: $\Sigma\tau = 0$. Substitute *mg* for the gravitational force and $k\Delta x$ for the spring force into the torque equation, using the given lever arms ($L/4$ each) — 1 point: $\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$. Solve for a correct expression for Δx — 1 point: $\Delta x = \frac{mg}{k}$ (earned regardless of an overall sign).

Solution 3

(c)(i)

Mark scheme

- Sketch of α vs. t from $t = 0$ to $t = t_1$, 1 point each: (1) The curve begins at a maximum (positive) value at $t = 0$ and monotonically decreases to a minimum of zero at $t = t_1$ (when the board reaches horizontal/equilibrium, so the net torque and hence α is zero there). (2) The curve is concave down over the entire interval $0 < t < t_1$ (not a straight line or concave-up decrease). Behavior of the sketch beyond t_1 is not scored.

Solution 3

(c)(ii) Mark scheme

- Write the rotational form of Newton's second law with the spring and gravitational torques in opposite directions — 1 point: $\tau_{spring} - \tau_g = I\alpha$ (i.e. $I_B\alpha_0$). Include $\sin(90 - \theta_0)$, $\sin(90 + \theta_0)$, or $\cos\theta_0$ in an expression for the net torque (the correct perpendicular lever-arm factor for the small angle θ_0) — 1 point: $F_{spring}d_{spring}(\cos\theta_0) - F_g d_g(\cos\theta_0) = I_B\alpha_0$. Substitute mg for the force of gravity and $k\Delta x_2$ for the spring force — 1 point: $d_{spring}k\Delta x_2(\cos\theta_0) - d_g\, mg(\cos\theta_0) = I_B\alpha_0$. Substitute the correct lever arms $d_{spring} = d_g = L/4$ — 1 point: $\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$, giving $\alpha_0 = \frac{L}{4I_B}\left(k\Delta x_2\cos\theta_0 - mg\cos\theta_0\right)$.

Solution 3

Solution 3

(d) Mark scheme

- Answer: $\alpha' < \alpha_0$ — select this choice with an attempt at a relevant justification (1 point). Justification must indicate the net torque on the system does not change — 1 point (the spring and gravity on the board still exert the same torques at the same lever arms; the two added equal masses sit at equal distances on opposite sides of the pivot, so their gravitational torques cancel and add no net torque). Justification must also indicate the rotational inertia of the system increases — 1 point (the two added point masses increase the total moment of inertia I about the pivot). Example full justification: The net torque on the system is unchanged (the spring and the board's own weight exert the same torques as before, and the additional torques from the two added masses cancel

Solution 3

since they sit at equal distances on opposite sides of the pivot), but the rotational inertia of the system is now larger. Since $\alpha = \tau_{net}/I$, with τ_{net} unchanged and I larger, the angular acceleration is now smaller: $\alpha' < \alpha_0$.