Exercise walk-through

Rotational Inertia ▪ 5.4

eXercise

You must be able to

- describe **rotational inertia 转动惯量** I as resistance to a change in rotation
- use $I = \sum m_i r_i^2$ for point masses
- explain that mass farther from the axis contributes more
- recall that I depends on the mass distribution and the axis chosen

Method — Rotational inertia of point masses

$$I = \sum m_i r_i^2 .$$

A 2 kg mass at $r = 3$ m from the axis has $I = 2 \times 3^2 = 18$ kg m^2.

Method — Distance matters most

Because r is squared, mass far from the axis dominates. Moving a mass from 1 m to 2 m quadruples its contribution.

Method — It depends on the axis

The same object has different I about different axes. A rod is easier to spin about its centre than about one end.

Practice 2.1 ▪ 2 marks

A 3 kg mass sits 2 m from the axis. Find its rotational inertia.

Solution 2.1

Method: Rotational inertia of point masses

Worked steps

$I = mr^2 = 3 \times 2^2 = 12$ kg m^2 M1 A1.

Practice 2.2 ▪ 1 mark

State what rotational inertia measures.

Solution 2.2

Method: Rotational inertia of point masses

Worked steps

An object's resistance to a change in its rotation B1.

Practice 2.3 ▪ 2 marks

Two 1 kg masses sit at $r = 1$ m and $r = 2$ m. Find the total I.

Solution 2.3

Method: Rotational inertia of point masses

Worked steps

$I = 1(1)^2 + 1(2)^2 = 1 + 4 = 5$ kg m^2 M1 A1.

Practice 2.4 ▪ 1 mark

If a mass is moved twice as far from the axis, by what factor does its contribution to I change?

Solution 2.4

Method: Distance matters most

Worked steps

Four times (r is squared) B1.

Exam-style 3.1 ▪ 1 mark

Rotational inertia depends on

A only the total mass

B the mass and how it is distributed about the axis

C only the speed

D the colour of the object

Solution 3.1

Method: Rotational inertia of point masses

A only the total mass

B the mass and how it is distributed about the axis ✓

C only the speed

D the colour of the object

Worked steps

B — mass and its distribution about the axis.

Exam-style 3.2 ▪ 1 mark

A point mass's contribution to rotational inertia is

A mr

B mr^2

C $\frac{1}{2}mr^2$

D m/r

Solution 3.2

Method: Rotational inertia of point masses

A mr

B mr^2 ✓

C $\frac{1}{2}mr^2$

D m/r

Worked steps

B — mr^2.

Exam-style 3.3 ▪ 2 marks

A 4 kg mass sits at $r = 1.5$ m and a 2 kg mass at $r = 3$ m on a light rod.
(a) Find the total rotational inertia.
(b) State which mass contributes more, and why.

Solution 3.3

Method: Rotational inertia of point masses

Worked steps

(a) $I = 4(1.5)^2 + 2(3)^2 = 9 + 18 = 27$ kg m^2 M1 A1. (b) The 2 kg mass, because it is farther out (r^2 dominates) B1.

Exam-style 3.4 ▪ 1 mark

A solid disc and a ring have the same mass and radius.
(a) State which has the larger rotational inertia.
(b) Explain why, in terms of mass distribution.

Solution 3.4

Method: Rotational inertia of point masses

Worked steps

(a) The ring B1. (b) All the ring's mass is at the rim (maximum r), while the disc's mass is spread inward, lowering its I M1 A1.