Past-paper walk-through

Connecting Linear and Rotational Motion ▪ 5.2

eXercise

Questions in this set

- 2021 Set 1 Q3
- 2019 Set 1 Q3
- 2017 Q3
- 2015 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

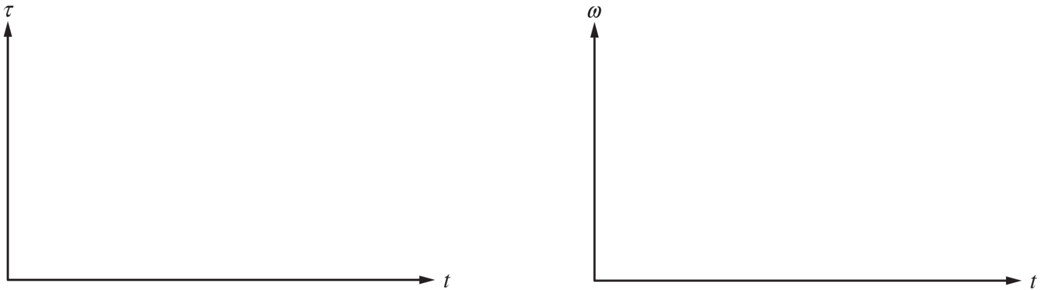
2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

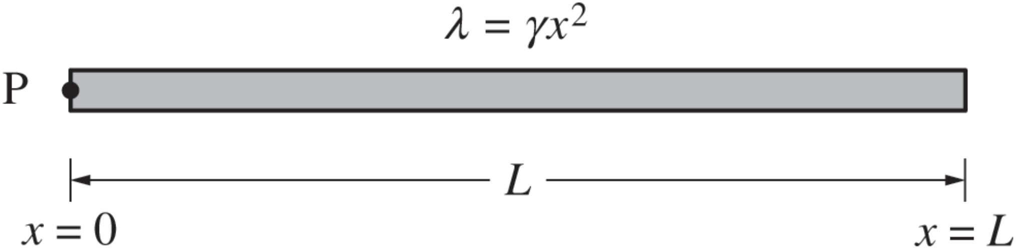
A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\dfrac{3}{5}ML^2$.

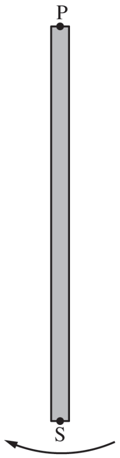
Question 3



Question 3



Question 3



Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

_______ Greater than _______ Less than _______ Equal to

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(d) The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

On the axes below, sketch graphs of the magnitude of the net torque τ on the rod and the angular speed ω of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.

[Two blank axes: left axis labeled τ (vertical) vs. t (horizontal); right axis labeled ω (vertical) vs. t (horizontal). Both currently blank, to be sketched by the student.]

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?

_______ Increasing _______ Decreasing _______ Not changing

Justify your answer.

Question 3

2021 Set 1 ▪ Q3

[Figure: A triangular (wedge-shaped) rod lies horizontally, thicker at the left end (point P, $x = 0$) and tapering to a point at the right end ($x = L$), with linear mass density $\lambda = \gamma x^2$ shown increasing along its length. The length L is marked from $x = 0$ to $x = L$.]

A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$, where $\gamma = \dfrac{3M}{L^3}$ and x is the distance from point P at the left end of the rod.

(f) [Figure: The rod hangs vertically, pivoted at point P at the top, with point S marked at the bottom end. A curved arrow beneath S indicates the rod swinging.]
The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m. Calculate the linear speed v of point S as the rod swings through the vertical position shown.

Solution 3

(a) Mark scheme

- Set up the rotational-inertia integral about point P: $I = \int r^2\, dm$, with the given linear mass density $\lambda = \gamma x^2$ so $dm = \lambda\, dx = \gamma x^2\, dx$ (1 point). Substitute and integrate from $x = 0$ to $x = L$, using $\gamma = 3M/L^3$ (from normalizing the total mass $M = \int_0^L \gamma x^2\, dx = \gamma L^3/3$) (1 point):
$$I = \int_0^L x^2(\gamma x^2\, dx) = \gamma \left[\frac{x^5}{5}\right]_0^L = \left(\frac{3M}{L^3}\right)\left(\frac{L^5}{5}\right) = \frac{3}{5}ML^2.$$

Solution 3

(b) Mark scheme

- Set up the center-of-mass integral: $X_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\int x\,dm}{\int dm}$ (1 point). Substitute $dm = \gamma x^2\,dx$ into the numerator (1 point), and either substitute M directly for the denominator or evaluate $\int dm$ to get the rod's mass (1 point): $X_{CM} = \frac{\int_0^L x(\gamma x^2)\,dx}{M} = \frac{[\gamma x^4/4]_0^L}{M} = \frac{(3M/L^3)(L^4/4)}{M} = \frac{3}{4}L$. The center of mass is located $\frac{3}{4}L$ from point P.

Solution 3

(c)

Mark scheme

- Answer: Greater than (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) more of the rod's mass is concentrated at the end opposite P (farther from the rotation axis), so the rotational inertia about P exceeds that about the center of mass; or (ii) by the parallel-axis theorem $I = I_{cm} + md^2$, an axis displaced from the center of mass ($d > 0$) always gives a larger rotational inertia than the axis through the center of mass.

Solution 3

(d) Mark scheme

- Sketch two graphs vs. time t, from release (rod horizontal) to when the center of mass reaches its lowest point (rod vertical). Torque τ vs. t (1 point): starts at its maximum value at $t = 0$ and decreases along a concave-down curve to zero (torque vanishes as the rod becomes vertical, when the weight acts along the line to the pivot). Angular speed ω vs. t (1 point): starts at 0 and increases along a concave-down curve that flattens, approaching a horizontal asymptote (constant ω) as t increases (since $\alpha = \tau/I \to 0$ as $\tau \to 0$). Consistency (1 point): ω must keep rising as long as $\tau > 0$ and level off exactly as $\tau \to 0$, matching $d\omega/dt = \tau/I$.

Solution 3

(e)

Mark scheme

- Answer: Decreases (1 point for selecting it with an attempted justification). Correct justification (1 point), either of: (i) from $\tau = rF\sin\theta$, as the rod swings downward the angle θ between the weight and the lever arm decreases, so the torque decreases; or (ii) the horizontal lever arm between P and the rod's center of mass continuously shrinks as the rod rotates downward, so the torque decreases. Since I is constant and $\alpha = \tau/I$, angular acceleration decreases correspondingly.

Solution 3

(f) Mark scheme

- Use conservation of energy for the rod swinging from horizontal (at rest) to vertical: $U_i + K_i = U_f + K_f \Rightarrow U_i = K_f$ (1 point). Substitute the gravitational PE (using the center-of-mass drop $h_i = \frac{3}{4}L$ found in part (b)) and the rotational KE about P (1 point): $Mgh_i = \frac{1}{2}I\omega_f^2 \Rightarrow Mg\left(\frac{3}{4}L\right) = \frac{1}{2}\left(\frac{3}{5}ML^2\right)\omega_f^2$. Using $I = \frac{3}{5}ML^2$ from part (a) and $\omega_f = v/L$, where v is the linear speed of point S (the free end, distance L from P) (1 point): $\frac{3}{4}MgL = \frac{3}{10}Mv^2 \Rightarrow v = \sqrt{\frac{5}{2}gL} = \sqrt{\frac{5}{2}(9.8\ \mathrm{m/s^2})(1.0\ \mathrm{m})} = 4.9\ \mathrm{m/s}$. (Given $M = 3.0$ kg, $L = 1.0$ m; note M cancels out of the final expression for v.)

Question 3

2019 Set 1 ▪ Q3

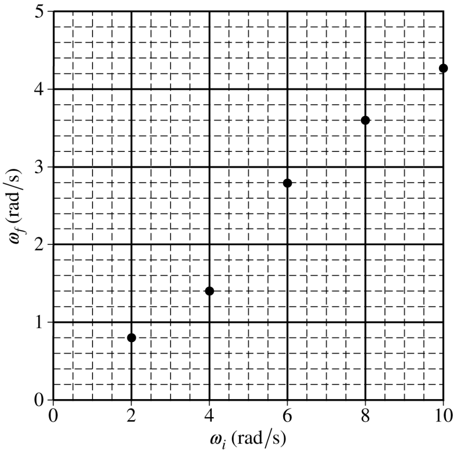
[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

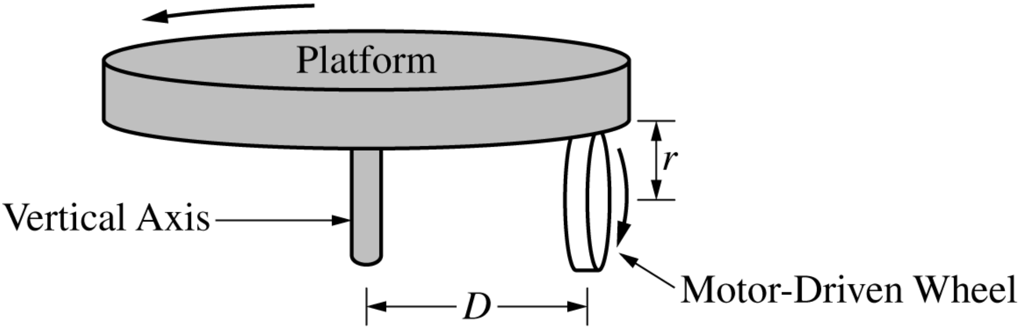
Question 3

(a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

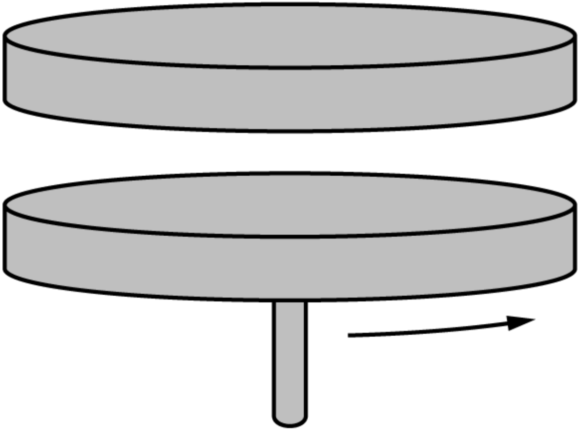
Question 3



Question 3



Question 3



Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

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Question 3

(c) Derive an expression for the angular speed ω_W of the wheel when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

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Question 3

(d) [Figure: the platform shown alone from a 3D perspective, spinning (arrow indicating rotation direction), with the motor-driven wheel removed.]

When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.

Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

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Question 3

(e)(i) A student now uses the rotating platform ($I_P = 3.1$ kg·m^2) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.
[Graph: ω_f (rad/s) vs. ω_i (rad/s); y-axis from 0 to 5 in increments of 1, x-axis from 0 to 10 in increments of 2. Data points roughly follow a straight line through the origin, e.g. near (2, 0.9), (4, 1.4), (6, 2.6), (8, 3.6), (10, 4.2).]
On the graph on the previous page, draw a best-fit line for the data.
(e)(ii) Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the platform until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

Question 3

(f) The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

______ $K_f < K_i$ ______ $K_f = K_i$ ______ $K_f > K_i$

Justify your answer.

Question 3

2019 Set 1 ▪ Q3

[Figure above the question: A horizontal circular "Platform" (rotational inertia I_P) mounted on a "Vertical Axis", shown from a 3D perspective with an arrow indicating rotation direction. A small "Motor-Driven Wheel" of radius r is mounted under the platform and touches it at a distance D from the vertical axis.]

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Question 3

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

______ Greater than   ______ Less than   ______ Equal to

Justify your answer.

Solution 3

(a)

Mark scheme

- Substitute into the rotational form of Newton's second law (1 pt): $\tau = I\alpha \Rightarrow FD = I_P\alpha \Rightarrow \alpha = \dfrac{FD}{I_P}$. Substitute into a rotational kinematic equation to find the angular speed (1 pt): $\omega_2 = \omega_1 + \alpha\Delta t = 0 + \dfrac{FD}{I_P}\Delta t$, giving $\omega_P = \dfrac{FD\Delta t}{I_P}$.

Solution 3

(b)

Mark scheme

- Use the rotational kinetic energy equation (1 pt): $K = \frac{1}{2} I \omega_P^2 = \frac{1}{2}(I)\left(\frac{FD\Delta t}{I}\right)^2$. Give an answer consistent with part (a) (1 pt): $K = \frac{(FD\Delta t)^2}{2I}$.

Solution 3

(c)

Mark scheme

- Indicate that the linear speed of the platform's edge equals the linear speed of the wheel's edge at the point of contact (1 pt): $v_P = v_W$ or $(r\omega)_P = (r\omega)_W$. Correctly relate the linear speeds to the angular speeds (1 pt):

$$D\omega_P = r\omega_W \Rightarrow \omega_W = \frac{D\omega_P}{r}.$$

Solution 3

(d)

Mark scheme

- Use conservation of angular momentum for the disk landing on the platform (1 pt): $L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2$. Correctly substitute (the disk has the same rotational inertia I_P as the platform, so the combined inertia doubles) (1 pt): $I_P\omega_P = (2I_P)\omega_f \Rightarrow \omega_f = \frac{1}{2}\omega_P$.

Solution 3

(e)(i)

Mark scheme

- Draw a reasonable straight best-fit line through the plotted ω_f vs. ω_i data points on the given graph (1 pt for an appropriate best-fit line).

Solution 3

(e)(ii)

Mark scheme

- Use conservation of angular momentum to derive an expression containing I_U (1 pt): $L_i = L_f \Rightarrow I_P\omega_i = (I_P + I_U)\omega_f \Rightarrow I_U = I_P\left(\frac{\omega_i}{\omega_f}\right) - I_P$. Substitute two points read from the best-fit line (1 pt): $I_U = (3.1\ \text{kg}\cdot\text{m}^2)\left(\frac{4.0 - 1.0\ \text{rad/s}}{9.3 - 2.4\ \text{rad/s}}\right) - (3.1\ \text{kg}\cdot\text{m}^2) = 4.1\ \text{kg}\cdot\text{m}^2$ (the point (0,0) may be used implicitly if the best-fit line passes through the origin).

Solution 3

(f)

Mark scheme

- Select $K_f < K_i$ with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the two disks end up rotating together at the same final angular speed, this is an inelastic collision, so kinetic energy is lost during the collision — therefore $K_f < K_i$.

Solution 3

(g)

Mark scheme

- Select 'Greater than' with an attempt at a relevant justification (1 pt), plus a fully correct justification (1 pt). Example: Because the object's center of mass is off the platform's axis, the parallel-axis theorem $I = I_{CM} + Mh^2$ applies, so the true (actual) rotational inertia about the axis through the center of mass is smaller than what was measured — the experimental value obtained in part (e) is therefore greater than the actual value, increased by Mh^2.

Question 3

2017 ▪ Q3

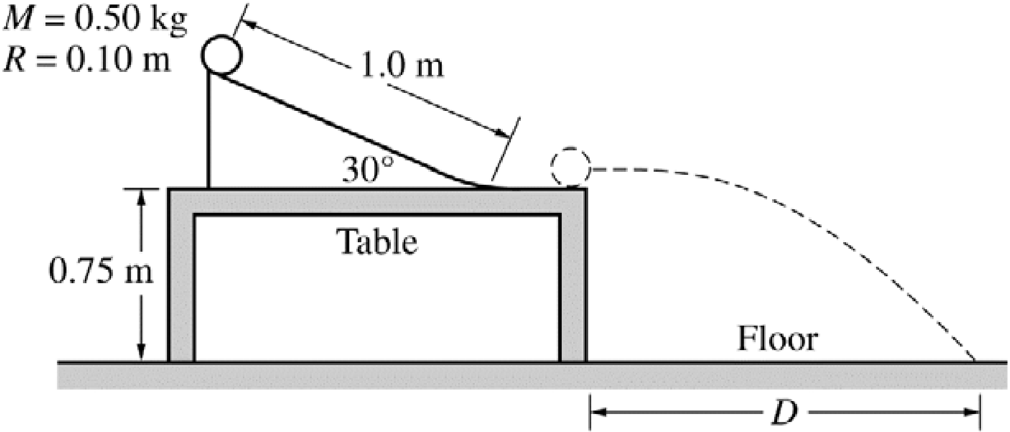
[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition

Question 3

from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.
(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

Question 3



Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the

Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the

Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the

Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

Question 3

2017 ▪ Q3

[Figure: A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m sits at the top of a 1.0 m long inclined plane making a $30°$ angle with the horizontal. The incline descends onto a table of height 0.75 m. The cylinder rolls down the incline, across the table, and launches horizontally off the edge of the table, following a projectile path (shown dashed) to land on the floor a horizontal distance D from the edge of the table. A coiled spring/bumper is shown mounted at the far right on the floor.]

A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the

Question 3

horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(e)(i) Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

(e)(ii) Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

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(e)(iii) Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Solution 3

(a)

Mark scheme

- Using conservation of energy from the top of the incline to the bottom (table level): $U_{g,top} = K_{table}$, i.e. $mgh = K_{table}$, where $h = L\sin\theta = (1.0\ \mathrm{m})\sin 30^\circ = 0.50\ \mathrm{m}$. So $K_{table} = mgh = (0.50\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(1.0\ \mathrm{m})(\sin 30^\circ) = 2.45\ \mathrm{J}$ (or $2.5\ \mathrm{J}$ using $g = 10\ \mathrm{m/s^2}$). 1 point for correctly applying conservation of energy, 1 point for the correct numeric answer with units.

Solution 3

(b) Mark scheme

- Total KE is translational plus rotational: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. For a cylinder, $I = \frac{1}{2}MR^2$ and $v = R\omega$, so $K = \frac{1}{2}M(R\omega)^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{2}M(R\omega)^2 + \frac{1}{4}M(R\omega)^2 = \frac{3}{4}M(R\omega)^2$. Solving for ω: $\omega = \sqrt{\frac{4K}{3MR^2}} = \sqrt{\frac{4(2.45\ \mathrm{J})}{3(0.50\ \mathrm{kg})(0.10\ \mathrm{m})^2}} = 25.6\ \mathrm{rad/s}$ (or $26.0\ \mathrm{rad/s}$ using $g = 10\ \mathrm{m/s^2}$ and $K = 2.5\ \mathrm{J}$). 1 point for correctly writing K as translational plus

Solution 3

rotational KE, 1 point for correctly substituting $v = R\omega$ and $I = \frac{1}{2}MR^2$, 1 point for the correct numeric answer.

Solution 3

(c) Mark scheme

- No torque acts on the cylinder during its fall off the table (only gravity, through the center of mass), so the rotational KE at the floor equals the rotational KE at the table edge: $K_{rot} = \dfrac{1}{4}M(R\omega)^2$. The total KE at the floor equals the total KE at the table ($K_{table} = \dfrac{3}{4}M(R\omega)^2$) plus the additional KE gained by falling the table height $h_{table} = 0.75\ \mathrm{m}$: $K_{tot} = \dfrac{3}{4}M(R\omega)^2 + Mgh_{table}$. So
$$\frac{K_{rot}}{K_{tot}} = \frac{(R\omega)^2}{3(R\omega)^2 + 4gh_{table}} = \frac{[(0.10\ \mathrm{m})(25.6\ \mathrm{rad/s})]^2}{3[(0.10)(25.6)]^2 + 4(9.81\ \mathrm{m/s^2})(0.75\ \mathrm{m})} = 0.133$$
(or 0.135 using $g = 10\ \mathrm{m/s^2}$). [Equivalent alternate approach given in the scoring

Solution 3

guideline: $K_{rot}/U_{total} = \frac{1}{4}(R\omega)^2/[g(h+y)]$, using the total incline-plus-table height drop, gives the same ≈ 0.13.] 1 point for a correct ratio expression, 1 point for the correct substituted numeric answer.

Solution 3

(d) Mark scheme

- Vertical motion (free fall from the table, height $y = 0.75\ \mathrm{m}$): $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(0.75\ \mathrm{m})}{9.81\ \mathrm{m/s^2}}} = 0.39\ \mathrm{s}$. Horizontal motion (constant velocity $v = R\omega$, no horizontal force during the fall): $D = (R\omega)t = (0.10\ \mathrm{m})(25.6\ \mathrm{rad/s})(0.39\ \mathrm{s}) = 1.0\ \mathrm{m}$. 1 point for correctly finding the fall time from vertical kinematics, 1 point for correctly using $D = R\omega t$ to get $D = 1.0\ \mathrm{m}$.

Solution 3

(e)(i)

Mark scheme

- Correct answer: Equal to. By conservation of energy, the total KE at the floor equals the total gravitational PE lost, $Mg(h+y)$. Since the sphere has the same mass and falls through the same total height as the cylinder, its total KE at the floor is the same as the cylinder's. 1 point for selecting 'Equal to' with an attempted relevant justification, 1 point for a fully correct justification.

Solution 3

(e)(ii) Mark scheme

- Correct answer: Less than. Because the sphere's rotational inertia is less than the cylinder's, the sphere rotates faster and, since $v = R\omega$ for rolling without slipping, moves with a greater linear speed. Because the mass is the same and the linear speed is greater, the sphere has greater linear (translational) kinetic energy. Since the total kinetic energies of the sphere and cylinder at the floor are equal (part i), the sphere must therefore have LESS rotational kinetic energy than the cylinder. 1 point for selecting 'Less than' with an attempted relevant justification, 1 point for a fully correct justification.

Solution 3

(e)(iii)

Mark scheme

- Correct answer: Greater than. Since the sphere has a greater linear speed than the cylinder as it leaves the table (from part ii), and both fall through the same height in the same time, the sphere travels a greater horizontal distance $D = vt$ before reaching the floor than the cylinder does. 1 point for selecting 'Greater than' with an attempted relevant justification, 1 point for a fully correct justification.

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.
[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]
(a) Using integral calculus, derive the rotational inertia for the rod around its end to show that it is $ML^2/3$.
[Diagram: A ceiling mount at the top left holding a horizontal rod via a pivot, with the rod extending to the right and labeled A at its free end. A curved arrow sweeps clockwise from the horizontal rod position down to a vertical dashed line, showing the

Question 3

rod falling from horizontal to vertical. The vertical dashed position is labeled B at its bottom end.]

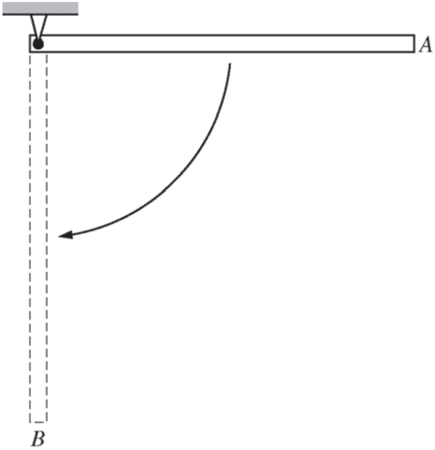
Question 3

Question 3

horizontal position A and uses photogates to measure the velocity of the free end at position B. T
recorded below.

Length (m)	0.25	0.50	0.75	1.00	1.25	1.50
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3

Question 3



Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(b) The rod is fixed at one end and allowed to fall from the horizontal position A through the vertical position B.

Derive an expression for the velocity of the free end of the rod at position B. Express your answer in terms of M, L, and physical constants, as appropriate.

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.
[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(c) An experiment is designed to test the validity of the expression found in part (b). A student uses rods of various lengths that all have a uniform mass distribution. The student releases each of the rods from the horizontal position A and uses photogates to measure the velocity of the free end at position B. The data are recorded below.

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Length (m)	**0.25**	**0.50**	**0.75**	**1.00**	**1.25**	**1.50**
Velocity (m/s)	2.7	3.8	4.6	5.2	5.8	6.3

[Two additional blank rows below the data table, with the same six columns, for the student to fill in derived quantities.]

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the acceleration due to gravity g.

Horizontal axis: ____________________

Question 3

Vertical axis: ____________________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given. Label each row you use and include units.

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(d) Plot the straight line data points on the grid below. Clearly scale and label all axes, including units as appropriate. Draw a straight line that best represents the data. [A blank grid divided into a 5×6 array of larger cells, each subdivided into finer dashed gridlines, for plotting data points and drawing a best-fit line. No axes are labeled and no data is plotted.]

Question 3

2015 ▪ Q3

A uniform, thin rod of length L and mass M is allowed to pivot about its end, as shown in the figure above.

[Diagram: A horizontal thin rod of length L pivoted at its left end (marked "Pivot Point" with a pivot symbol), extending to the right to a point labeled M (the mass) at its far end. The length L is marked with a double-headed arrow spanning the rod above it.]

(e)(i) Using your straight line, determine an experimental value for g.

(e)(ii) Describe two ways in which the effects of air resistance could be reduced.