Past-paper walk-through

Rotational Kinematics ▪ 5.1

eXercise

Questions in this set

- 2023 Set 2 Q3
- 2022 Set 1 Q3
- 2022 Set 2 Q3
- 2021 Set 2 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2023 Set 2 ▪ Q3

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.
[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]
[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

Question 3

(a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M, L, and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{cm} = \frac{1}{18}ML^2$.

Question 3

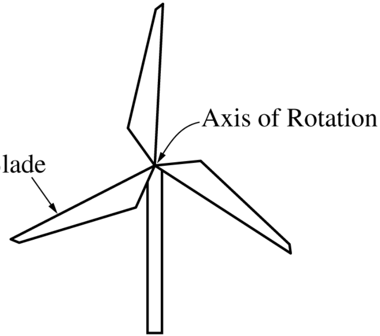

Figure 1

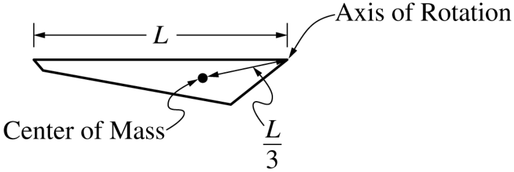

Figure 2

Question 3

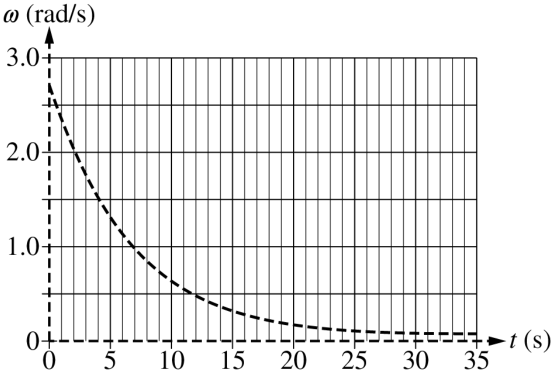

Figure 4

Question 3

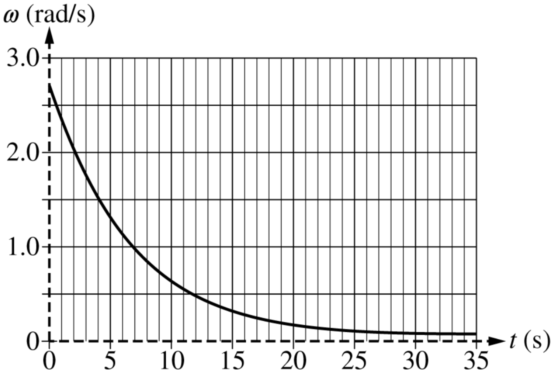

Figure 3

Question 3

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A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M, with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

[Figure 1: "Three-Blade System" — a turbine with three blades extending from a central hub, with the "Axis of Rotation" labeled through the hub and one "Blade" labeled.]

[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{sys} = 6.7 \times 10^6$ kg·m^2. Calculate the time T it takes the outer edge of a single blade to complete one revolution.

Question 3

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[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(c)(i) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

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[Figure 3: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; curve starts at $(0, 2.6)$ and decays exponentially toward 0 as t increases, with a dashed vertical line at $t = 0$ marking the initial value.]
Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.
(c)(ii) Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.
(c)(iii) Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t. Express your answer in terms of β_0, ω_0, M, L, I_{sys}, and physical constants, as appropriate.
The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the

Question 3

equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.
[Figure 4: Graph of ω (rad/s) vs. t (s); y-axis from 0 to 3.0 in increments of 1.0, x-axis from 0 to 35 in increments of 5; a dashed curve shows the original $\omega_0 e^{-\beta_0 t}$ decay from $(0, 2.6)$ toward 0.]

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2023 Set 2 ▪ Q3

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[Figure 2: "Single Blade" — a blade of length L shown horizontally, with the "Axis of Rotation" at one end and the "Center of Mass" marked at a distance $\frac{L}{3}$ from the axis of rotation.]

(d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t.

Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
[Diagram, Figure 2: The same disk-and-stand setup, now rotated clockwise through a small angle θ (marked at the axle). The string on the right side of the disk now goes over the top and down to a hanging "Block" on the right side. The spring on the left side is now shown stretched, labeled "Stretched Spring", connecting down to the ground.]
Note: Figures not drawn to scale.

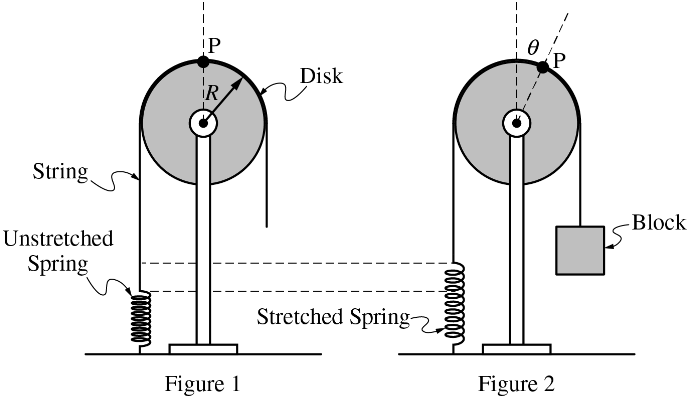
Question 3

A solid uniform disk is supported by a vertical stand. The disk is able to rotate with negligible friction about an axle that passes through the center of the disk. The mass and radius of the disk are given by M_d and R, respectively. The rotational inertia of the disk is $I_d = \frac{1}{2}M_dR^2$. A string of negligible mass is draped over the disk and attached to the top of the disk at point P. One end of the string is connected to an unstretched ideal spring of spring constant k, which is fixed to the ground as shown in Figure 1.
A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.
Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

Question 3

(a) Derive an expression for the mass m_B of the block.

Question 3



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Question 3

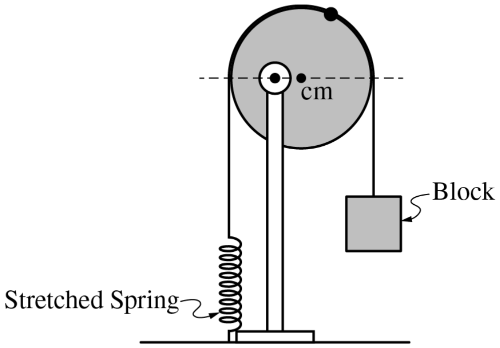

Figure 3

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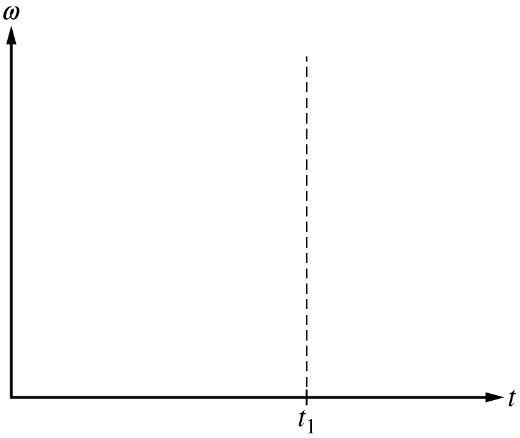
Question 3



Question 3

2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
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Question 3

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A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(b) At time $t = 0$, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

Question 3

[Diagram: A circle representing the disk, with a dot at its center marking the axle, and a diagonal dashed line through the center, on which forces should be drawn.]

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2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
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Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(c) Derive an expression for the angular acceleration α of the disk immediately after the string is cut.

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2022 Set 1 ▪ Q3

[Diagram, Figure 1: A solid disk mounted on a vertical stand, able to rotate about a horizontal axle through its center (marked with a dot). Point P is marked near the top-left edge of the disk with an arrow labeled "Disk" pointing to the disk. A string labeled "String" is draped over the top of the disk and hangs down the left side, attached at the bottom to an unstretched spring labeled "Unstretched Spring", which is anchored to the ground.]
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A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(d) At $t = t_1$, the disk has rotated and point P is again directly above the axle. Sketch a graph of the magnitude of the angular velocity ω of the disk as a function of time t from $t = 0$ to $t = t_1$.

[Graph: axes labeled "ω" (vertical) vs. "t" (horizontal); a dashed vertical gridline is marked at t_1 on the time axis; blank axes for the student to sketch on.]

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2022 Set 1 ▪ Q3

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Question 3

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A block of mass m_B is then attached to the string on the right side of the disk. The block is slowly lowered until the spring-disk-block system reaches equilibrium, as shown in Figure 2. In this equilibrium position, the disk has rotated clockwise through a small angle θ.

Give all algebraic answers in terms of M_d, R, k, θ, and physical constants, as appropriate.

(e) [Diagram, Figure 3: The disk-and-stand setup, now mounted so the axle (marked with a dot, off-center within the disk, labeled "axle") does not pass through the center of the disk (marked "com" for center of mass). A string goes over the top of the disk down to a hanging "Block" on the right side. A "Stretched Spring" connects from the left side down to the ground.]

Note: Figure not drawn to scale.

Question 3

The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

Solution 3

(a)

Mark scheme

- Derivation of m_B. Indicate the disk is in rotational (or translational) equilibrium before the string is cut: $\Sigma\tau_{\text{on disk}} = 0 \Rightarrow \tau_g = \tau_s$ (or equivalently $\Sigma F = 0 \Rightarrow F_g = F_s$) (1 point). Substitute the force expressions (both forces act at the same radius R, gravity via the block's weight through the string, spring force $F_s = k\Delta x$): $F_g R = F_s R \Rightarrow m_B g R = k\Delta x R \Rightarrow m_B g = k\Delta x$ (1 point). Substitute $\Delta x = R\theta$ (arc length for the disk's rotation angle θ): $m_B g = kR\theta \Rightarrow m_B = \dfrac{kR\theta}{g}$ (1 point).

Solution 3

(b) Mark scheme

- Free-body diagram of the disk immediately after the string on the right is cut (block falling, only the left string/spring remains attached): draw and label (1) the tension/spring force F_s (or F_{spring}, T_{spring}) tangent to the disk, anywhere from point P to the left edge of the disk inclusive (1 point); (2) the force of gravity F_g located at the center, pointing straight down (1 point); (3) the force from the axle F_{axle} (or F_N, F_n) located at the center, directed so the disk remains in translational equilibrium, i.e. $\Sigma F = 0$ — typically straight up with magnitude balancing F_g and the vertical component of F_s (1 point). A response with extraneous forces/vectors caps at a maximum of 2 of the 3 points.

Solution 3

(c) Mark scheme

- Derivation of angular acceleration α. Indicate the net torque on the disk is due only to the tension/spring force, via the rotational form of Newton's second law: $\tau_s = I_d\alpha$ (1 point). Express that torque as the spring force times its lever arm R: $F_sR = I_d\alpha$ (1 point). Substitute $F_s = -k\Delta x$: $-k\Delta xR = I_d\alpha$ (1 point). Substitute the disk's moment of inertia $I_d = \frac{1}{2}M_dR^2$ and $\Delta x = R\theta$ (consistent with part (a)): $-k(R\theta)R = \frac{1}{2}M_dR^2\alpha \Rightarrow \alpha = -\dfrac{2k\theta}{M_d}$ (1 point). The negative sign is not required to earn this point.

Solution 3

(d)

Mark scheme

- Sketch of ω vs. t from $t = 0$ to $t = t_1$ (point P returns directly above the axle at t_1): the curve must start at zero and increase monotonically over the whole interval (1 point), and be concave down throughout — i.e. increasing but at a decreasing rate (1 point). Only the portion of the graph from 0 to t_1 is scored.

Solution 3

(e) Mark scheme

- Setup: the disk is remounted so its axle no longer passes through its center of mass; a stretched spring pulls down-left and a string over the top holds a hanging block on the right. As the configuration is adjusted (disk rotated / axle offset from center of mass), the response must indicate three torque changes, each earning 1 point: (1) the torque due to gravity ON THE DISK increased, because the disk's weight now acts with a nonzero lever arm about the (now off-center) axle, where previously it had none; (2) the torque due to the tension from the hanging block (transmitted via the string on the right) increased, because that force's lever arm about the axle grew longer; (3) the torque due to the spring's tension (on the left) increased, because it is the only force that can supply the

Solution 3

extra counterclockwise torque needed to counteract the increased clockwise torques from (1) and (2) and keep the disk in equilibrium. Scoring note: a response that also states the torque from the axle force stays the same can still earn all 3 points; a response that says the axle-force torque changes, or invokes additional torques, caps at a maximum of 2 of the 3 points.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.
[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]
[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(a) On the rectangle below, which represents the board, draw and label the forces (not components) that act on the board while the board-spring system is in equilibrium. Each force should be represented by an arrow away from the board, and should represent the location at which that force acts.
[A blank horizontal rectangle representing the board, with a dashed vertical centerline and dashed horizontal centerline, for the student to draw and label force vectors acting on the board.]

Question 3

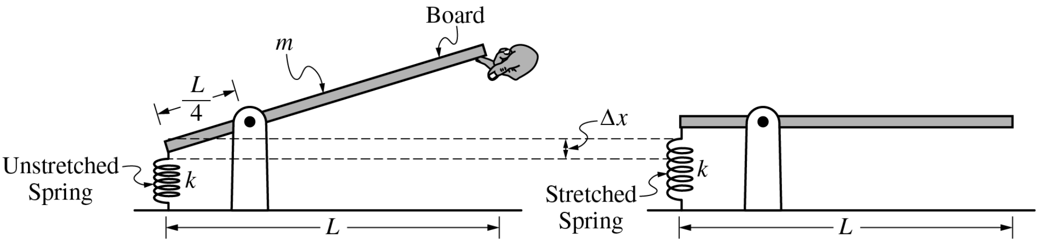
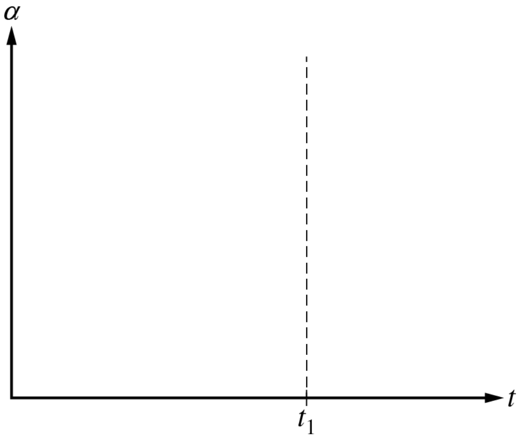


Figure 1

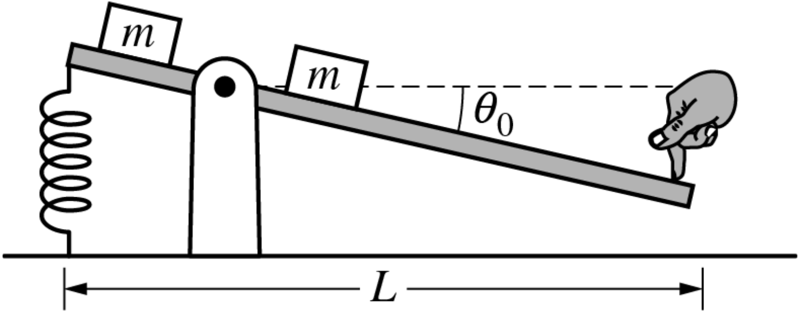
Figure 2

Note: Figures not drawn to scale.

Question 3



Question 3



Note: Figure not drawn to scale.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(b) Derive an expression for the distance the spring stretches, Δx, when the board is in equilibrium. Express your answer in terms of k, L, m, and physical constants, as appropriate.

Question 3

2022 Set 2 ▪ Q3

A uniform board of length L and mass m is attached to a pivot $\frac{L}{4}$ from the left end of the board. The left end of the board is attached to an ideal spring of spring constant k that is attached to the ground. The student slowly lowers and then releases the board. The board remains at rest in the horizontal position, with the spring unstretched, as shown in Figure 1. The rotational inertia of the board about the pivot is I.

[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(c)(i) [Figure: A board pushed down on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest. Note: Figure not drawn to scale.]
A student pushes down the board on the right side, stretching the spring a new distance Δx_2 from the unstretched position. The board is held at a small angle θ_0 with the horizontal, as shown. The student then releases the board from rest.
At time $t = 0$, the board is released. At time t_1, the board first crosses the horizontal.
Sketch a graph of the magnitude of the angular acceleration α of the board as a function of time t from $t = 0$ to $t = t_1$.
[A blank graph with vertical axis α and horizontal axis t, with a marked point t_1 on the time axis, for the student to sketch angular acceleration vs. time.]

Question 3

(c)(ii) Derive an expression for the angular acceleration α_0 of the board immediately after the board is released. Express your answer in terms of k, L, m, I, Δx_2, θ_0, and physical constants.

Question 3

2022 Set 2 ▪ Q3

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[Figure 1: A board of length L resting horizontally, pivoted at a point $L/4$ from its left end (pivot labeled at the fulcrum). The left end of the board connects down to an "Unstretched Spring" anchored to the ground. Note: Figures not drawn to scale.]

[Figure 2: The board is shown lowered at an angle on the left side, with the spring now stretched by an amount Δx ("Stretched Spring"), anchored to the ground; the board length L is marked on the right/upper side.]

Question 3

(d) [Figure: The same board-spring-pivot setup, with two blocks of equal mass *m* attached to the board equal distances from the pivot point, as shown. Note: Figure not drawn to scale.]

Two blocks of equal mass *m* are attached to the board equal distances from the pivot point, as shown. The board is again pushed down on the right side so that the spring stretches the same distance Δx_2 as in part (c). The board is then released. The new angular acceleration α' when the blocks are attached compare to the angular acceleration α_0 from part (c) ?

$\alpha' > \alpha_0$ $\alpha' < \alpha_0$ $\alpha' = \alpha_0$

Justify your answer.

Solution 3

(a) Mark scheme

- Free-body diagram on the board rectangle, 1 point each: (1) Spring force F_{spring} (or F_S, "spring force") drawn as an arrow pointing downward at the location where the spring attaches (the right end of the board, $L/4$ from the pivot). (2) Force of gravity F_g (or mg, F_G, W, "grav force" — not "G"/"g" alone) drawn pointing downward at the center of the board (its center of mass). (3) Pivot force F_{pivot} (or F_N, F_n, N, "normal force") drawn pointing upward at the pivot location (the fixed support at the board's center). Each force must be correctly located and labeled; extraneous forces cap the score at 2 points.

Solution 3

(b) Mark scheme

- Indicate the sum of torques about the pivot equals zero (equilibrium) — 1 point: $\Sigma\tau = 0$. Substitute mg for the gravitational force and $k\Delta x$ for the spring force into the torque equation, using the given lever arms ($L/4$ each) — 1 point: $\frac{L}{4}k\Delta x - \frac{L}{4}mg = 0$. Solve for a correct expression for Δx — 1 point: $\Delta x = \frac{mg}{k}$ (earned regardless of an overall sign).

Solution 3

(c)(i)

Mark scheme

- Sketch of α vs. t from $t = 0$ to $t = t_1$, 1 point each: (1) The curve begins at a maximum (positive) value at $t = 0$ and monotonically decreases to a minimum of zero at $t = t_1$ (when the board reaches horizontal/equilibrium, so the net torque and hence α is zero there). (2) The curve is concave down over the entire interval $0 < t < t_1$ (not a straight line or concave-up decrease). Behavior of the sketch beyond t_1 is not scored.

Solution 3

(c)(ii) Mark scheme

- Write the rotational form of Newton's second law with the spring and gravitational torques in opposite directions — 1 point: $\tau_{spring} - \tau_g = I\alpha$ (i.e. $I_B\alpha_0$). Include $\sin(90-\theta_0)$, $\sin(90+\theta_0)$, or $\cos\theta_0$ in an expression for the net torque (the correct perpendicular lever-arm factor for the small angle θ_0) — 1 point: $F_{spring}d_{spring}(\cos\theta_0) - F_g d_g(\cos\theta_0) = I_B\alpha_0$. Substitute mg for the force of gravity and $k\Delta x_2$ for the spring force — 1 point: $d_{spring}k\Delta x_2(\cos\theta_0) - d_g\, mg(\cos\theta_0) = I_B\alpha_0$. Substitute the correct lever arms $d_{spring} = d_g = L/4$ — 1 point: $\frac{L}{4}(\cos\theta_0)k\Delta x_2 - \frac{L}{4}(\cos\theta_0)mg = I_B\alpha_0$, giving $\alpha_0 = \frac{L}{4I_B}\left(k\Delta x_2\cos\theta_0 - mg\cos\theta_0\right)$.

Solution 3

Solution 3

(d) Mark scheme

- Answer: $\alpha' < \alpha_0$ — select this choice with an attempt at a relevant justification (1 point). Justification must indicate the net torque on the system does not change — 1 point (the spring and gravity on the board still exert the same torques at the same lever arms; the two added equal masses sit at equal distances on opposite sides of the pivot, so their gravitational torques cancel and add no net torque). Justification must also indicate the rotational inertia of the system increases — 1 point (the two added point masses increase the total moment of inertia I about the pivot). Example full justification: The net torque on the system is unchanged (the spring and the board's own weight exert the same torques as before, and the additional torques from the two added masses cancel

Solution 3

since they sit at equal distances on opposite sides of the pivot), but the rotational inertia of the system is now larger. Since $\alpha = \tau_{net}/I$, with τ_{net} unchanged and I larger, the angular acceleration is now smaller: $\alpha' < \alpha_0$.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.

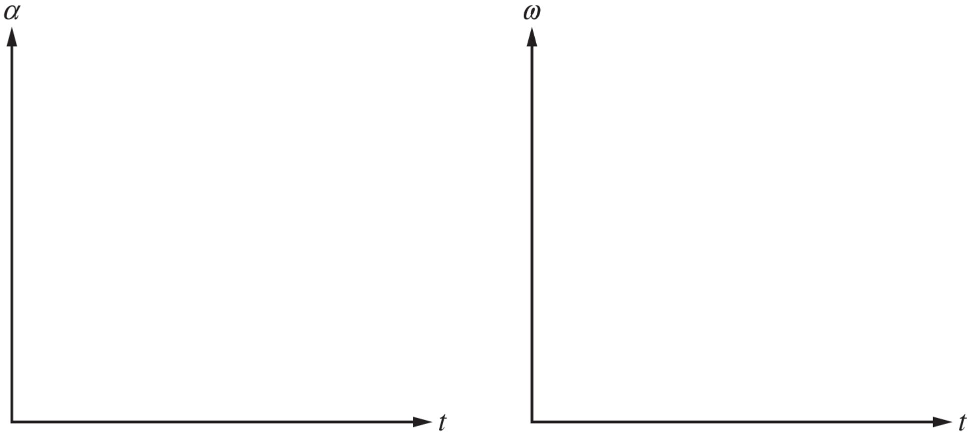
[Figure: "Object B" — an L-shaped (right-angle) object formed of two rods, shown on an x-y coordinate system with origin O at the bottom-left. A pivot is marked at the top-left corner (labelled "Pivot", on the y-axis). A horizontal rod of length L extends

Question 2

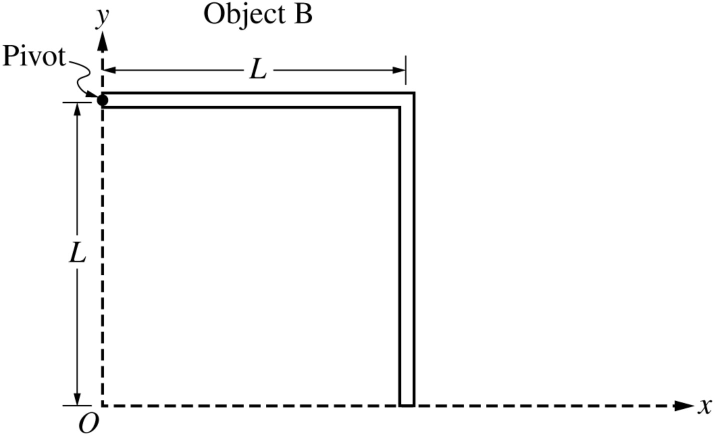
right from the pivot along the top (labelled "L"), and a vertical rod of length L extends down from the pivot to O along the y-axis (labelled "L"). The x-axis points right from O, the y-axis points up from O through the pivot.]

Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L.

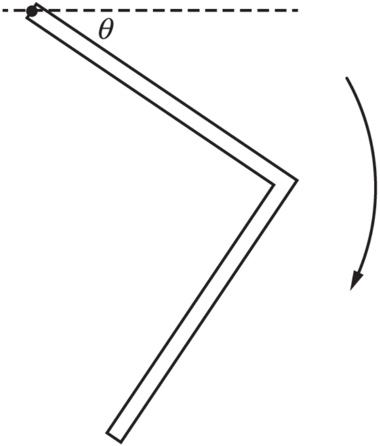
Question 2



Question 2



Question 2



Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(b)(i) Determine the following for the given coordinate system shown in the figure.

The x-coordinate of the center of mass of object B

(b)(ii) The y-coordinate of the center of mass of object B

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(c) Object B has a rotational inertia of I_B about its pivot.

Is the value of I_B greater than, less than, or equal to I_A?

______Greater than ______Less than ______Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.

[Two blank sets of axes are provided: left axis labelled α (vertical) vs. t (horizontal); right axis labelled ω (vertical) vs. t (horizontal). Both axes are otherwise unmarked (no gridlines or scale), for the student to sketch curves.]

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(e) [Figure: The L-shaped object B is shown rotated partway from its initial horizontal/vertical position, tilted through an angle θ (marked at the top between a dashed horizontal reference line and the now-tilted top rod), with a curved arrow indicating the direction of rotation (downward/clockwise) at the lower end of the shape.]

Question 2

While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

$I^{ncreasing}{}_{D^{ecreasing}{}_{N^{otchanging}}}$

Justify your answer.

[Figure: Object B is shown in a new orientation — the vertical rod of length L now lies along the bottom (horizontal), and the horizontal rod of length L extends straight up from its right end (vertical), forming an upside-down L / right-angle shape resting on the ground.]

Object B rotates through the position shown above.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

Solution 2

(a) Mark scheme

- Using integral calculus with λ the linear mass density of the rod (length $2L$, mass M): $I = \int_{r=0}^{r=2L} \lambda r^2 \, dr = \lambda \left[\frac{r^3}{3}\right]_0^{2L} = \frac{\lambda}{3}(2L)^3$ (1 pt, for correct limits/constant of integration). Since $\lambda = m/\ell = M/(2L)$: $I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)(8L^3) = \frac{4}{3}ML^2$ (1 pt, for correctly relating λ to M and L). This confirms $I_A = \frac{4}{3}ML^2$.

Solution 2

(b)(i) Mark scheme

- $X_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} = \dfrac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)}{\frac{M}{2} + \frac{M}{2}} = \dfrac{\frac{ML}{4} + \frac{ML}{2}}{M} = \dfrac{3}{4}L$. Final answer: $X_{CM} = \dfrac{3}{4}L$.

Solution 2

(b)(ii) Mark scheme

- $Y_{CM} = \dfrac{\sum m_i y_i}{\sum m_i} = \dfrac{\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)}{\frac{M}{2} + \frac{M}{2}} = \dfrac{\frac{ML}{2} + \frac{ML}{4}}{M} = \dfrac{3}{4}L$. Final answer: $Y_{CM} = \dfrac{3}{4}L$.

Solution 2

(c)

Mark scheme

- Select ‘Less than’ (1 pt for selection + attempted relevant justification) with a correct justification (1 pt). Example: ‘Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.’ I.e. $I_B < I_A$.

Solution 2

(d)

Mark scheme

- Sketch two graphs covering the interval from release to the moment the center of mass reaches its lowest point: the angular acceleration α vs. *t* graph must be concave down and begin horizontally (start at a nonzero maximum value with zero slope) (1 pt); the angular speed ω vs. *t* graph must be concave down and end horizontally (rise from zero to a plateau with zero final slope) (1 pt); the two graphs must be consistent with each other — ω increasing exactly while $\alpha > 0$, and both approaching their limiting behavior together as $\alpha \to 0$ near the lowest point (1 pt).

Solution 2

(e)

Mark scheme

- Select ‘Decreasing’ (1 pt for selection + attempted relevant justification) with a justification that identifies the lever arm for the torque as decreasing (1 pt). Example: ‘Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.’

Solution 2

(f) Mark scheme

- Conservation of energy: $U_{g1} = K_2$ (1 pt). Relate the change in rotational KE to the change in gravitational PE: $mgh = \frac{1}{2}I\omega^2$ (1 pt). Substitute $h = L/2$ (the drop in height of the center of mass): $Mg\left(\frac{L}{2}\right) = \frac{1}{2}I_B\omega^2$ (1 pt). Solve for ω in terms of the allowed symbols M, L, I_B: $\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}} = \sqrt{\frac{MgL}{I_B}}$ (1 pt).

  Final answer: $\omega = \sqrt{\frac{MgL}{I_B}}$.