Past-paper walk-through

Elastic and Inelastic Collisions ▪ 4.4

eXercise

Questions in this set

- 2025 Q1
- 2024 Set 1 Q1
- 2022 Set 1 Q2
- 2019 Set 1 Q2
- 2018 Q2
- 2016 Q2
- 2015 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2025 ▪ Q1

Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed. [Figure 1: Horizontal surface with a $+x$ arrow pointing right. Block 1 (mass m) on the left, moving in the $+x$-direction with speed $2v_0$ (arrow pointing right labeled $2v_0$). Block 2 (mass $6m$) on the right, moving in the $-x$-direction with speed v_0 (arrow pointing left labeled v_0).]
(a)(i) The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

Question 1

[Figure 2: A table titled "Momentum Before Collision" / "Momentum After Collision" with rows for Block 1, Block 2, and Two-Block System. Each cell shows a horizontal zero-momentum grid line labeled "0". The "Block 1, Before Collision" cell already shows an arrow starting at 0 and pointing right (+x-direction), representing the momentum of Block 1 before the collision. All other cells (Block 2 before collision; Block 1 after collision [shaded, not applicable]; Block 2 after collision [shaded, not applicable]; Two-Block System before collision; Two-Block System after collision) are blank grids to be filled in.]

Draw arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

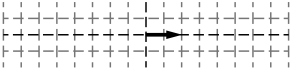
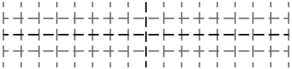
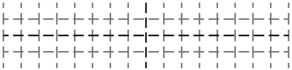
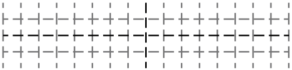
- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.

Question 1

- Represent an arrow of zero length by drawing a dot at zero.

(a)(ii) During the time interval $0 \leq t \leq t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{\max}\sin(At)$, where A is a positive constant and $F_{\max}$ is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.
Derive an expression for $F_{\max}$. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 1

	Momentum Before Collision	Momentum After Collision
Block 1	 0	
Block 2	 0	
Two-Block System	 0	 0

Question 1

Question 1

2025 ▪ Q1

Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

[Figure 1: Horizontal surface with a $+x$ arrow pointing right. Block 1 (mass m) on the left, moving in the $+x$-direction with speed $2v_0$ (arrow pointing right labeled $2v_0$). Block 2 (mass $6m$) on the right, moving in the $-x$-direction with speed v_0 (arrow pointing left labeled v_0).]

(b) Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Question 1

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Solution 1

(a)(i) Mark scheme

- Momentum vector diagram (Figure 2). Using the scale already set by the given Block 1 arrow (2 units right, i.e. magnitude $2v_0$ for mass m, since Block 1's momentum is $m(2v_0)$): draw ONE arrow for Block 2's momentum before the collision pointing in the $-x$ direction (leftward) and 6 units long (magnitude $6mv_0$, consistent with Block 2's mass $6m$ and speed v_0) [Point A1]. Draw identical arrows – same length and direction – for the momentum of the two-block system both BEFORE and AFTER the collision, since total momentum is conserved; the length equals the vector sum of the Block 1 and Block 2 arrows: 2 units right + 6 units left = 4 units left, i.e. pointing in the $-x$ direction with magnitude $4mv_0$ (matches $m(2v_0) + 6m(-v_0) = -4mv_0 = 7m(-\frac{4}{7}v_0)$) [Point

Solution 1

A2]. Award A1 for the correctly-directed 6-unit Block 2 arrow; award A2 for two IDENTICAL two-block-system arrows (before and after) of the correct length/direction, independent of A1.

Solution 1

(a)(ii) Mark scheme

- Derive $F_{\max}$. Begin with the impulse-momentum theorem or Newton's second law: $\vec{J} = \int_{t_1}^{t_2} \vec{F}_{net}(t)\,dt = \Delta\vec{p}$, or equivalently $\vec{F}_{net} = \dfrac{d\vec{p}}{dt}$ [Point A3: multistep derivation using the integral form of impulse or the differential form of N2L]. Apply conservation of momentum to the original collision: $\sum \vec{p}_0 = \sum \vec{p}_f \Rightarrow (m)(2v_0) + (6m)(-v_0) = (m+6m)(v_f) \Rightarrow v_f = -\frac{4}{7}v_0$ [Point A4: award for stating any ONE equivalent fact – final speed of Block 1 or 2 is $\frac{4}{7}v_0$, $\Delta p_{Block2} = +\frac{18}{7}mv_0$, $\Delta p_{Block1} = -\frac{18}{7}mv_0$, $\Delta v_{Block2} = +\frac{3}{7}v_0$, or $\Delta v_{Block1} = -\frac{18}{7}v_0$]. Then $\Delta p_{Block2} = -\Delta p_{Block1} = 6m\left(-\frac{4}{7}v_0 - (-v_0)\right) = \frac{18}{7}mv_0$. Substitute the given force into the impulse integral:

Solution 1

$\Delta p_{Block2} = \int_0^{t_c} F_{\max} \sin(At)\, dt$ [Point A5: substituting F(t) into the impulse/N2L expression]. Evaluate: $\frac{18}{7} m v_0 = \frac{F_{\max}}{A}\left[-\cos(At)\right]_0^{t_c} = \frac{F_{\max}}{A}\left[1-\cos(At_c)\right]$ [Point A6: definite integral with correct limits, or an equivalent indefinite integral plus a constant of integration]. Solve: $F_{\max} = \frac{18}{7}\left(\frac{Amv_0}{1-\cos(At_c)}\right)$ [Point A7: final correct expression in terms of m, v_0, A, t_c].

Solution 1

(b) Mark scheme

- New scenario: Block 1 (mass m) slides at speed v_1 in the $+x$ direction, Block 2 (mass $6m$) slides at speed v_0 in the $-x$ direction; they collide and stick, and the combined system moves afterward at constant speed v_0. Using conservation of momentum (a multistep derivation) [Point B1]: $\sum \vec{p}_0 = \sum \vec{p}_f$. The final momentum magnitude of the combined $7m$ system is $7mv_0$ [Point B2]. $mv_1 + 6m(-v_0) = 7m(v_0) \Rightarrow mv_1 = 13mv_0 \Rightarrow v_1 = 13v_0$ [Point B3: correct final expression for v_1 in terms of v_0].

Question 1

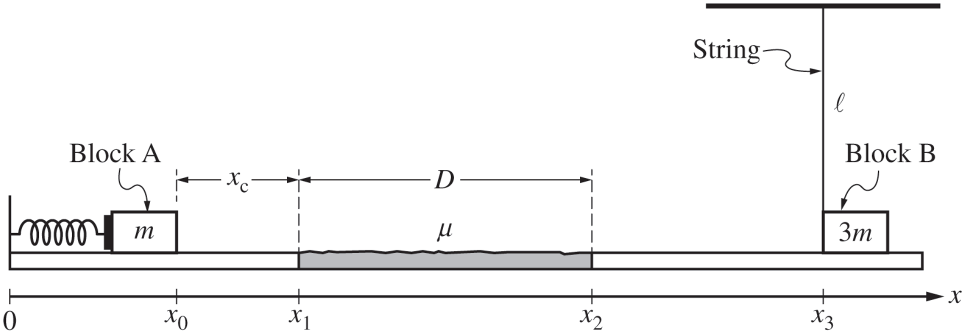
2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.
[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

Question 1

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.
- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.
(a)(i) Derive an expression for the speed v of Block A at time t_1.
(a)(ii) Derive an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

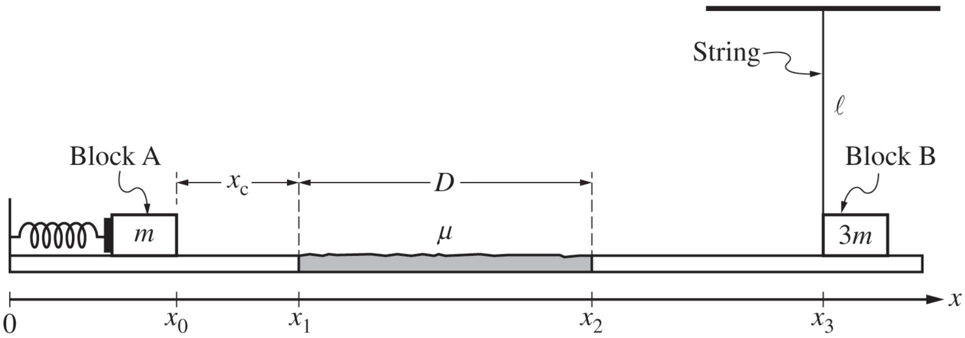
Question 1



Note: Figure not drawn to scale.

Figure 1

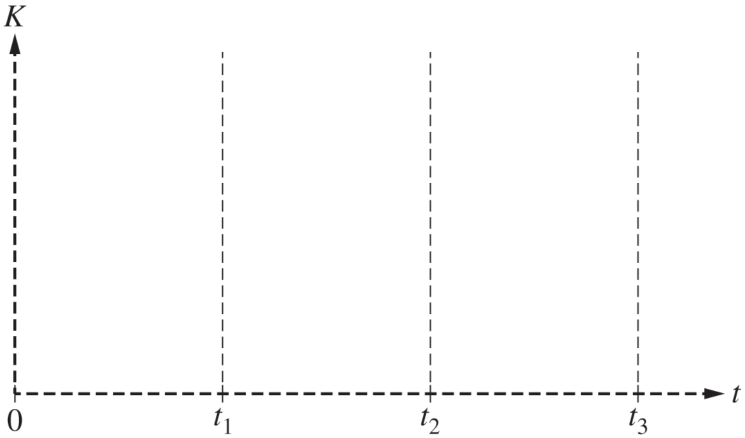
Question 1



Note: Figure not drawn to scale.

Figure 1

Question 1



Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.
[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]
At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure 1 repeated: same horizontal setup with Block A (mass m) against the spring, distance x_c, friction region of length D with coefficient μ between x_1 and x_2, and Block B (mass $3m$) hanging by a string of length ℓ at x_3. Positions marked $0, x_0, x_1, x_2, x_3$ along axis x. Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to t_3.

Question 1

[Blank graph: vertical axis K, horizontal axis t with gridlines/tick marks at t_1, t_2, t_3 (origin at 0); axes given with no curve drawn — student is to sketch the curve.]

(b)(ii) Use principles of work and energy to justify the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(c) [Figure 2: The two-block system (combined mass $m + 3m$) hangs from a string of length ℓ attached to a fixed support at the top. The string makes an angle θ_{max} with the vertical (dashed vertical reference line shown), with the block system displaced to one side at time t_4, having swung from its position at time t_3 (shown hanging straight down). Note: Figure not drawn to scale.]

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in

Question 1

Figure 2. For times $t > t_4$, the system oscillates with frequency f_i. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ. Indicate how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_i from the original procedure.

______ $f_{2\ell} > f_i$ ______ $f_{2\ell} < f_i$ ______ $f_{2\ell} = f_i$

Briefly justify your answer.

Solution 1

(a)(i)

Mark scheme

- Apply conservation of mechanical energy: all of the initial energy is stored as spring potential energy U_s, so $E_{initial} = E_{final}$ gives $\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$. Solving for v: $v = x_c\sqrt{k/m}$. Point 1: multi-step energy derivation that identifies the initial energy as entirely spring PE U_s. Point 2: correct algebraic solution for v.

Solution 1

(a)(ii) Mark scheme

- Step 1 (energy or kinematics to get the speed v_2 at x_2, after friction acts over distance D): Energy method — $K_{x_1} - \Delta E_{friction} = K_{before\ collision}$: $\frac{1}{2}m\left(x_c\sqrt{k/m}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2 \Rightarrow v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$. (Equivalently by kinematics: $a = -\mu g$ from $-\mu mg = ma$, then $v_2^2 = v^2 + 2aD = \left(x_c\sqrt{k/m}\right)^2 - 2\mu gD$, same result.) Step 2 (perfectly inelastic collision, conservation of momentum): $\Sigma p_{before} = \Sigma p_{after}$: $mv_2 = (m+3m)v_{A,B} = 4m\,v_{A,B}$. Solving: $v_{A,B} = \frac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m} = \frac{1}{4}\sqrt{\frac{kx_c^2}{m} - 2\mu gD}$. Points: (1) energy or kinematics

Solution 1

application to find speed at x_2; (2) correct expression for friction energy loss or deceleration ($\Delta E_{friction} = \mu mgD$ or $a = -\mu g$); (3) attempt at conservation of momentum $m_{initial}v_{initial} = m_{A,B}v_{A,B}$; (4) correctly substituting the consistent speed and masses into the momentum equation.

Solution 1

(b)(i) Mark scheme

- Sketch: $K = 0$ at $t = 0$; the curve rises nonlinearly and increases throughout $0 \leq t \leq t_1$, reaching a maximum at t_1 (when Block A leaves the spring). From t_1 to t_2 (the friction region) the curve decreases but does NOT reach zero, and is concave up (a constant friction deceleration makes v decrease linearly in time, so $K = \frac{1}{2}mv^2$ decreases at a decreasing rate). From t_2 to t_3 (after leaving the friction region, coasting to the collision) the curve is a horizontal line at a constant value greater than zero, and the whole curve from t_1 to t_3 is continuous (no jump at t_2). Points: (1) nonlinear sketch beginning at zero and increasing throughout $0 \leq t \leq t_1$; (2) decreasing throughout $t_1 \leq t \leq t_2$ without reaching

Solution 1

zero; (3) concave up on $t_1 \leq t \leq t_2$; (4) a continuous function on $t_1 \leq t \leq t_3$ that is a horizontal line greater than zero on $t_2 \leq t \leq t_3$.

Solution 1

(b)(ii) Mark scheme

- From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate (it decreases as the spring relaxes), the kinetic energy of the block does not increase at a constant rate — hence the nonlinear shape. Points: (1) a statement about the change in KE consistent with the graph (KE increasing on this interval); (2) a correct explanation for why KE is increasing (positive work done on the block / mechanical energy conserved as spring PE converts to KE); (3) a correct explanation for why the graph is

Solution 1

nonlinear (the spring force — and hence the rate of work done — changes as the block moves, so KE does not increase at a constant rate).

Solution 1

(c)

Mark scheme

- Select $f_{2\ell} < f_\ell$ (doubling the pendulum length decreases its frequency). Justification: the period of a pendulum is $T = 2\pi\sqrt{\ell/g}$. As the length increases, the period increases. Because frequency and period are inversely related ($f = 1/T$), an increase in period results in a decrease in frequency, so $f_{2\ell} < f_\ell$. Points: (1) selecting $f_{2\ell} < f_\ell$ with an attempt at a relevant justification; (2) correctly applying the pendulum period/frequency-length relationship $T = 2\pi\sqrt{\ell/g}$.

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2.

Question 2

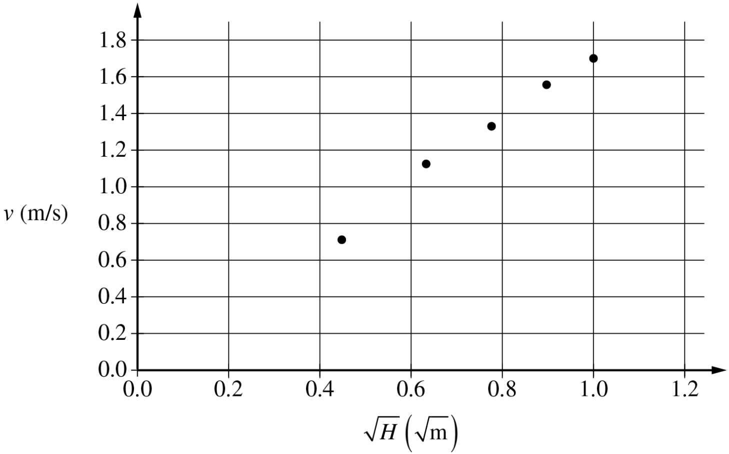
After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

______ Greater than　　______ Less than　　______ Equal to

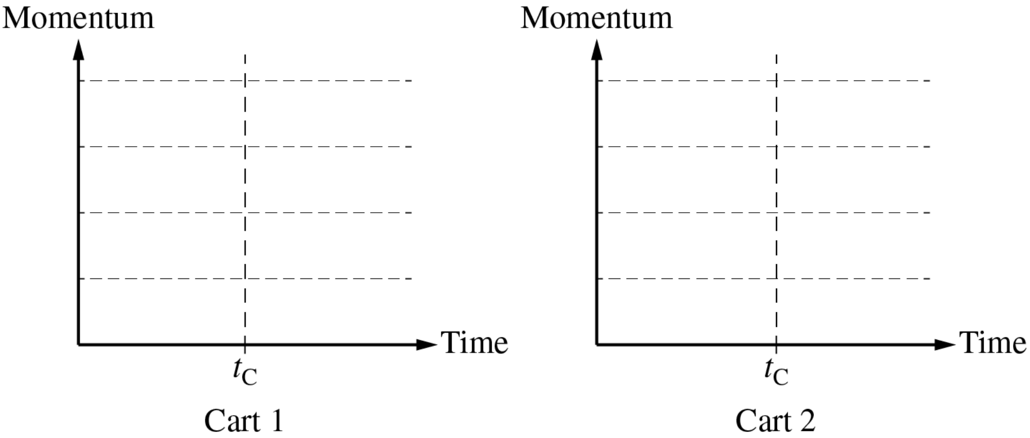
Justify your answer.

Question 2

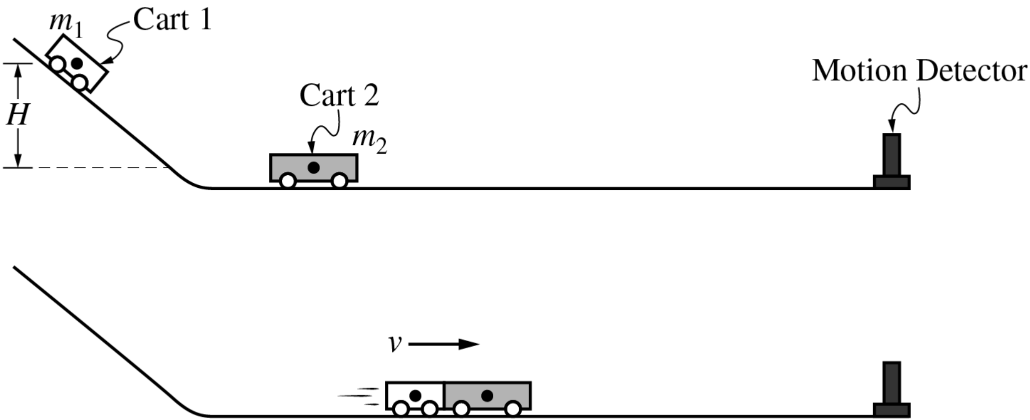


1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
v (m/s)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$\sqrt{H}\left(\sqrt{\mathrm{m}}\right)$

Question 2



Question 2



Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

[Graph 1, titled "Cart 1": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at t_C on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]

[Graph 2, titled "Cart 2": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at t_C on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(d)(i) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.

[Graph: scatter plot; vertical axis "v (m/s)" ranges from 0.0 to 1.8 in increments of 0.2; horizontal axis "$\sqrt{H}$ ($\sqrt{\text{m}}$)" ranges from 0.0 to 1.2 in increments of 0.2; plotted points approximately at $(0.45, 0.75)$, $(0.65, 1.05)$, $(0.65, 1.25)$, $(0.85, 1.35)$, $(1.0, 1.55)$, $(1.0, 1.65)$, showing a roughly linear increasing trend; no line drawn yet.]

Draw a line that represents the best fit to the data points shown.

(d)(ii) Use the best-fit line to calculate the mass of Cart 2.

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

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Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg?

$\mathrm{m}_1' > 0.250$ kg $\quad$ $\mathrm{m}_1' < 0.250$ kg $\quad$ $\mathrm{m}_1' = 0.250$ kg

Justify your answer.

Solution 2

(a)

Mark scheme

- Answer: Equal to (1 point for selecting this with an attempted justification). Justification: By Newton's third law, the force Cart 2 exerts on Cart 1 and the force Cart 1 exerts on Cart 2 are equal in magnitude (and opposite in direction) at every instant during the collision; since both carts experience the collision over the same time interval, the impulses ($\int F\,dt$) on each cart must be equal in magnitude (1 point).

Solution 2

(b) Mark scheme

- Momentum (p) vs. time (t) graphs, before/after t_C (collision assumed instantaneous). Cart 1: for $0 < t < t_C$, a straight line increasing linearly from zero (Cart 1 accelerates down the incline); at t_C its momentum drops abruptly to a smaller constant (horizontal) value for $t > t_C$ (1 point for the $0 < t < t_C$ portion: linear increasing for Cart 1 and zero for Cart 2; 1 point for Cart 1's post-collision horizontal line being smaller in magnitude than its momentum at $t = t_C$). Cart 2: zero (flat on the axis) for $0 < t < t_C$, then jumps at t_C to a constant (horizontal) value for $t > t_C$ that is GREATER in magnitude than Cart 1's post-collision momentum (1 point). The magnitude of Cart 1's momentum loss at the collision

Solution 2

must equal the magnitude of Cart 2’s momentum gain (equal and opposite changes), consistent with the equal-impulse answer in part (a) (1 point).

Solution 2

(c) Mark scheme

- Derivation of $v = \sqrt{2g}\left(\dfrac{m_1}{m_1 + m_2}\right)\sqrt{H}$. Conservation of energy (or equivalently kinematics) gives Cart 1's speed at the bottom of the incline: $m_1 gH = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gH}$ (1 point). Conservation of momentum for the perfectly inelastic collision: $m_1 v_1 = (m_1 + m_2)v \Rightarrow v = \dfrac{m_1}{m_1 + m_2} v_1$ (1 point). Combining: $v = \dfrac{m_1}{m_1 + m_2}\sqrt{2gH} = \sqrt{2g}\left(\dfrac{m_1}{m_1 + m_2}\right)\sqrt{H}$, as required (1 point).

Solution 2

(d)(i)

Mark scheme

- Draw a straight best-fit line through the v vs. $\sqrt{H}$ scatter plot with approximately equal numbers of data points above and below the line (through or near the origin).

Solution 2

(d)(ii) Mark scheme

- Use the best-fit line to find m_2. Calculate the slope using two well-separated points on the best-fit line, e.g. slope $= \dfrac{(1.20-0.60)\ \text{m/s}}{(0.70-0.35)\sqrt{\text{m}}} = 1.72\ \sqrt{\text{m}}/\text{s}$ (1 point). Relate the slope to the masses via the part (c) result, $slope = \dfrac{m_1}{m_1+m_2}\sqrt{2g}$, and solve for m_2: $m_2 = \dfrac{m_1\sqrt{2g}}{slope} - m_1$ (1 point). Substituting $m_1 = 0.250$ kg and $g = 9.8\ \text{m/s}^2$: $m_2 = \dfrac{(0.25\ \text{kg})\sqrt{2(9.8)}}{1.72} - 0.25\ \text{kg} \approx 0.39\ \text{kg}$ (1 point). Any value in the range 0.30–0.60 kg is acceptable given a valid best-fit slope.

Solution 2

(e)

Mark scheme

- Answer: $m_1' < 0.250$ kg (1 point for selecting this with an attempted justification). Justification: The directly-measured mass of Cart 2 came out smaller than the value calculated in part (d). A smaller true m_2 corresponds to a smaller initial energy/momentum for the same drop height H; since the best-fit slope (and hence the computed relationship) stays the same, this discrepancy is explained if the actual mass of Cart 1 is smaller than the recorded 0.250 kg — i.e., $m_1' < 0.250$ kg (1 point).

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 $= 3M$", "Mass of Block 2 $= M$". Note: Figure not drawn to scale.]

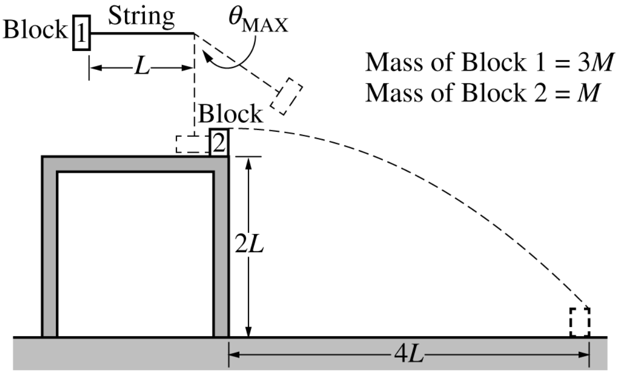
A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a

Question 2

distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

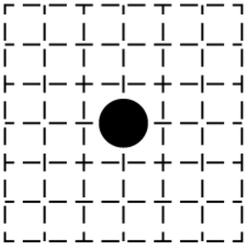
Question 2



Mass of Block 1 = $3M$
Mass of Block 2 = M

Note: Figure not drawn to scale.

Question 2



Question 2

2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.

[A dot on a dashed grid, representing block 1, for the student to draw force vectors on.]

Question 2

2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

Question 2

2019 Set 1 ▪ Q2

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Question 2

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(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

Question 2

2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(e) Calculate the speed of block 2 as it leaves the table.

Question 2

2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(f) Calculate the speed of block 1 just after it collides with block 2.

Question 2

2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

Solution 2

(a)

Mark scheme

- Energy conservation for block 1's swing:
  $U_{g1} = K_2 \Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow gL = \frac{1}{2}v^2 \Rightarrow v = \sqrt{2gL}$. Full credit even if no work is shown.

Solution 2

(b)

Mark scheme

- Draw two vectors from the dot: the weight F_W pointing straight down, and the string tension F_T pointing straight up, with the tension arrow drawn noticeably LONGER than the weight arrow (net force must be upward/centripetal). 1 pt for correctly drawing and labeling both the weight and the tension; 1 pt for drawing the tension longer than the weight. Only a maximum of 1 point can be earned if any extraneous vectors are included.

Solution 2

(c)

Mark scheme

- Use Newton's second law along the string direction, tension minus weight equals centripetal force (1 pt):
$F_T = mg + ma_c = mg + \frac{mv^2}{r} = 3Mg + \frac{(3M)(\sqrt{2gL})^2}{L}$. Correct final answer (1 pt): $F_T = 9Mg$.

Solution 2

(d) Mark scheme

- Use a correct kinematic equation for the vertical fall from table height $2L$ (1 pt): $\Delta y = v_i t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{4(0.75\text{ m})}{9.8\text{ m/s}^2}}$. Correct final answer (1 pt): $t = 0.55$ s.

Solution 2

(e)

Mark scheme

- Use a correct kinematic equation for the horizontal (projectile) motion (1 pt): $\Delta x = vt \Rightarrow v = \dfrac{\Delta x}{t} = \dfrac{4L}{t}$. Substitute the time from part (d) (1 pt): $v = \dfrac{(4)(0.75\ \text{m})}{0.55\ \text{s}} = 5.45\ \text{m/s}$.

Solution 2

(f) Mark scheme

- Use conservation of momentum for the collision (1 pt): $p_i = p_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$. Correctly substitute the masses (1 pt): $(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$. Substitute the speeds found in parts (a) and (e) (1 pt): $v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{3M} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{3M}$, giving $v_{1f} = \dfrac{3\sqrt{2(9.8)(0.75)} - (1)(5.45)}{3} = 2.02$ m/s (or $v_{1f} = 2.06$ m/s using $g = 10$ m/s^2).

Solution 2

(g) Mark scheme

- Use conservation of energy for block 1's rise after the collision (1 pt): $K_1 = U_{g2} \Rightarrow \frac{1}{2}mv^2 = mgh$. Correctly substitute $h = L(1 - \cos\theta)$ (1 pt): $\frac{1}{2}mv^2 = mgL(1 - \cos\theta) \Rightarrow \dfrac{v^2}{2gL} = 1 - \cos\theta$. Substitute the speed found in part (f) (1 pt): $\theta = \cos^{-1}\left(1 - \dfrac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \dfrac{(2.02)^2}{2(9.8)(0.75)}\right) = 43.5^\circ$ (or $\theta = 44.1^\circ$ using $g = 10$ m/s^2).

Question 2

2018 ▪ Q2

Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200 \text{ N/s}^2\right) t^2 - (500 \text{ N/s})t$.

[Diagram: Cart 1 (mass $m = 0.500$ kg) with a Force Sensor attached at its right side, moving right with $v_0 = 3.00$ m/s toward a spring ($k = 130$ N/m) connected to Cart 2 (mass $m = 1.05$ kg, initially at rest, $v = 0$), all on a horizontal track.]

Question 2

[Graph: Force F (N) on the vertical axis (marked at 5, 0, -5, -10, -15, -20, -25) versus time t (s) on the horizontal axis (marked 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18). Data points start near $F = 0$ at small t, dip down to a minimum of about -20 N around $t = 0.08$–0.10 s, then rise back up toward $F = 0$ by about $t = 0.16$–0.18 s, tracing a roughly parabolic (U-shaped, inverted) curve consistent with the quadratic fit given.]

(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)(i) Calculate the speed of cart 1 after the collision.

(b)(ii) In which direction does cart 1 move after the collision?

______ Left ______ Right

______ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)(i) Calculate the speed of cart 2 after the collision.

Question 2

(c)(ii) Show that the collision between the two carts is elastic.
(d)(i) Calculate the speed of the center of mass of the two-cart-spring system.
(d)(ii) Calculate the maximum elastic potential energy stored in the spring.

Question 2

a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are sho

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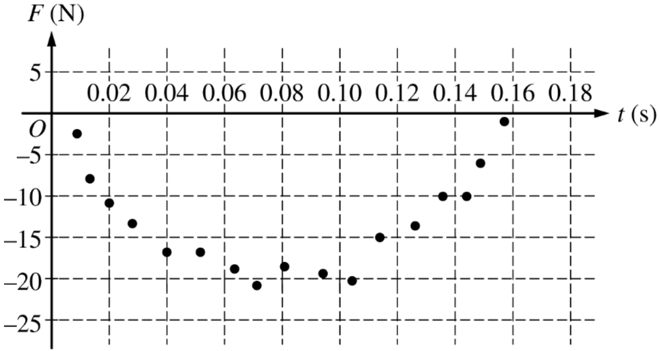
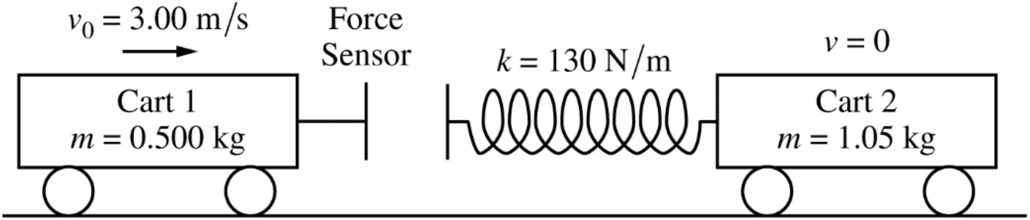


Question 2



Solution 2

(a)

Mark scheme

- $J = \int F\,dt = \int_0^{0.16} (3200t^2 - 500t)\,dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_0^{0.16} = \frac{3200}{3}(0.16)^3 - 250(0.16)^2 = -2.03$ N·s. 1 point for using the given force equation in the impulse integral; 1 point for integrating with correct limits (or constant of integration); 1 point for the correct final numeric answer $J = -2.03$ N·s.

Solution 2

(b)(i)

Mark scheme

- $J = m_1(v_{1f} - v_{1i}) \;\Rightarrow\; v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{-2.03\ \mathrm{N\cdot s}}{0.500\ \mathrm{kg}} + 3.00\ \mathrm{m/s} = -1.06\ \mathrm{m/s}.$

Solution 2

(b)(ii)

Mark scheme

- Left. Since $v_{1f} = -1.06$ m/s is negative (opposite the original direction of motion), cart 1 moves to the left after the collision.

Solution 2

(c)(i) Mark scheme

- By Newton's third law on the spring force, $-J = -m_1(v_{1f} - v_{1i}) = m_2 v_{2f} \Rightarrow v_{2f} = \frac{-J}{m_2} = \frac{2.03\ \mathrm{N \cdot s}}{1.05\ \mathrm{kg}} = 1.93\ \mathrm{m/s}$. (Equivalent alternate via momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \Rightarrow v_{2f} = \frac{m_1(v_{1i} - v_{1f})}{m_2} = \frac{(0.500)(3.00 - (-1.06))}{1.05} = 1.93$ m/s.) 1 point for a correct equation to find v_{2f}; 1 point for correct substitution.

Solution 2

(c)(ii)

Mark scheme

- An elastic collision requires initial KE = final KE: $K_{ii} = K_{1f} + K_{2f}$. Check: $\frac{1}{2}(0.500)(3.00)^2 = \frac{1}{2}(0.500)(-1.06)^2 + \frac{1}{2}(1.05)(1.93)^2 \Rightarrow 2.25$ J ≈ 2.24 J — equal within rounding, so the collision is elastic. 1 point for indicating that initial and final kinetic energies must be equal; 1 point for correct substitution/numeric verification.

Solution 2

(d)(i)

Mark scheme

- $m_1 v_{1i} = (m_1 + m_2)v_{cm} \;\Rightarrow\; v_{cm} = \dfrac{m_1 v_{1i}}{m_1 + m_2} = \dfrac{(0.500\text{ kg})(3.00\text{ m/s})}{0.500\text{ kg} + 1.05\text{ kg}} = 0.97$ m/s. 1 point for the correct equation for the center-of-mass speed; 1 point for correct substitution.

Solution 2

(d)(ii) Mark scheme

- Using conservation of energy: $K_i = K_f + U_{Sf} \Rightarrow U_{Sf} = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}^2 = \frac{1}{2}(0.500)(3.00)^2 - \frac{1}{2}(1.55)(0.97)^2 = 1.52$ J. (Alternate: from $dF/dt = 0$, $t = 0.078$ s gives $|F_{max}| = 19.5$ N (accept 20-21 N from the graph); $x_{max} = F_{max}/k = 0.15$ m (accept 0.15-0.16 m); $U_S = \frac{1}{2}kx_{max}^2 = 1.46$ J, accept 1.46-1.70 J if estimated from the graph.) Scoring: 1 point for using conservation of energy to find the max elastic PE; 1 point for using the center-of-mass speed for the system's translational KE; 1 point for correct substitution/final numeric answer. Q2 also carries an overall 1-point "units" credit (correct units on all calculated answers through the question) — folded into this final part so Q2's 15 points are fully accounted for.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

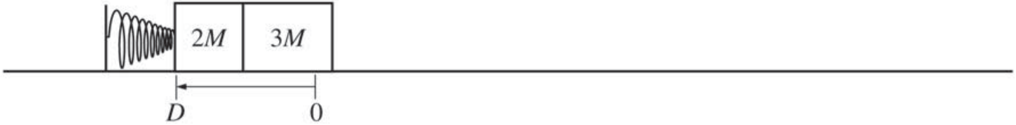
A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Question 2

[Two dots are shown side by side, labeled "Block of Mass $2M$" (left dot) and "Block of Mass $3M$" (right dot), for the student to draw force vectors on.]
[Figure: The spring (coil) anchored at the wall on the left is now attached to a combined block labeled "2M" next to "3M" (the two blocks shown adjacent, having moved together). A distance D is marked from the wall/spring's compressed end to a reference position "0".]
The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

Question 2



Question 2

d pointing away from, the appropriate dot.

Block of Mass $2M$ | Block of Mass $3M$

Question 2

Question 2

2016 ▪ Q2

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(b) Derive an expression for the speed of the blocks immediately after the collision.

Question 2

2016 ▪ Q2

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(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

Question 2

2016 ▪ Q2

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(d) Derive an expression for the maximum distance D that the spring is compressed.

Question 2

2016 ▪ Q2

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A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(e)(i) In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

_____ Left _____ Right _____ The net force on the block of mass $2M$ is zero.

Question 2

Justify your answer.

(e)(ii) Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

_____ The magnitude of the net force is greater on the block of mass $2M$. _____ The magnitude of the net force is greater on the block of mass $3M$. _____ The magnitude of the net force on each block has the same nonzero value. _____ The magnitude of the net force on each block is zero.

Justify your answer.

Question 2

2016 ▪ Q2

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A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

______ Yes   ______ No

Justify your answer.

Solution 2

(a)

Mark scheme

- Draw two force diagrams (blocks not yet touching the spring, frictionless surface): on the $2M$ block, normal force F_{N1} up and weight $2Mg$ down; on the $3M$ block, normal force F_{N2} up and weight $3Mg$ down (distinct symbols since the two blocks have different masses/normal forces). 1 pt for correctly drawing/labeling the vectors on the $2M$ block; 1 pt for correctly drawing/labeling the vectors on the $3M$ block, using different, physically correct symbols. Note: a maximum of 1 of the 2 points can be earned if any extraneous vectors are drawn.

Solution 2

(b)

Mark scheme

- The $3M$ block moving at v_0 collides and sticks to the stationary $2M$ block (perfectly inelastic). Conservation of momentum: $p_1 = p_2 \Rightarrow 3Mv_0 = (3M + 2M)v_f$. 1 pt for correctly substituting into the momentum equation; 1 pt for the correct final answer $v_f = \frac{3}{5}v_0$.

Solution 2

(c)

Mark scheme

- Kinetic energy of the combined system: $K = \frac{1}{2}mv^2 = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2 = \frac{9}{10}Mv_0^2$. 1 pt for an answer consistent with the speed found in part (b).

Solution 2

(d) Mark scheme

- By conservation of energy, $\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$, so $K_0 = U_{\text{final}}$: the kinetic energy right after the collision converts entirely to spring potential energy at maximum compression. With nonlinear spring force $F = Bx^3$: $\frac{9}{10}Mv_0^2 = \int_0^D Bx^3\,dx = \left[\frac{Bx^4}{4}\right]_0^D = \frac{BD^4}{4}$. Solving: $D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$. 1 pt for a correct conservation-of-energy expression ($\Delta K + \Delta U = 0$, $K_0 = U_{\text{final}}$); 1 pt for attempting to integrate the spring-force equation; 1 pt for using correct limits of integration (0 to D) or an appropriate constant of integration; 1 pt for a final answer consistent with the speed from (b) or the kinetic energy from (c). Correct answer: $D = \sqrt[4]{18Mv_0^2/(5B)}$.

Solution 2

(e)(i)

Mark scheme

- Answer: Right. At maximum compression the two-block system is instantaneously at rest, but the (still-compressed) spring continues to exert a rightward force on the system, so the block of mass $2M$ has a net force/acceleration to the right at that instant. 1 pt for selecting 'Right' with a reasonable justification attempt (no credit for the justification if the wrong choice is made); 1 pt for indicating that at maximum compression the $2M$ block has a rightward acceleration, due either to the forces acting on it directly or to the external spring force on the system.

Solution 2

(e)(ii)

Mark scheme

- Answer: The magnitude of the net force is greater on the block of mass $3M$. Because the blocks are stuck together they share the same acceleration; since $F = ma$, the block with more mass ($3M$) experiences the greater net force at that same acceleration. 1 pt for indicating both blocks have the same acceleration; 1 pt for a correct justification that the net force is greater on the $3M$ block (no credit if the wrong block is selected).

Solution 2

(f) Mark scheme

- Answer: No. Although the spring exerts a restoring force on the stuck-together blocks, the spring is nonlinear ($F = -Bx^3$, not $F = -kx$), so the force is not proportional to displacement from equilibrium. SHM requires a linear restoring force ($a = -\frac{k}{m}\Delta x$), so the blocks do not exhibit simple harmonic motion. 1 pt for selecting 'No' with a reasonable justification attempt (the selection point is not earned if 'No' is chosen with no justification; no credit for the justification if the wrong answer is selected); 1 pt for indicating the spring's force is not linear and that SHM specifically requires a linear restoring force.

Question 2

2015 ▪ Q2

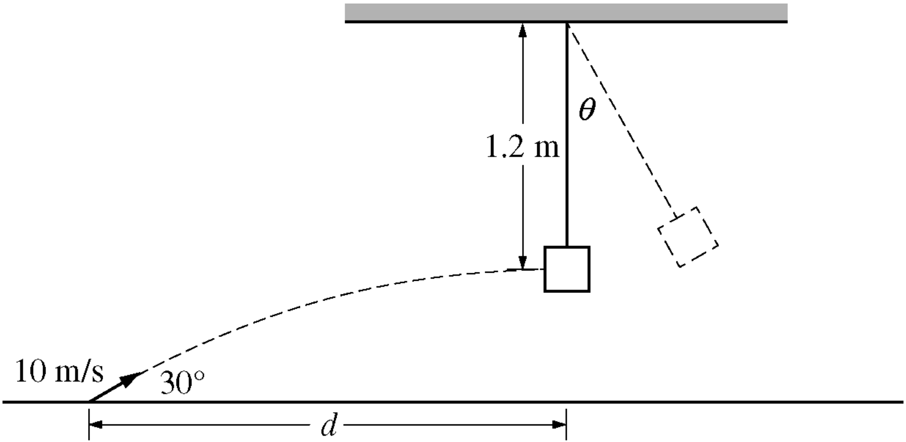
A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible. [Diagram: A ceiling (hatched bar) with a string of length 1.2 m hanging straight down to a small square block. A dashed line at angle θ from the vertical string shows the block's swung-up position (dashed square), with θ marked at the pivot between the vertical string and the dashed string. Below, a dart is launched from the ground at 10 m/s at 30° above the horizontal (arrow labeled "10 m/s" with angle "30°" marked from the horizontal ground line); a dashed curved trajectory arcs up from the launch

Question 2

point to the block, reaching the block moving horizontally at the top of its arc. A horizontal distance d is marked along the ground from the dart's launch point to the point on the floor directly below the block.]

(a) Determine the speed of the dart just before it strikes the block.

Question 2



Question 2

2015 ▪ Q2

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Question 2

moving horizontally at the top of its arc. A horizontal distance d is marked along the ground from the dart's launch point to the point on the floor directly below the block.]

(b) Calculate the horizontal distance d between the launching point of the dart and a point on the floor directly below the block.

Question 2

2015 ▪ Q2

A small dart of mass 0.020 kg is launched at an angle of 30° above the horizontal with an initial speed of 10 m/s. At the moment it reaches the highest point in its path and is moving horizontally, it collides with and sticks to a wooden block of mass 0.10 kg that is suspended at the end of a massless string. The center of mass of the block is 1.2 m below the pivot point of the string. The block and dart then swing up until the string makes an angle θ with the vertical, as shown above. Air resistance is negligible.

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Question 2

moving horizontally at the top of its arc. A horizontal distance d is marked along the ground from the dart's launch point to the point on the floor directly below the block.]

(c) Calculate the speed of the block just after the dart strikes.

Question 2

2015 ▪ Q2

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Question 2

moving horizontally at the top of its arc. A horizontal distance d is marked along the ground from the dart's launch point to the point on the floor directly below the block.]

(d) Calculate the angle θ through which the dart and block on the string will rise before coming momentarily to rest.

Question 2

2015 ▪ Q2

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Question 2

moving horizontally at the top of its arc. A horizontal distance d is marked along the ground from the dart's launch point to the point on the floor directly below the block.]

(e) The block then continues to swing as a simple pendulum. Calculate the time between when the dart collides with the block and when the block first returns to its original position.

Question 2

2015 ▪ Q2

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Question 2

moving horizontally at the top of its arc. A horizontal distance d is marked along the ground from the dart's launch point to the point on the floor directly below the block.]

(f) In a second experiment, a dart with more mass is launched at the same speed and angle. The dart collides with and sticks to the same wooden block.

(f)(i) Would the angle θ that the dart and block swing to increase, decrease, or stay the same?

$I^{ncrease}_{D^{ecrease}_{S^{taythesame}}}$

Justify your answer.

(f)(ii) Would the period of oscillation after the collision increase, decrease, or stay the same?

_______ Increase _______ Decrease _______ Stay the same

Justify your answer.