Past-paper walk-through

Conservation of Energy ▪ 3.4

eXercise

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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

The following information applies to parts A and B.

A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

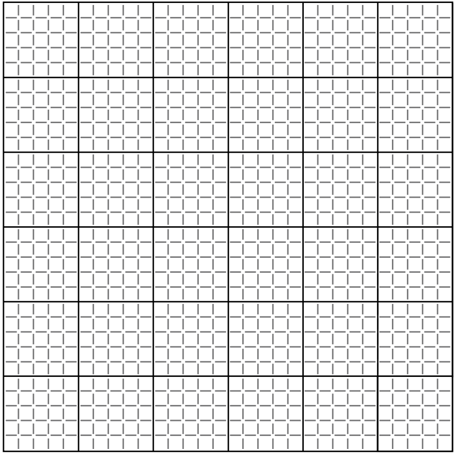
[Figure 1: A horizontal diagram with a $+x$ axis arrow pointing right at the top. A spring with spring constant k is attached to a wall on the left and connects to a block labeled m. To the right of the block (for $x > 0$), the surface is marked with a squiggly friction symbol and labeled μ_k. Below the block is the label $x = 0$.]

A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

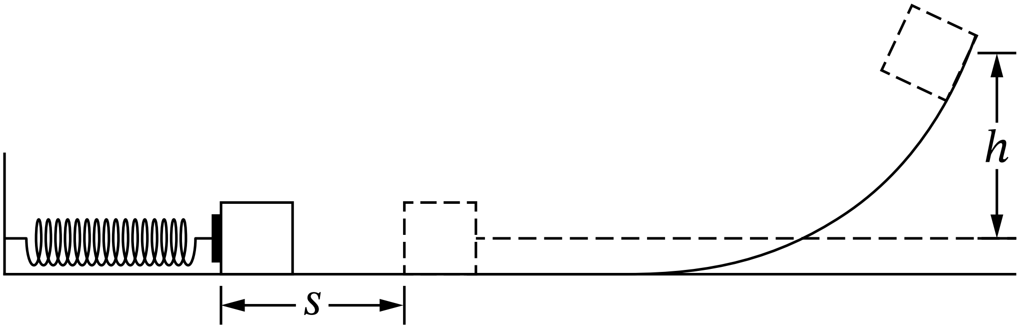
Question 3

(a)(i) Indicate quantities that could be measured by the student that would allow them to determine μ_k using a graph.
(a)(ii) Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

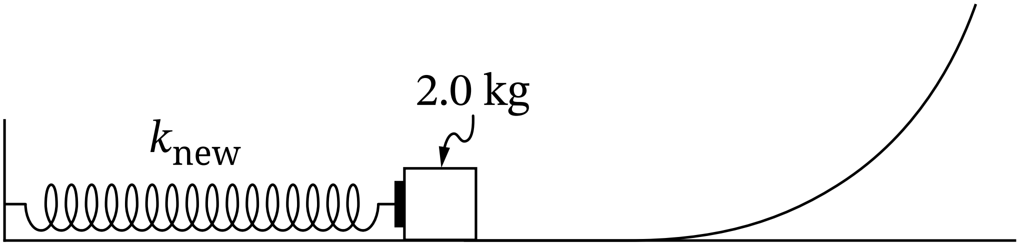
Question 3



Question 3



Question 3



Question 3

2026 ▪ Q3

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A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Question 3

(b)(i) Indicate what quantities the student could graph on the horizontal and vertical axes to create a graph that can be used to determine μ_k.

(b)(ii) Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

Question 3

2026 ▪ Q3

The following information applies to parts A and B.

A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

[Figure 1: A horizontal diagram with a $+x$ axis arrow pointing right at the top. A spring with spring constant k is attached to a wall on the left and connects to a block labeled m. To the right of the block (for $x > 0$), the surface is marked with a squiggly friction symbol and labeled μ_k. Below the block is the label $x = 0$.]

A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

(c)(i)(n)(t)(r)(o) The following information applies to parts C and D.

Question 3

In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass 2.0 kg is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.

[Figure 2: A diagram showing a spring labeled k_{new} attached to a wall on the left, connected to a block labeled "2.0 kg" with an arrow "f" curving above it. The horizontal surface to the right of the block slopes upward into a curved ramp rising to the upper right.]

The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.

[Figure 3: A diagram showing the spring compressed, attached to the wall on the left and to the block, with a distance s marked by a double-arrow between the wall-side

Question 3

spring end and the block's initial (uncompressed) dashed outline position. To the right, a dashed outline of the block is shown partway up the curved ramp at height h above the horizontal surface level, with h marked by a vertical double-arrow.]

The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

s (m)	h (m)
0.020	0.02
0.040	0.04
0.060	0.13
0.080	0.18
0.100	0.33

(c)(i) **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine k_{new}.

(c)(ii) On the grid provided, create a graph of the quantities indicated in part C (i).

Question 3

- Clearly **label** the vertical and horizontal axes with numerical scales.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

[Figure: Two blank labeled-axis lines reading “Quantity (units, if appropriate)” each with two blank underlines for axis labels, flanking a blank square grid for plotting, printed twice (before and after the grid) for the vertical and horizontal axes.]

(c)(iii) Draw a best-fit line for the data graphed in part C (ii).

Question 3

2026 ▪ Q3

The following information applies to parts A and B.

A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

[Figure 1: A horizontal diagram with a $+x$ axis arrow pointing right at the top. A spring with spring constant k is attached to a wall on the left and connects to a block labeled m. To the right of the block (for $x > 0$), the surface is marked with a squiggly friction symbol and labeled μ_k. Below the block is the label $x = 0$.]

A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Question 3

(d) Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for k_{new}.

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.
[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]
The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.
[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis

Question 2

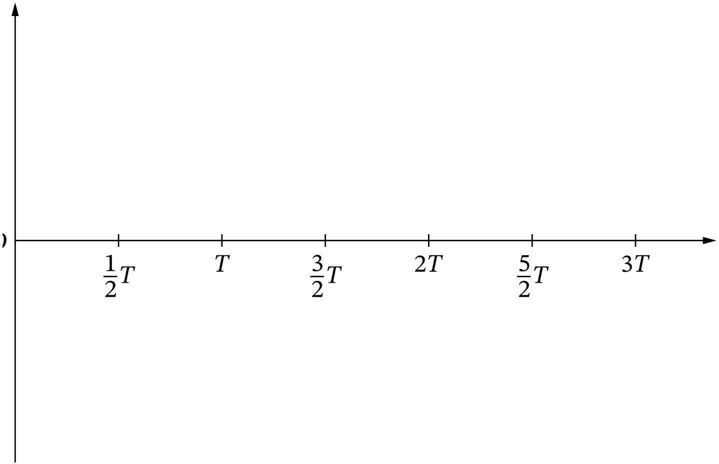
is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

(a) An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

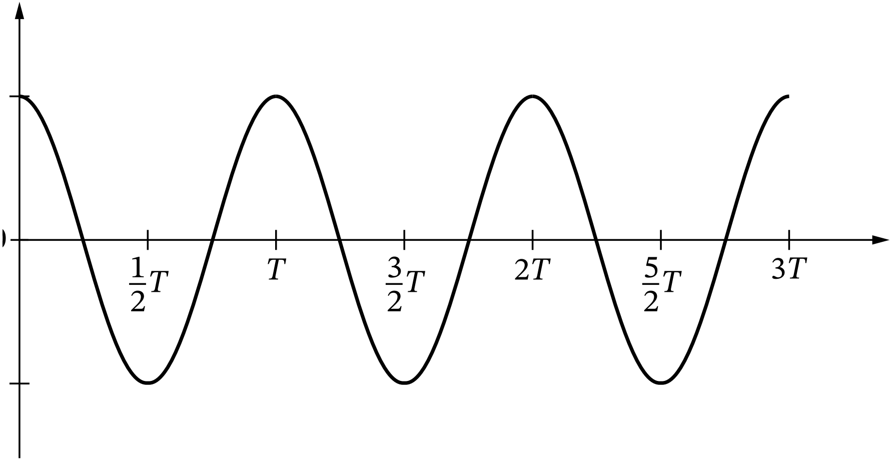
- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}. - Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

[Figure 3: A blank bar-chart grid titled "Energy" (vertical axis) with three labeled columns: U_A, U_B, K_{block}, and a horizontal "0" line partway up the grid, with gridlines above and below for plotting bar heights.]

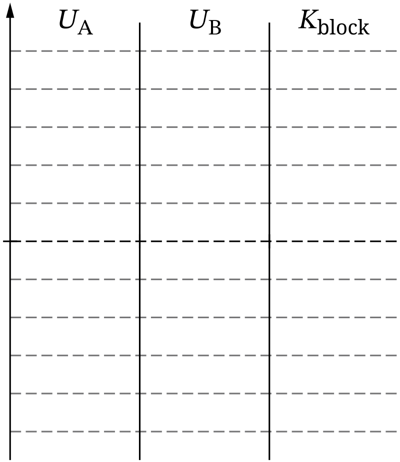
Question 2



Question 2



Question 2



Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(b) The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(c)(i) In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.
[Figure 4: Graph titled "Scenario 1" of position x vs. time t. Vertical axis x with values $+x_1$ marked above 0 and $-x_1$ marked below 0; horizontal axis t with gridmarks at $\frac{1}{2}T$, T, $\frac{3}{2}T$, $2T$, $\frac{5}{2}T$, $3T$. The curve is a cosine-like oscillation: starts at $+x_1$ at $t = 0$, crosses zero at $\frac{1}{2}T$ going down to $-x_1$ at around $\frac{1}{2}T$ to T, back up through zero, reaching $+x_1$ again at T, and repeating this pattern periodically through $3T$.]
In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.
On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.

Question 2

[Figure 5: Blank axes titled "Scenario 2" with vertical axis K and horizontal axis t with gridmarks at $\frac{1}{2}T$, T, $\frac{3}{2}T$, $2T$, $\frac{5}{2}T$, $3T$, origin labeled O.]

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(d) In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Solution 2

(a) Mark scheme

- Energy bar chart at $x = x_1$ (Figure 3), where the block is momentarily at rest (released from rest at x_1, so all mechanical energy is stored in the two springs): the K_{block} bar has zero height [Point A1]. Both U_A and U_B bars are drawn with positive height, since elastic PE stored in a spring is never negative [Point A2]. Spring B has spring constant $2k$ (twice Spring A's k) and both springs share displacement x_1: $U_A = \frac{1}{2}kx_1^2$ and $U_B = \frac{1}{2}(2k)x_1^2 = 2U_A$, so the U_B bar is drawn exactly twice the height of the U_A bar (this point is awarded on the height ratio regardless of which sign/direction the bars are drawn in) [Point A3].

Solution 2

(b) Mark scheme

- Derive v at $x = \frac{1}{2}x_1$ using energy conservation (an equivalent SHM method is also accepted) [Point B1: multistep derivation that includes energy conservation or simple harmonic motion]. The system behaves as a single effective spring combining both springs acting on the block together, $k_{eff} = k + 2k = 3k$ [Point B2: relates the presence of both springs to the system's behavior]. Initial state at $x = x_1$: $E_{tot,0} = U_A(x_1) + U_B(x_1) + K_{block}(x_1) = \frac{1}{2}kx_1^2 + \frac{1}{2}(2k)x_1^2 + 0 = \frac{3}{2}kx_1^2$. Final state at $x = \frac{1}{2}x_1$ [Point B3: relates positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block]: $E_{tot,f} = \frac{1}{2}k\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{block,\frac{1}{2}x_1} = \frac{3}{8}kx_1^2 + K_{block,\frac{1}{2}x_1}$. Setting $E_{tot,0} = E_{tot,f}$:

Solution 2

$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{block,\frac{1}{2}x_1} \Rightarrow K_{block,\frac{1}{2}x_1} = \frac{9}{8}kx_1^2 \Rightarrow \frac{1}{2}mv^2 = \frac{9}{8}kx_1^2 \Rightarrow v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$ [Point B4: correct final expression for v in terms of m, k, x1]. (Equivalent SHM route: $\omega = \sqrt{3k/m}$, $x(t) = x_1 \cos\omega t$, solve $\cos\omega t = \frac{1}{2} \Rightarrow \omega t = \pi/3$, then $v = x_1\omega\sin(\pi/3) = \frac{3}{2}x_1\sqrt{k/m}$ – same result.)

Solution 2

(c)(i) Mark scheme

- Sketch K (kinetic energy of the block) vs. t for Scenario 2 (the block oscillating on the two-spring system WITH friction/damping), on axes marked at $\frac{1}{2}T, T, \frac{3}{2}T, 2T, \frac{5}{2}T, 3T$ (Figure 5), where T is the period from the frictionless Scenario 1. The block is released from rest at x_1, so $K = 0$ at $t = 0$, and the curve must start at zero and remain ≥ 0 for all t (kinetic energy cannot be negative) [Point C1]. Because the block passes through $x = 0$ (where K is maximum) twice per full oscillation period T and momentarily has $K = 0$ at each turning point (also twice per period T), the sketched curve should be periodic with zeros spaced $\frac{1}{2}T$ apart – i.e. humps peaking near $\frac{1}{4}T, \frac{3}{4}T, \frac{5}{4}T, \ldots$ and touching zero at $0, \frac{1}{2}T, T, \frac{3}{2}T, 2T, \ldots$ [Point C2]. Because friction dissipates

Solution 2

mechanical energy each cycle, the successive peak heights of K must decrease over time, i.e. the curve is a periodic wave of decreasing amplitude [Point C3].

Solution 2

(d) Mark scheme

- Scenario 3: the block is replaced with a new, LARGER-mass block (same coefficient of kinetic friction as Scenario 2). Compare the K vs. t graph of Scenario 3 to that of Scenario 2 (part C) and describe ONE feature that would differ, with a correct justification. Acceptable answers (state ONE, with matching reasoning): (1) 'The period increases' – because the period of the oscillator is $T = 2\pi\sqrt{m/k_{eff}}$, and increasing the mass m (with k_{eff} unchanged) increases T [Point D1 + Point D2]. (2) 'The rate at which the graph approaches zero amplitude increases' – because a larger mass under the same coefficient of friction experiences a larger friction force, so the block loses mechanical energy at a greater rate. (3) 'The maximum value of K over each period is less than the

Solution 2

graph drawn [in part C]' – a valid alternative feature with matching reasoning (more energy lost to friction per cycle). Award Point D1 for indicating any ONE of these three features, and Point D2 for a justification that correctly supports the specific feature chosen.

Question 3

2025 ▪ Q3

A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

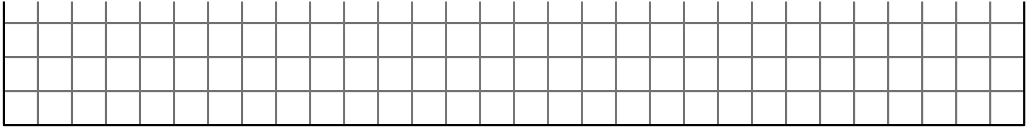
[Figure 1: A pivot at top connected by a rod to a box (containing a block) displaced to the left and up, with the center of mass of the block shown a vertical distance h above

Question 3

the box's lowest-point position (dashed horizontal line). A stopper is shown attached near the pivot.]
[Figure 2: The rod now hangs vertically straight down from the pivot, with the box (and block) at its lowest point, aligned with the dashed horizontal line.]
(a) Students are asked to experimentally determine the acceleration due to gravity g using a linear graph. To determine g, the students are permitted to use measurements from only a meterstick and the motion sensor.
Describe an experimental procedure using the described setup to collect data that would allow the students to determine an experimental value of g using a linear graph. Include any steps necessary to reduce experimental uncertainty.

Question 3

Question 3



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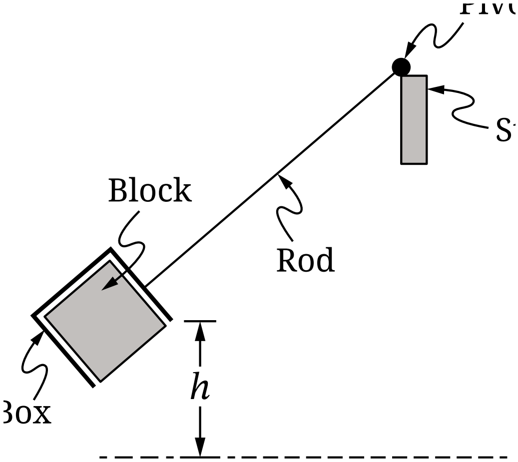


Question 3

h (m)	x_{max} (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

Question 3

2025 ▪ Q3

A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

[Figure 1: A pivot at top connected by a rod to a box (containing a block) displaced to the left and up, with the center of mass of the block shown a vertical distance h above the box's lowest-point position (dashed horizontal line). A stopper is shown attached near the pivot.]

Question 3

[Figure 2: The rod now hangs vertically straight down from the pivot, with the box (and block) at its lowest point, aligned with the dashed horizontal line.]

(b) Describe how the data collected in part A could be graphed and how that graph would be analyzed to determine the value of g.

Question 3

2025 ▪ Q3

A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.

The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.

[Figure 1: A pivot at top connected by a rod to a box (containing a block) displaced to the left and up, with the center of mass of the block shown a vertical distance h above the box's lowest-point position (dashed horizontal line). A stopper is shown attached near the pivot.]

Question 3

[Figure 2: The rod now hangs vertically straight down from the pivot, with the box (and block) at its lowest point, aligned with the dashed horizontal line.]

(c)(i) The experiment is repeated, but the horizontal surface on which the block slides is replaced with a new rough surface, as shown in Figure 3. The coefficient of kinetic friction between the block and the new surface is μ.

[Figure 3: A rod hanging vertically from a pivot with the box/block at its lowest point on a rough (wavy-line) horizontal surface at position $x = 0$; a dashed square outline is shown further right at position $x = x_{\max}$, indicating where the block comes to rest.]

The block-box system is pulled aside so that the center of mass of the block is displaced various vertical distances h and then released from rest. For each vertical distance, students measure the position $x = x_{\max}$ at which the block comes to rest. The students' measurements of h and $x_{\max}$ are shown in Table 1.

Table 1

Question 3

h (m)	$x_{\max}$ (m)
0.30	0.76
0.45	1.10
0.60	1.40
0.75	1.90
0.90	2.30

Indicate two quantities, either measured quantities from Table 1 or additional calculated quantities, that could be graphed to produce a straight line that could be used to determine μ.

Vertical axis:

Horizontal axis:

(c)(ii) On the grid provided, create a graph of the quantities indicated in part C (i).

- Use Table 2 to record the measured or calculated quantities that you will plot.
- Clearly **label** the axes, including units as appropriate.
- **Plot** the points you recorded in Table 2.

[A blank graphing grid (square-ruled) is provided for plotting the chosen quantities.]

(c)(iii) **Draw** a best-fit line to the data graphed in part C (ii).

Question 3

2025 ▪ Q3

A box is connected to one end of a rigid rod. Both the box and the rod have negligible mass. The other end of the rod is connected to a pivot. The box is open on one side, and a block is placed inside the box.
The center of mass of the block is displaced a vertical distance h, as shown in Figure 1. The block-box system is then released from rest and swings downward. There is negligible friction about the pivot. When the system is at the lowest point of its swing, the rod collides with a rigid stopper, as shown in Figure 2. The box comes to rest, and the block is launched horizontally out of the box. The block moves across a horizontal surface toward a motion sensor that measures the speed of the block. All frictional forces are negligible.
[Figure 1: A pivot at top connected by a rod to a box (containing a block) displaced to the left and up, with the center of mass of the block shown a vertical distance h above the box's lowest-point position (dashed horizontal line). A stopper is shown attached near the pivot.]

Question 3

[Figure 2: The rod now hangs vertically straight down from the pivot, with the box (and block) at its lowest point, aligned with the dashed horizontal line.]

(d) Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for μ.

Solution 3

(a) Mark scheme

- Describe an experimental procedure (using only a meterstick and a motion sensor) to collect data allowing determination of g from a linear graph, including a step to reduce experimental uncertainty. Model answer: measure the height h from which the block(-box) system is released (using the meterstick). Measure the resulting speed v of the block as it moves across the horizontal surface (using the motion sensor) [Point A1: procedure includes measuring h and the block's speed]. Repeat the speed measurement for multiple different release heights h (or repeat multiple trials at the same h and average) to reduce experimental/random uncertainty [Point A2: any reasonable uncertainty-reduction method, e.g. repeating trials at one h, or repeating across several values of h].

Solution 3

(b) Mark scheme

- Describe how the data from part A could be graphed/analyzed to determine g. Any of the following linear relationships is acceptable, along with their reciprocals: v^2 vs. $2h$ (slope $= g$); $\frac{1}{2}v^2$ vs. h (slope $= g$); v^2 vs. h (slope $= 2g$); v vs. $\sqrt{h}$ (slope $= \sqrt{2g}$) [Point B1: describes a graph with a linear trend usable to find g; may be earned independently of part A]. Model answer: plot v^2 on the vertical axis and $2h$ on the horizontal axis; the slope of the best-fit line equals g [Point B2: correctly relates the best-fit slope to g, per whichever pairing is chosen – see the graph/analysis table above for the correct slope-to-g relationship for each valid axis choice].

Solution 3

(c)(i)

Mark scheme

- New scenario: the horizontal surface is replaced by a rough surface with kinetic friction coefficient μ; the block (released from height h) slides and comes to rest at $x_{\max}$. Indicate quantities that could be plotted to produce a LINEAR graph usable to determine μ. Model answer: plot h vs. $x_{\max}$ (the reciprocal, $x_{\max}$ vs. h, is equally acceptable) [Point C1: any correct linear pairing of measurable quantities that yields μ from the slope, e.g. h vs. x$_max$].

Solution 3

(c)(ii) Mark scheme

- Graph the quantities identified in part C(i) using the data in Table 2. Label both axes, including units, with a linear scale – e.g. h (m) on the vertical axis and x_{max} (m) on the horizontal axis, with evenly-spaced gridlines from 0 upward [Point C2: correctly labeled linear-scale axes with units]. Plot each data point from Table 2 as a dot at its (x_{max}, h) coordinates on the grid, consistent with the axes chosen in part C(i) – the example data trend roughly linearly from about $(0.3 \text{ m}, 0.15 \text{ m})$ through $(1.4, 0.6)$, $(1.9, 0.75)$, up to about $(2.9 \text{ m}, 1.1 \text{ m})$ [Point C3: plotted points consistent with the part-C(i) quantities, Table 2's values, or the grid's own labeled axes].

Solution 3

(c)(iii)

Mark scheme

- Draw a best-fit straight line through the data points plotted in part C(ii) – a single straight line that approximates the overall increasing linear trend of the plotted (x_{max}, h) points (not simply connecting the dots), extending across the range of the data [Point C4: any reasonable line/curve that approximates the trend of the plotted data].

Solution 3

(d) Mark scheme

- Using the best-fit line from part C(iii), calculate an experimental value of μ. Derivation: by conservation of energy, $mgh - F_f x_{\max} = 0$ where $F_f = \mu mg$ (kinetic friction on a horizontal surface, normal force $= mg$), so $mgh = \mu mg\, x_{\max} \Rightarrow h = \mu x_{\max}$. Comparing to $y = mx + b$ with h vs. $x_{\max}$: the slope of the best-fit line equals μ directly [Point D1: correctly relates the best-fit slope to μ]. Reading two well-separated points off the best-fit line, e.g. $(0.70\ \text{m}, 0.30\ \text{m})$ and $(2.4\ \text{m}, 0.95\ \text{m})$: slope $= \dfrac{0.95 - 0.30}{2.4 - 0.70} = 0.38$, so $\mu = 0.38$ [Point D2: award for any value of μ within the acceptable range 0.30 to 0.50, since the exact value depends on the student's own graphed data/line].

Question 1

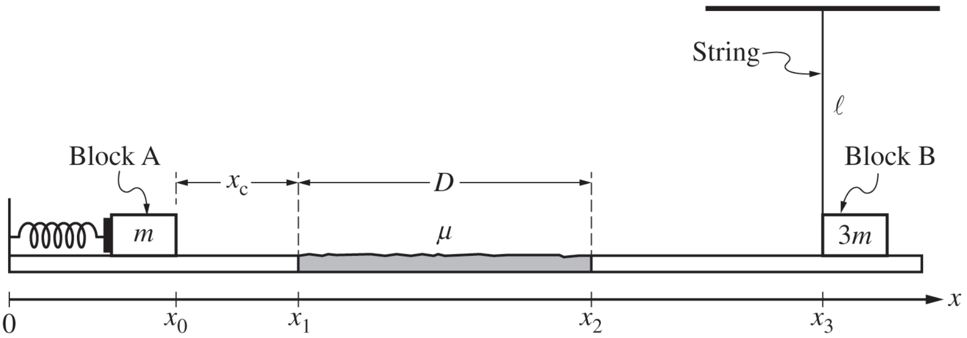
2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.
[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

Question 1

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.
- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.
(a)(i) Derive an expression for the speed v of Block A at time t_1.
(a)(ii) Derive an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

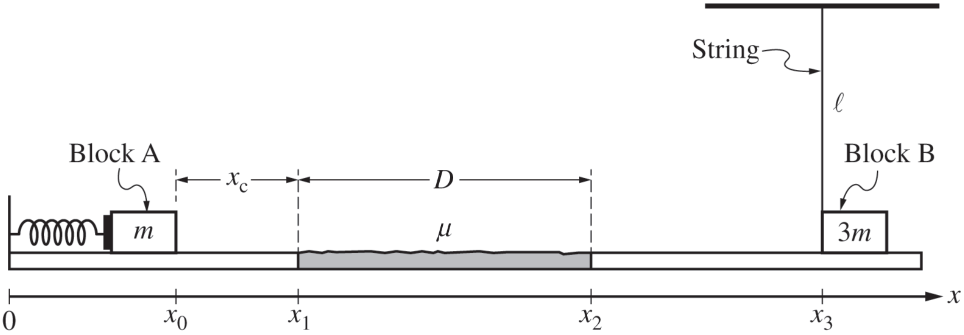
Question 1



Note: Figure not drawn to scale.

Figure 1

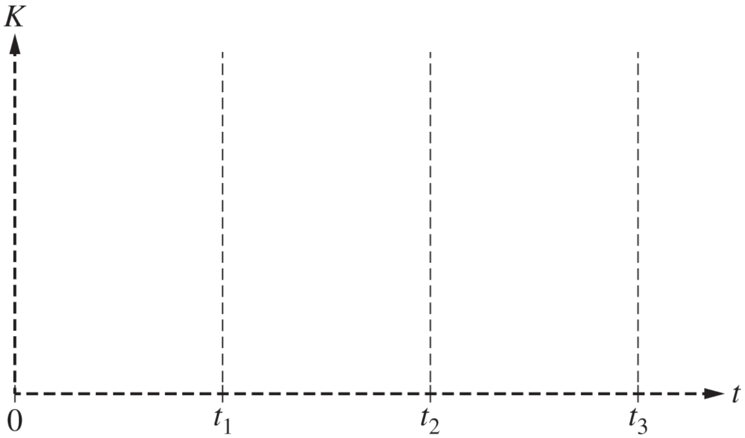
Question 1



Note: Figure not drawn to scale.

Figure 1

Question 1



Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure 1 repeated: same horizontal setup with Block A (mass m) against the spring, distance x_c, friction region of length D with coefficient μ between x_1 and x_2, and Block B (mass $3m$) hanging by a string of length ℓ at x_3. Positions marked $0, x_0, x_1, x_2, x_3$ along axis x. Note: Figure not drawn to scale.]
On the following axes, sketch a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to t_3.

Question 1

[Blank graph: vertical axis K, horizontal axis t with gridlines/tick marks at t_1, t_2, t_3 (origin at 0); axes given with no curve drawn — student is to sketch the curve.]

(b)(ii) Use principles of work and energy to justify the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.
[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]
At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(c) [Figure 2: The two-block system (combined mass $m + 3m$) hangs from a string of length ℓ attached to a fixed support at the top. The string makes an angle θ_{max} with the vertical (dashed vertical reference line shown), with the block system displaced to one side at time t_4, having swung from its position at time t_3 (shown hanging straight down). Note: Figure not drawn to scale.]

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in

Question 1

Figure 2. For times $t > t_4$, the system oscillates with frequency f_i. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ. Indicate how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_i from the original procedure.

______ $f_{2\ell} > f_i$ ______ $f_{2\ell} < f_i$ ______ $f_{2\ell} = f_i$

Briefly justify your answer.

Solution 1

(a)(i)

Mark scheme

- Apply conservation of mechanical energy: all of the initial energy is stored as spring potential energy U_s, so $E_{initial} = E_{final}$ gives $\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$. Solving for v: $v = x_c\sqrt{k/m}$. Point 1: multi-step energy derivation that identifies the initial energy as entirely spring PE U_s. Point 2: correct algebraic solution for v.

Solution 1

(a)(ii) Mark scheme

- Step 1 (energy or kinematics to get the speed v_2 at x_2, after friction acts over distance D): Energy method — $K_{x_1} - \Delta E_{friction} = K_{before\ collision}$: $\frac{1}{2}m\left(x_c\sqrt{k/m}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2 \Rightarrow v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$. (Equivalently by kinematics: $a = -\mu g$ from $-\mu mg = ma$, then $v_2^2 = v^2 + 2aD = \left(x_c\sqrt{k/m}\right)^2 - 2\mu gD$, same result.) Step 2 (perfectly inelastic collision, conservation of momentum): $\Sigma p_{before} = \Sigma p_{after}$: $mv_2 = (m+3m)v_{A,B} = 4m\,v_{A,B}$. Solving: $v_{A,B} = \dfrac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m} = \dfrac{1}{4}\sqrt{\dfrac{kx_c^2}{m} - 2\mu gD}$. Points: (1) energy or kinematics

Solution 1

application to find speed at x_2; (2) correct expression for friction energy loss or deceleration ($\Delta E_{friction} = \mu mgD$ or $a = -\mu g$); (3) attempt at conservation of momentum $m_{initial}v_{initial} = m_{A,B}v_{A,B}$; (4) correctly substituting the consistent speed and masses into the momentum equation.

Solution 1

(b)(i) Mark scheme

- Sketch: $K = 0$ at $t = 0$; the curve rises nonlinearly and increases throughout $0 \leq t \leq t_1$, reaching a maximum at t_1 (when Block A leaves the spring). From t_1 to t_2 (the friction region) the curve decreases but does NOT reach zero, and is concave up (a constant friction deceleration makes v decrease linearly in time, so $K = \frac{1}{2}mv^2$ decreases at a decreasing rate). From t_2 to t_3 (after leaving the friction region, coasting to the collision) the curve is a horizontal line at a constant value greater than zero, and the whole curve from t_1 to t_3 is continuous (no jump at t_2). Points: (1) nonlinear sketch beginning at zero and increasing throughout $0 \leq t \leq t_1$; (2) decreasing throughout $t_1 \leq t \leq t_2$ without reaching

Solution 1

zero; (3) concave up on $t_1 \leq t \leq t_2$; (4) a continuous function on $t_1 \leq t \leq t_3$ that is a horizontal line greater than zero on $t_2 \leq t \leq t_3$.

Solution 1

(b)(ii) Mark scheme

- From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate (it decreases as the spring relaxes), the kinetic energy of the block does not increase at a constant rate — hence the nonlinear shape. Points: (1) a statement about the change in KE consistent with the graph (KE increasing on this interval); (2) a correct explanation for why KE is increasing (positive work done on the block / mechanical energy conserved as spring PE converts to KE); (3) a correct explanation for why the graph is

Solution 1

nonlinear (the spring force — and hence the rate of work done — changes as the block moves, so KE does not increase at a constant rate).

Solution 1

(c)

Mark scheme

- Select $f_{2\ell} < f_\ell$ (doubling the pendulum length decreases its frequency). Justification: the period of a pendulum is $T = 2\pi\sqrt{\ell/g}$. As the length increases, the period increases. Because frequency and period are inversely related ($f = 1/T$), an increase in period results in a decrease in frequency, so $f_{2\ell} < f_\ell$. Points: (1) selecting $f_{2\ell} < f_\ell$ with an attempt at a relevant justification; (2) correctly applying the pendulum period/frequency-length relationship $T = 2\pi\sqrt{\ell/g}$.

Question 1

2024 Set 2 ▪ Q1

Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

[Figure: A ramp inclined at angle θ to a horizontal table. Block A (mass $2m$) sits on the ramp at a distance D up the incline, with the region of length D on the ramp labeled with friction coefficient μ. The ramp meets the table at the origin 0. On the horizontal table, Block B (mass m) rests at position x_1. Further right along the table, a spring of constant k is fixed to a wall, with its uncompressed (equilibrium) position at x_3; a distance x_c is marked from x_3 back toward the wall/spring, near the wall. The

Question 1

horizontal axis is labeled with positions 0, x_1, x_3 increasing to the right toward the wall.]

Note: Figure not drawn to scale.

Continue your response to QUESTION 1 on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

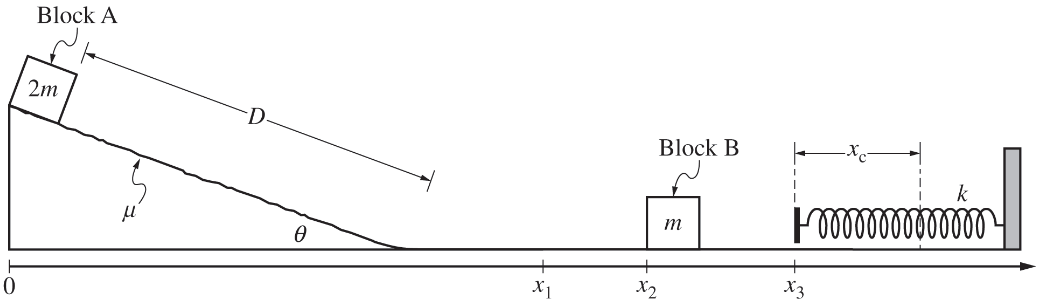
Question 1

For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

(a)(i) Derive an expression for the speed v of Block A at time t_1.

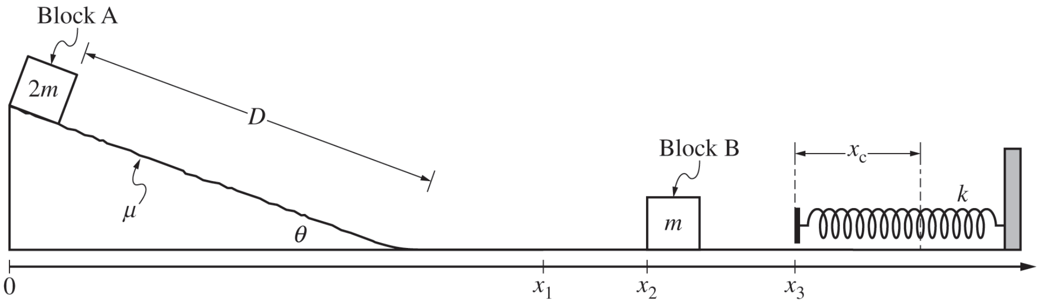
(a)(ii) Derive an expression for the spring constant k of the spring.

Question 1



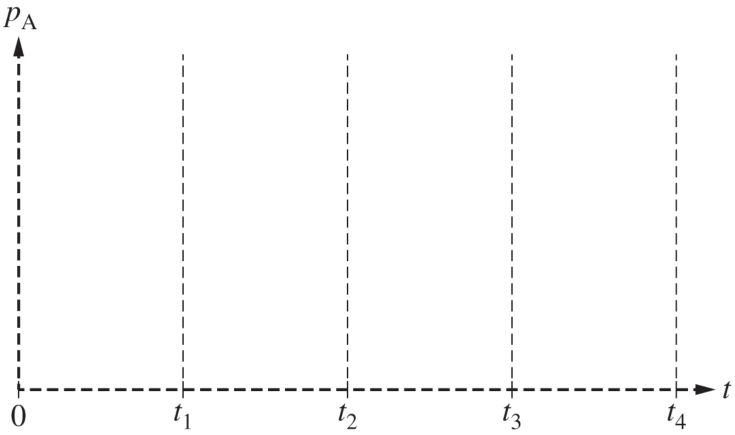
Note: Figure not drawn to scale.

Question 1



Note: Figure not drawn to scale.

Question 1



Question 1

2024 Set 2 ▪ Q1

Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

[Figure: A ramp inclined at angle θ to a horizontal table. Block A (mass $2m$) sits on the ramp at a distance D up the incline, with the region of length D on the ramp labeled with friction coefficient μ. The ramp meets the table at the origin 0. On the horizontal table, Block B (mass m) rests at position x_1. Further right along the table, a spring of constant k is fixed to a wall, with its uncompressed (equilibrium) position at x_3; a distance x_c is marked from x_3 back toward the wall/spring, near the wall. The horizontal axis is labeled with positions 0, x_1, x_3 increasing to the right toward the wall.]

Question 1

Note: Figure not drawn to scale.

Continue your response to QUESTION 1 on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure repeated: the same ramp-table-spring setup with Block A (mass $2m$) on the ramp, Block B (mass m) on the table, and the spring of constant k near the wall;

Question 1

axis positions 0, x_1, x_2, x_3 marked left to right, with x_c marked near the spring. Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4.

[Graph: vertical axis labeled p_A (unlabeled scale), horizontal axis labeled t with tick marks at 0, t_1, t_2, t_3, t_4 (unlabeled scale/blank grid for the student to draw on).]

(b)(ii) Use principles of forces to justify the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 2 ▪ Q1

Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

[Figure: A ramp inclined at angle θ to a horizontal table. Block A (mass $2m$) sits on the ramp at a distance D up the incline, with the region of length D on the ramp labeled with friction coefficient μ. The ramp meets the table at the origin 0. On the horizontal table, Block B (mass m) rests at position x_1. Further right along the table, a spring of constant k is fixed to a wall, with its uncompressed (equilibrium) position at x_3; a distance x_c is marked from x_3 back toward the wall/spring, near the wall. The horizontal axis is labeled with positions 0, x_1, x_3 increasing to the right toward the wall.]

Question 1

Note: Figure not drawn to scale.
Continue your response to QUESTION 1 on this page.
At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.
- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.
For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

Question 1

(c) For times $t > t_4$, the two-block-spring system oscillates with period T_O. The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

Indicate how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

____ $T_N > T_O$ ____ $T_N < T_O$ ____ $T_N = T_O$

Briefly justify your answer.

Solution 1

(a)(i) Mark scheme

- Using energy conservation for Block A sliding down the ramp (mass $2m$, ramp length D, angle θ, coefficient of friction μ): $U_g - \Delta E_{friction} = K$, i.e. $m_A g D \sin\theta - \mu m_A g D \cos\theta = \frac{1}{2} m_A v^2$, giving $v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$. (Equivalently via kinematics: Newton's second law gives $a = g\sin\theta - \mu g\cos\theta$, then $v^2 = v_0^2 + 2a\Delta x = 2(g\sin\theta - \mu g\cos\theta)D$.) Points: 1 for a correct multi-step derivation start (energy-conservation statement with U_g, or Newton's second law including the signed friction force), 1 for a consistent next step (friction-energy expression with correct sign, or substituting acceleration into a kinematics equation), 1 for the final correct expression $v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$.

Solution 1

(a)(ii) Mark scheme

- Momentum conservation for the collision between Block A (mass $2m$, speed v from part (a)(i)) and Block B (mass m, at rest): $2mv = (2m+m)v_{A,B}$, so $v_{A,B} = \frac{2}{3}v = \frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}$. Then energy conservation from just after the collision to maximum spring compression: $K_{after\ collision} = U_{s,max}$, i.e. $\frac{1}{2}(3m)v_{A,B}^2 = \frac{1}{2}kx_c^2$ (using that the combined blocks' speed just before contacting the spring equals $v_{A,B}$). Solving: $k = \frac{3m}{x_c^2}v_{A,B}^2 = \frac{8}{3}\frac{mgD(\sin\theta - \mu\cos\theta)}{x_c^2}$. Points: 1 for using momentum conservation to find $v_{A,B}$, 1 for equating post-collision kinetic energy to the maximum elastic potential energy of the spring

Solution 1

$(K_{after\ collision} = U_{s,max})$, 1 for indicating the speed just before the spring contact equals $v_{A,B}$.

Solution 1

(b)(i) Mark scheme

- Sketch of p_A (magnitude of Block A's momentum) vs. t from $t = 0$ to t_4, with four required features: (1) a straight line increasing from 0 at $t = 0$ to a value $p_1 = m_A v$ at $t = t_1$ (Block A accelerates down the ramp under the net gravity-minus-friction force); (2) a horizontal line from t_1 to t_2 at the same level p_1, continuous at t_1 (Block A crosses the frictionless table at constant velocity before the collision); (3) a horizontal line from t_2 to t_3 at a smaller constant level $p_2 = m_A v_{A,B} < p_1$ (the perfectly inelastic collision with Block B at t_2 reduces Block A's speed to $v_{A,B} = \frac{2}{3}v$, then A+B move together at constant velocity toward the spring); (4) a concave-down curve from t_3 to t_4, continuous with level p_2 at t_3, decreasing to 0 at t_4 (the spring's restoring force grows as compression

Solution 1

increases, decelerating the blocks at an increasing rate until they momentarily stop at maximum compression).

Solution 1

(b)(ii) Mark scheme

- For $t_3 \leq t \leq t_4$: the momentum of Block A decreases because the spring exerts a force on the blocks directed opposite to their velocity, decelerating them toward rest. As the spring compresses further, the spring force increases in magnitude, so the rate of decrease of momentum (the magnitude of the slope of the p_A-vs-t graph) increases with time – this is why the curve is concave down, becoming steeper as $t \to t_4$, consistent with the sketch in part (b)(i). Points: 1 for a statement about the change in momentum consistent with the (b)(i) graph, 1 for identifying that a decreasing graph means the net force on Block A is opposite its motion, 1 for explicitly relating the changing steepness of the curve to the increasing magnitude of the spring force.

Solution 1

(c)

Mark scheme

- $T_N = T_O$ (the period is unchanged). Justification: the period of a spring-block oscillator depends only on the oscillating mass and the spring constant ($T = 2\pi\sqrt{m/k}$), not on friction, amplitude, velocity, or compression distance. Removing friction on the new ramp changes only the compression distance/amplitude of the oscillation, not m or k, so the period stays the same.

Question 1

2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in

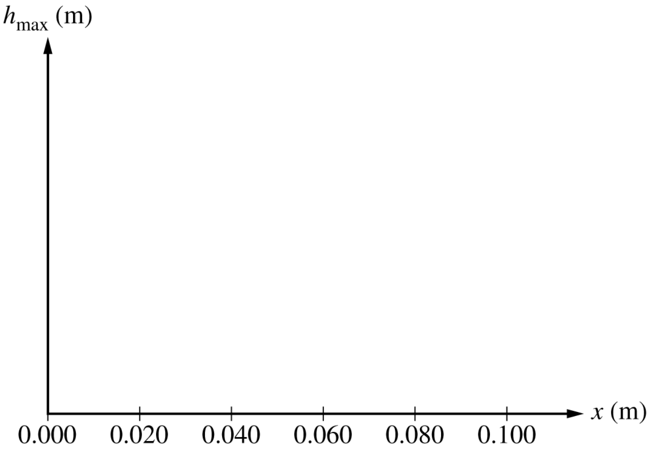
Question 1

Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]
(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010$ m. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following representations of the block in Trials 1 and 2, draw and label the forces (non-components) that are exerted on the block at the instant the block is released. Each force must be

Question 1

represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.
[Two grid diagrams are shown, each with a dot at the center representing the block: "Trial 1 (Spring P)" on the left and "Trial 2 (Spring Q)" on the right. Each grid is for the student to draw labeled force vectors.]

Question 1



Question 1

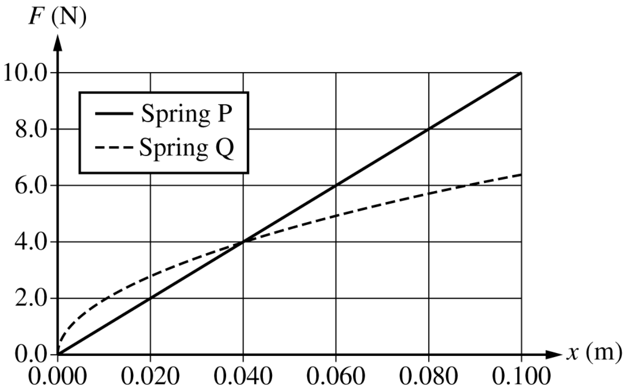
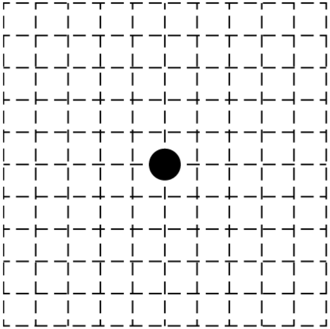


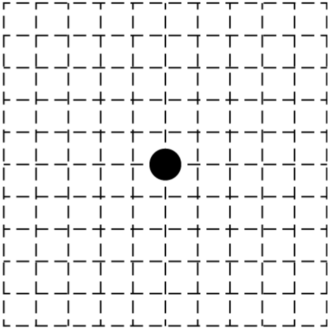
Figure 2

Question 1

Trial 1
(Spring P)



Trial 2
(Spring Q)



Question 1

2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.

[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

Question 1

straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(b)(i) What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

(b)(ii) There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

(b)(iii) On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance x each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.

Question 1

[Blank axes provided: y-axis labeled h_{max} (m), x-axis labeled x (m) from 0.000 to 0.100 in increments of 0.020, for the student to sketch two curves.]

Question 1

2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.
A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

Question 1

straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(c)(i) [Figure 3: Block A rests at the top of an inclined ramp of height H, connected by a string over the top of the ramp; Block B rests on the horizontal surface at the bottom of the ramp, connected to Spring Q which is attached to a wall, with $x = 0$ marked at the natural length position of the spring.]

Spring Q is attached to the wall and at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp on the horizontal surface. The change in vertical height of Block A is $H = 0.75$ m. The student releases Block A, and it moves down the ramp and collides with Block B. After the collision, the blocks stick together and move to the right,

Question 1

compressing the spring. Frictional forces between the blocks and all surfaces are negligible.
Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.
(c)(ii) Calculate the maximum compression of the spring.

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[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.
A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0\ \text{N/m}^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

Question 1

straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D?

______ $C < D$ ______ $C > D$ ______ $C = D$

Briefly justify your answer.

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

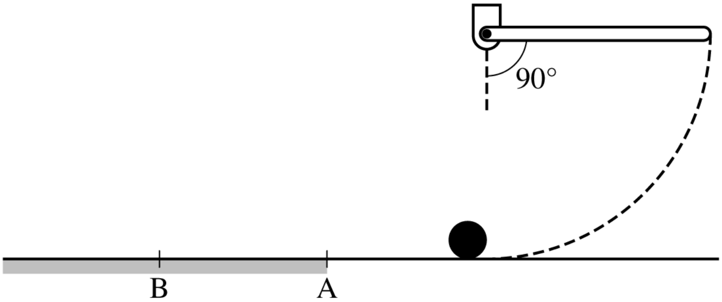
A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total

Question 3

rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

Question 3



Note: Figure not drawn to scale.

Figure 1

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate. After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

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2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.

[A circle is shown representing the sphere, for the student to draw and label force vectors starting on the circle.]

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(d)(i) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

The linear velocity v of the center of mass of the sphere as a function of time t

(d)(ii) The angular velocity ω of the sphere as a function of time t

Question 3

2023 Set 1 ▪ Q3

[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

Question 3

the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(e)(i) Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

(e)(ii) Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2.

Question 2

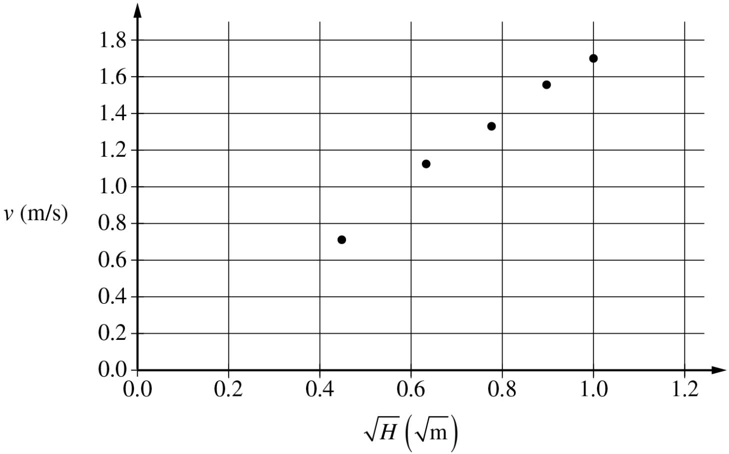
After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

______ Greater than   ______ Less than   ______ Equal to

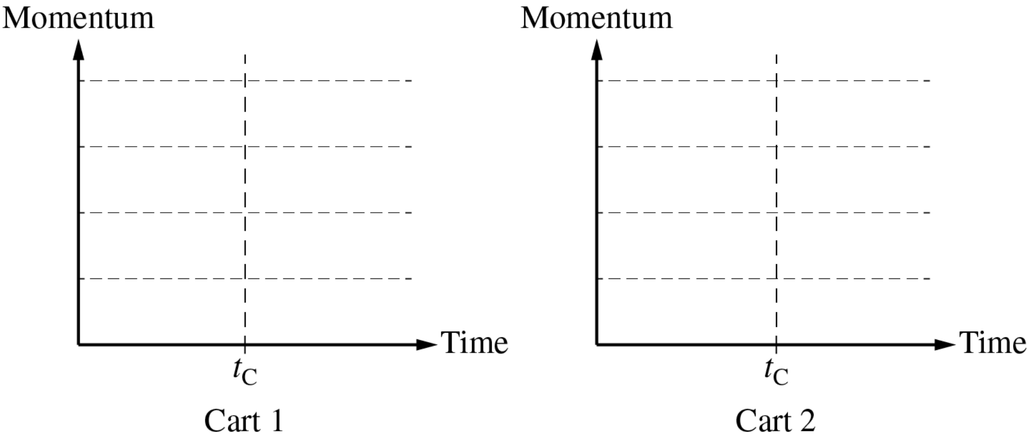
Justify your answer.

Question 2

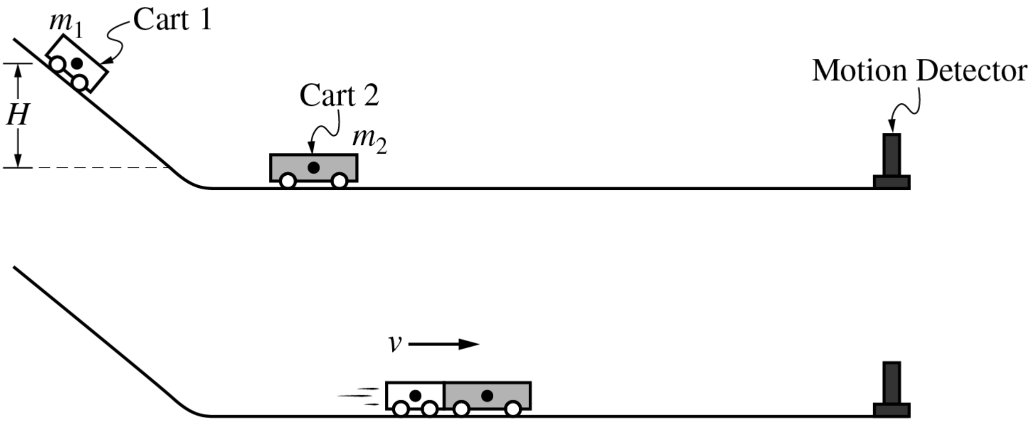


1.8
1.6
1.4
1.2
1.0
v (m/s)
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$\sqrt{H}\ (\sqrt{\mathrm{m}})$

Question 2



Question 2



Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

[Graph 1, titled "Cart 1": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at t_C on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]

[Graph 2, titled "Cart 2": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at t_C on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(c) Show that the velocity v of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

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incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(d)(i) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of $\sqrt{H}$, as shown below.

[Graph: scatter plot; vertical axis "v (m/s)" ranges from 0.0 to 1.8 in increments of 0.2; horizontal axis "$\sqrt{H}\,(\sqrt{\text{m}})$" ranges from 0.0 to 1.2 in increments of 0.2; plotted points approximately at $(0.45, 0.75)$, $(0.65, 1.05)$, $(0.65, 1.25)$, $(0.85, 1.35)$, $(1.0, 1.55)$, $(1.0, 1.65)$, showing a roughly linear increasing trend; no line drawn yet.]

Draw a line that represents the best fit to the data points shown.

(d)(ii) Use the best-fit line to calculate the mass of Cart 2.

Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass m_1 sits at the top of an incline (marked "Cart 1"), held at rest at a height H above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass m_2, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity v (arrow labeled "v" pointing right) toward the Motion Detector.]

Cart 1 of mass m_1 is held at rest above the bottom of the incline. Cart 2 has mass m_2, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance H, as shown. At time t_C, Cart 1 reaches the bottom of the

Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg?

$\mathrm{m}_1' > 0.250$ kg $\quad$ $\mathrm{m}_1' < 0.250$ kg $\quad$ $\mathrm{m}_1' = 0.250$ kg

Justify your answer.

Solution 2

(a)

Mark scheme

- Answer: Equal to (1 point for selecting this with an attempted justification). Justification: By Newton's third law, the force Cart 2 exerts on Cart 1 and the force Cart 1 exerts on Cart 2 are equal in magnitude (and opposite in direction) at every instant during the collision; since both carts experience the collision over the same time interval, the impulses ($\int F\,dt$) on each cart must be equal in magnitude (1 point).

Solution 2

(b) Mark scheme

- Momentum (p) vs. time (t) graphs, before/after t_C (collision assumed instantaneous). Cart 1: for $0 < t < t_C$, a straight line increasing linearly from zero (Cart 1 accelerates down the incline); at t_C its momentum drops abruptly to a smaller constant (horizontal) value for $t > t_C$ (1 point for the $0 < t < t_C$ portion: linear increasing for Cart 1 and zero for Cart 2; 1 point for Cart 1's post-collision horizontal line being smaller in magnitude than its momentum at $t = t_C$). Cart 2: zero (flat on the axis) for $0 < t < t_C$, then jumps at t_C to a constant (horizontal) value for $t > t_C$ that is GREATER in magnitude than Cart 1's post-collision momentum (1 point). The magnitude of Cart 1's momentum loss at the collision

Solution 2

must equal the magnitude of Cart 2's momentum gain (equal and opposite changes), consistent with the equal-impulse answer in part (a) (1 point).

Solution 2

(c) Mark scheme

- Derivation of $v = \sqrt{2g}\left(\frac{m_1}{m_1+m_2}\right)\sqrt{H}$. Conservation of energy (or equivalently kinematics) gives Cart 1's speed at the bottom of the incline: $m_1 gH = \frac{1}{2}m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gH}$ (1 point). Conservation of momentum for the perfectly inelastic collision: $m_1 v_1 = (m_1+m_2)v \Rightarrow v = \frac{m_1}{m_1+m_2}v_1$ (1 point). Combining: $v = \frac{m_1}{m_1+m_2}\sqrt{2gH} = \sqrt{2g}\left(\frac{m_1}{m_1+m_2}\right)\sqrt{H}$, as required (1 point).

Solution 2

(d)(i)

Mark scheme

- Draw a straight best-fit line through the v vs. $\sqrt{H}$ scatter plot with approximately equal numbers of data points above and below the line (through or near the origin).

Solution 2

(d)(ii) Mark scheme

- Use the best-fit line to find m_2. Calculate the slope using two well-separated points on the best-fit line, e.g. slope $= \dfrac{(1.20 - 0.60)\ \text{m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\ \sqrt{\text{m}}/\text{s}$ (1 point). Relate the slope to the masses via the part (c) result, $\mathit{slope} = \dfrac{m_1}{m_1 + m_2}\sqrt{2g}$, and solve for m_2: $m_2 = \dfrac{m_1\sqrt{2g}}{\mathit{slope}} - m_1$ (1 point). Substituting $m_1 = 0.250$ kg and $g = 9.8\ \text{m/s}^2$: $m_2 = \dfrac{(0.25\ \text{kg})\sqrt{2(9.8)}}{1.72} - 0.25\ \text{kg} \approx 0.39\ \text{kg}$ (1 point). Any value in the range 0.30–0.60 kg is acceptable given a valid best-fit slope.

Solution 2

(e)

Mark scheme

- Answer: $m_1' < 0.250$ kg (1 point for selecting this with an attempted justification). Justification: The directly-measured mass of Cart 2 came out smaller than the value calculated in part (d). A smaller true m_2 corresponds to a smaller initial energy/momentum for the same drop height H; since the best-fit slope (and hence the computed relationship) stays the same, this discrepancy is explained if the actual mass of Cart 1 is smaller than the recorded 0.250 kg — i.e., $m_1' < 0.250$ kg (1 point).

Question 1

2022 Set 2 ▪ Q1

A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.

[Figure 1: A sled on a horizontal table, moving right with initial velocity v_0. Positions $x = 0$ and $x = L$ are marked on the table top. Below the table, a string of vertical extent y connects down to a "Device" on the ground.]

Question 1

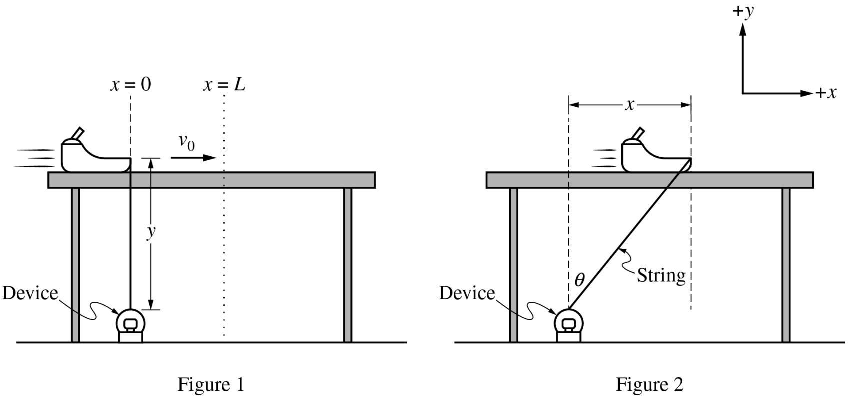
[Figure 2: Top-down/side view with axes $+y$ up and $+x$ to the right. The sled starts at the left, and a dashed line of horizontal length x extends from the sled's starting position to its current position. A "String" is shown connecting the sled diagonally down to the Device on the ground.]

Note: Figures not drawn to scale.

(a) On the dot below that represents the sled, draw and label the forces (not components) that are exerted on the sled a short time after $t = 0$ but before the sled has come to rest. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

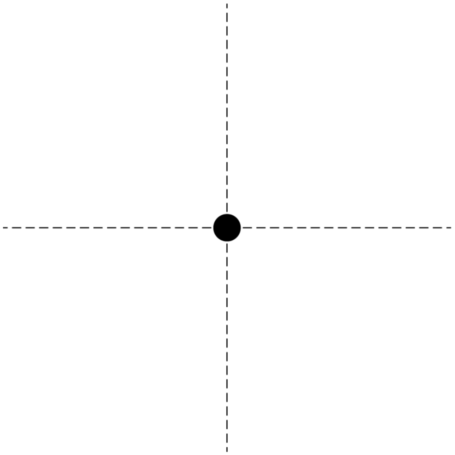
[A diagram showing a single dot with a horizontal dashed line through it and a vertical dashed line through it, crossing at the dot, for the student to draw force vectors on.]

Question 1



Note: Figures not drawn to scale.

Question 1



Question 1

2022 Set 2 ▪ Q1

A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.
[Figure 1: A sled on a horizontal table, moving right with initial velocity v_0. Positions $x = 0$ and $x = L$ are marked on the table top. Below the table, a string of vertical extent y connects down to a "Device" on the ground.]
[Figure 2: Top-down/side view with axes $+y$ up and $+x$ to the right. The sled starts at the left, and a dashed line of horizontal length x extends from the sled's starting position to its

Question 1

current position. A "String" is shown connecting the sled diagonally down to the Device on the ground.]

Note: Figures not drawn to scale.

(b) Determine an expression for the angle θ that the string makes with the vertical when the sled has traveled a horizontal distance x.

Question 1

2022 Set 2 ▪ Q1

A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.
[Figure 1: A sled on a horizontal table, moving right with initial velocity v_0. Positions $x = 0$ and $x = L$ are marked on the table top. Below the table, a string of vertical extent y connects down to a "Device" on the ground.]
[Figure 2: Top-down/side view with axes $+y$ up and $+x$ to the right. The sled starts at the left, and a dashed line of horizontal length x extends from the sled's starting position to its

Question 1

current position. A "String" is shown connecting the sled diagonally down to the Device on the ground.]

Note: Figures not drawn to scale.

(c)(i) Derive an expression for the normal force F_N exerted on the sled by the table as a function of the position x.

(c)(ii) Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the sled as a function of the position x.

Question 1

2022 Set 2 ▪ Q1

A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.
[Figure 1: A sled on a horizontal table, moving right with initial velocity v_0. Positions $x = 0$ and $x = L$ are marked on the table top. Below the table, a string of vertical extent y connects down to a "Device" on the ground.]
[Figure 2: Top-down/side view with axes $+y$ up and $+x$ to the right. The sled starts at the left, and a dashed line of horizontal length x extends from the sled's starting position to its

Question 1

current position. A "String" is shown connecting the sled diagonally down to the Device on the ground.]

Note: Figures not drawn to scale.

(d) Derive an expression for the work W done by the string on the sled as the sled moves from $x = 0$ to $x = L$.

Question 1

2022 Set 2 ▪ Q1

A small sled slides across a rough horizontal table with an initial velocity v_0. The coefficient of kinetic friction between the sled and the table is μ_k. A string connects the sled to a device on the ground. The device maintains constant tension F_T in the string by unwinding the string as the sled slides to the right. The total mass of the sled is m. The string is attached to the device at $x = 0$ and at a height of y, as shown in Figure 1. The horizontal position of the sled is represented by x, as shown in Figure 2. Express all algebraic answers in terms of m, μ_k, F_T, x, y, and physical constants, as appropriate.
[Figure 1: A sled on a horizontal table, moving right with initial velocity v_0. Positions $x = 0$ and $x = L$ are marked on the table top. Below the table, a string of vertical extent y connects down to a "Device" on the ground.]
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Question 1

current position. A "String" is shown connecting the sled diagonally down to the Device on the ground.]

Note: Figures not drawn to scale.

(e) The sled comes to rest after traveling a horizontal distance $x = 2L$. As the system slides from $x = 0$ to $x = L$, the energy dissipated by friction is E_1. As the sled slides from $x = L$ to $x = 2L$, the energy dissipated by friction is E_2. Is E_1 greater than, less than, or equal to E_2 ?

____ $E_1 > E_2$ ____ $E_1 < E_2$ ____ $E_1 = E_2$

Justify your answer.

Solution 1

(a) Mark scheme

- Free-body diagram at the dot: draw three distinct force arrows starting at the dot. (1) Force of gravity F_g (or mg, F_G, W, "grav force" — not "G"/"g" alone) pointing straight down, AND the normal force F_N (or F_n, N, "normal force") pointing straight up — 1 point together. (2) Force of friction F_f pointing horizontally, opposite the sled's direction of motion (the sled is decelerating) — 1 point. (3) Force of tension F_T (or F_s, $F_{Tension}$, T, "string force") pointing up and backward along the string toward its attachment point (diagonal, not purely vertical or horizontal) — 1 point. Each force must be a distinct labeled arrow starting on and pointing away from the dot; components alone do not earn credit. A response with extraneous forces/vectors caps at 2 points.

Solution 1

(b) Mark scheme

- Using the right triangle formed by the string's vertical rise y and horizontal displacement x, any correct trig expression for θ earns the point: $\sin\theta = \dfrac{x}{\sqrt{y^2+x^2}} \Rightarrow \theta = \sin^{-1}\left(\dfrac{x}{\sqrt{y^2+x^2}}\right)$, equivalently $\theta = \cos^{-1}\dfrac{y}{\sqrt{y^2+x^2}}$ or $\theta = \tan^{-1}\dfrac{x}{y}$.

Solution 1

(c)(i)

Mark scheme

- Start from Newton's second law in the vertical direction (must explicitly begin with $\Sigma F = ma$, consistent with part (a)'s forces) — 1 point: $\Sigma F = ma \Rightarrow F_N - F_{T,y} - F_g = 0 \Rightarrow F_N = F_{T,y} + F_g$. Then substitute the vertical component of tension using part (b)'s geometry — 1 point:

  $$F_N = mg + F_T \cos\theta = mg + F_T \left(\frac{y}{\sqrt{y^2 + x^2}} \right).$$

Solution 1

(c)(ii) Mark scheme

- Sum forces in the horizontal direction, consistent with parts (a) and (b) — 1 point: $F_{net} = -F_{T,x} - F_f$. Substitute the horizontal component of the tension force — 1 point: $F_{net} = -F_T \sin\theta - F_f = -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) - F_f$. Substitute the normal force from part (c)(i) into $F_f = \mu_k F_N$ — 1 point: $F_{net} = -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) - \mu_k \left(mg + F_T \left(\frac{y}{\sqrt{y^2 + x^2}} \right) \right)$.

Solution 1

(d) Mark scheme

- Use the integral definition of work — 1 point: $W = \int F \cdot dx$. Indicate the work on the sled is due only to the horizontal component of tension — 1 point: $W = \int F_{T,x}\, dx$. Substitute the horizontal component of tension consistent with part (b)/(c) — 1 point: $W = \int_{x=0}^{x=L} -F_T \left(\frac{x}{\sqrt{y^2 + x^2}} \right) dx$. Use correct limits $x = 0$ to $x = L$ and show the work is negative — 1 point. Solution: let $u = y^2 + x^2$, $du = 2x\, dx$:
  $W = -F_T \int_{x=0}^{x=L} \frac{1}{2} \left(\frac{1}{\sqrt{u}} \right) du = -F_T \left(\sqrt{y^2 + x^2} \right) \Big|_{x=0}^{x=L} = -F_T \left(\sqrt{y^2 + L^2} - \sqrt{y^2} \right)$.

Solution 1

(e)

Mark scheme

- Answer: $E_1 > E_2$ — select this choice with an attempt at relevant justification (1 point), plus a justification that correctly relates the friction force to the normal force, and the normal force to position/angle (1 point). Example justification: The vertical component of the string tension is largest when the string is more vertical, which occurs over $0 < x < L$ (compared to $L < x < 2L$). A larger vertical component of tension gives a larger normal force F_N, and hence (since $F_f = \mu_k F_N$) a larger friction force and more energy dissipated over $0 < x < L$ than over $L < x < 2L$, so $E_1 > E_2$.

Question 2

2021 Set 1 ▪ Q2

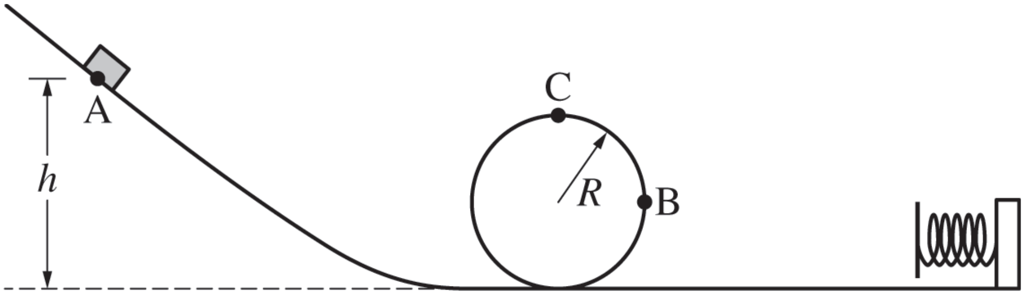
[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

Question 2

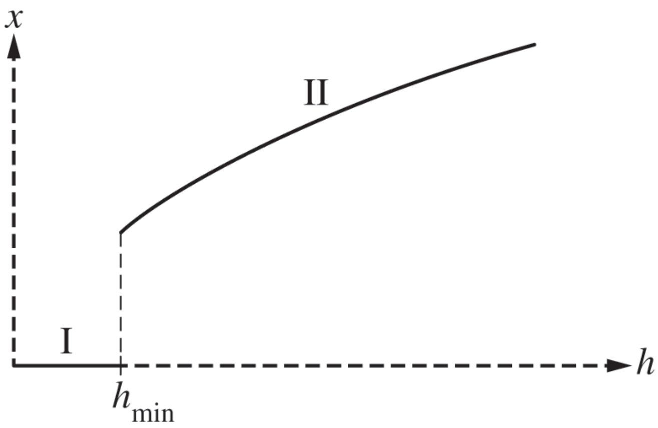
(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.
[Diagram: a single dot with a horizontal dashed line through it and a vertical dashed line through it, representing point B.]
Justify your answer.

Question 2

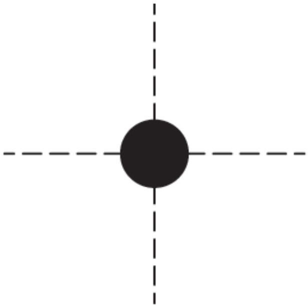


Note: Figure not drawn to scale.

Question 2



Question 2



Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(b)(i) Derive an expression for the speed v of the block at point B.

Question 2

(b)(ii) Derive an expression for the magnitude of the net force F on the block at point B.

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

Question 2

(d) It is determined that $h = 0.30$ m and $R = 0.10$ m. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(e)(i) A graph of the maximum compression of the spring as a function of height is shown below. The height $h_{\min}$ is the height calculated in part (c).

Question 2

[Graph: axes x (vertical) vs. h (horizontal). For h from 0 to $h_{\min}$, labeled region I, x is a horizontal line segment at a constant low value along the horizontal axis. For $h > h_{\min}$, labeled region II, the curve rises smoothly and continues increasing with an upward-curving-then-leveling shape.]

Explain why section I appears as a horizontal line segment on the horizontal axis.

(e)(ii) Explain the reason for the shape of section II on the graph.

Solution 2

(a)

Mark scheme

- Draw a single acceleration vector from the dot at point B pointing down and to the left (1 point for correct direction with an attempted justification). Justification (1 point): at point B the block moves along a circular arc, so it has a centripetal acceleration component directed toward the center of the circle (horizontally, toward the left in this geometry) plus the usual downward gravitational acceleration; the vector sum of these two components points down and to the left.

Solution 2

(b)(i)

Mark scheme

- Apply conservation of energy between the release point A (height h, at rest) and point B (height $= R$, bottom of the loop):
  $K_A + U_{gA} = K_B + U_{gB} \Rightarrow \frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$ (1 point).
  Substitute $v_A = 0,\ h_A = h,\ h_B = R$ (1 point):
  $mgh = \frac{1}{2}mv_B^2 + mgR \Rightarrow v_B = \sqrt{2g(h-R)}$.

Solution 2

(b)(ii) Mark scheme

- Substitute v_B into the centripetal-force expression (1 point): $F_c = \dfrac{mv_B^2}{R} = \dfrac{m\left(\sqrt{2g(h-R)}\right)^2}{R} = \dfrac{2mg(h-R)}{R}$. Combine F_c (horizontal) with weight mg (vertical) via vector addition to get the net-force magnitude at B (1 point): $F_{net} = \sqrt{F_c^2 + (mg)^2} = \sqrt{\left(\dfrac{2mg}{R}(h-R)\right)^2 + (mg)^2}$.

Solution 2

(c) Mark scheme

- Find the speed at point C (top of the loop, height $2R$) via conservation of energy (1 point): $0 + mgh = \frac{1}{2}mv_C^2 + mg(2R) \Rightarrow v_C = \sqrt{2g(h-2R)}$. Apply Newton's second law at C with the normal force set to zero (the minimum-speed / minimum-height condition, 1 point): $mg = \dfrac{mv_C^2}{R} \Rightarrow v_C = \sqrt{Rg}$. Combine the two expressions for v_C (1 point): $\sqrt{Rg} = \sqrt{2g(h-2R)} \Rightarrow R = 2(h-2R) \Rightarrow h_{\min} = 2.5R$.

Solution 2

(d) Mark scheme

- With $h = 0.30$ m, $R = 0.10$ m (above h_{min}), apply conservation of energy from release to maximum spring compression, where all gravitational PE converts to elastic PE (1 point for using energy conservation): $mgh = \frac{1}{2}kx_{MAX}^2$ (1 point for the correctly substituted energy equation). Using the spring-constant relation $k = mg/(2R)$ (established earlier in the problem) and solving for x_{MAX} (1 point): $x_{MAX} = \sqrt{\dfrac{2mgh}{k}} = \sqrt{\dfrac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{4(0.30\text{ m})(0.10\text{ m})} = 0.35$ m.

Solution 2

(e)(i)

Mark scheme

- Justification (1 point) for why section I (for $h \leq h_{\mathrm{min}}$) is flat at $x \approx 0$: because the block cannot make it around the full loop at these heights (it loses contact with the track before reaching the top), it never reaches the spring, so there is no compression.

Solution 2

(e)(ii)

Mark scheme

- For section II ($h > h_{\min}$): as h increases, the compression x of the spring increases (1 point). From part (c)'s relation $x_{MAX} = \sqrt{4hR}$, x_{MAX} is proportional to $\sqrt{h}$, i.e. h is proportional to x^2 – so the curve has the shape of a sideways-opening (x-axis) parabola, rising from $h_{\min}$ and curving upward, rather than a straight line (1 point).

Question 2

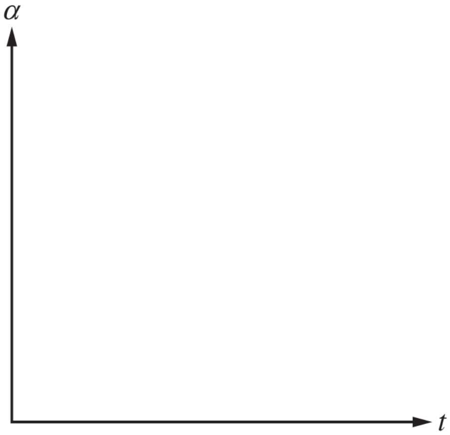
2021 Set 2 ▪ Q2

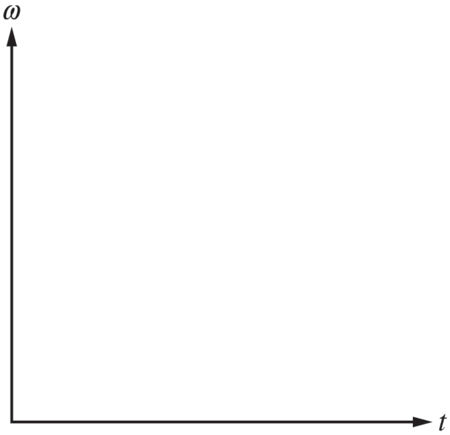
[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]
Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.
(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.
[Figure: "Object B" — an L-shaped (right-angle) object formed of two rods, shown on an x-y coordinate system with origin O at the bottom-left. A pivot is marked at the top-left corner (labelled "Pivot", on the y-axis). A horizontal rod of length L extends

Question 2

right from the pivot along the top (labelled "L"), and a vertical rod of length L extends down from the pivot to O along the y-axis (labelled "L"). The x-axis points right from O, the y-axis points up from O through the pivot.]
Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above.
Express your answers in part (b) in terms of L.

Question 2





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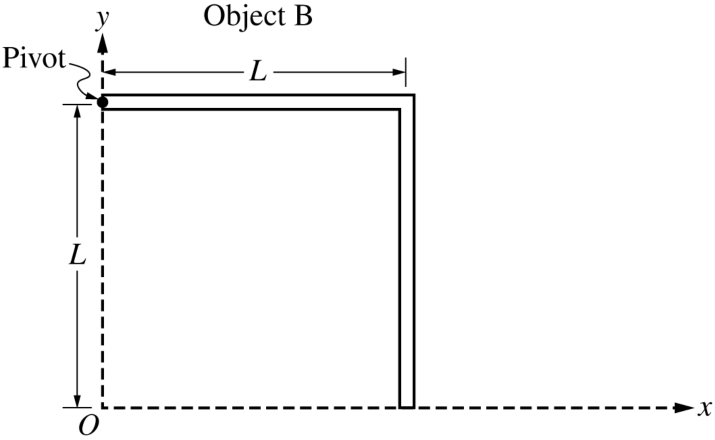
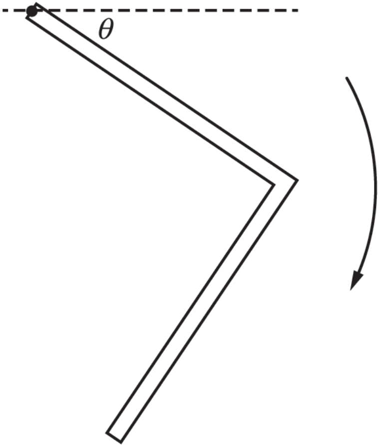


Question 2



Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(b)(i) Determine the following for the given coordinate system shown in the figure. The x-coordinate of the center of mass of object B

(b)(ii) The y-coordinate of the center of mass of object B

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(c) Object B has a rotational inertia of I_B about its pivot.

Is the value of I_B greater than, less than, or equal to I_A?

_______Greater than _______Less than _______Equal to

Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.

[Two blank sets of axes are provided: left axis labelled α (vertical) vs. t (horizontal); right axis labelled ω (vertical) vs. t (horizontal). Both axes are otherwise unmarked (no gridlines or scale), for the student to sketch curves.]

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(e) [Figure: The L-shaped object B is shown rotated partway from its initial horizontal/vertical position, tilted through an angle θ (marked at the top between a dashed horizontal reference line and the now-tilted top rod), with a curved arrow indicating the direction of rotation (downward/clockwise) at the lower end of the shape.]

Question 2

While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

$I^{ncreasing}D^{ecreasing}N^{otchanging}$

Justify your answer.

[Figure: Object B is shown in a new orientation — the vertical rod of length L now lies along the bottom (horizontal), and the horizontal rod of length L extends straight up from its right end (vertical), forming an upside-down L / right-angle shape resting on the ground.]

Object B rotates through the position shown above.

Question 2

2021 Set 2 ▪ Q2

[Figure: "Object A" — a long, thin, uniform rod of mass M and total length $2L$, drawn horizontally. A pivot is shown at the left end (marked "Pivot" with a bracket symbol), and the rod extends to the right; a bracket below the rod labels its full length as $2L$, with M labelling the mass above the rod.]

Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

Solution 2

(a) Mark scheme

- Using integral calculus with λ the linear mass density of the rod (length $2L$, mass M): $I = \int_{r=0}^{r=2L} \lambda r^2\, dr = \lambda \left[\frac{r^3}{3}\right]_0^{2L} = \frac{\lambda}{3}(2L)^3$ (1 pt, for correct limits/constant of integration). Since $\lambda = m/\ell = M/(2L)$: $I = \left(\frac{1}{3}\right)\left(\frac{M}{2L}\right)(8L^3) = \frac{4}{3}ML^2$ (1 pt, for correctly relating λ to M and L). This confirms $I_A = \frac{4}{3}ML^2$.

Solution 2

(b)(i) Mark scheme

- $X_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} = \dfrac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{M}{2}\right)(L)}{\frac{M}{2} + \frac{M}{2}} = \dfrac{\frac{ML}{4} + \frac{ML}{2}}{M} = \dfrac{3}{4}L$. Final answer: $X_{CM} = \dfrac{3}{4}L$.

Solution 2

(b)(ii) Mark scheme

- $Y_{CM} = \dfrac{\sum m_i y_i}{\sum m_i} = \dfrac{\left(\frac{M}{2}\right)(L) + \left(\frac{M}{2}\right)\left(\frac{L}{2}\right)}{\frac{M}{2} + \frac{M}{2}} = \dfrac{\frac{ML}{2} + \frac{ML}{4}}{M} = \dfrac{3}{4}L$. Final answer: $Y_{CM} = \dfrac{3}{4}L$.

Solution 2

(c)

Mark scheme

- Select 'Less than' (1 pt for selection + attempted relevant justification) with a correct justification (1 pt). Example: 'Because object B has more of its mass closer to the pivot than object A, the rotational inertia of object B must be less than that of object A.' I.e. $I_B < I_A$.

Solution 2

(d)

Mark scheme

- Sketch two graphs covering the interval from release to the moment the center of mass reaches its lowest point: the angular acceleration α vs. t graph must be concave down and begin horizontally (start at a nonzero maximum value with zero slope) (1 pt); the angular speed ω vs. t graph must be concave down and end horizontally (rise from zero to a plateau with zero final slope) (1 pt); the two graphs must be consistent with each other — ω increasing exactly while $\alpha > 0$, and both approaching their limiting behavior together as $\alpha \to 0$ near the lowest point (1 pt).

Solution 2

(e)

Mark scheme

- Select 'Decreasing' (1 pt for selection + attempted relevant justification) with a justification that identifies the lever arm for the torque as decreasing (1 pt). Example: 'Because the horizontal position of the center of mass for the object is moving closer to the pivot, the lever arm for the force of gravity is decreasing so the angular acceleration decreases.'

Solution 2

(f) Mark scheme

- Conservation of energy: $U_{g1} = K_2$ (1 pt). Relate the change in rotational KE to the change in gravitational PE: $mgh = \frac{1}{2}I\omega^2$ (1 pt). Substitute $h = L/2$ (the drop in height of the center of mass): $Mg\left(\frac{L}{2}\right) = \frac{1}{2}I_B\omega^2$ (1 pt). Solve for ω in terms of the allowed symbols M, L, I_B: $\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2Mg(L/2)}{I_B}} = \sqrt{\frac{MgL}{I_B}}$ (1 pt). Final answer: $\omega = \sqrt{\frac{MgL}{I_B}}$.

Question 3

2021 Set 2 ▪ Q3

[Figure: A block of mass m sits on top of a vertical spring (spring constant k) which is compressed a distance Δx from its natural length, at the bottom of a vertical track. The track curves upward and to the right in a smooth arc, reaching a highest point labelled A at height $3R$ above the block's release point, where the track has constant radius of curvature R (shown with a radius arrow labelled R from a center point to point A). The track continues curving over the top and down to point B, at the same height $3R$, where the track becomes horizontal (radius R again, shown with a radius arrow labelled R from a center point to B) and the block exits horizontally moving to the right and slightly upward along a dashed projectile trajectory, landing a horizontal distance D from the end of the track (measured from directly below B). A note states "Figure not drawn to scale."]

Question 3

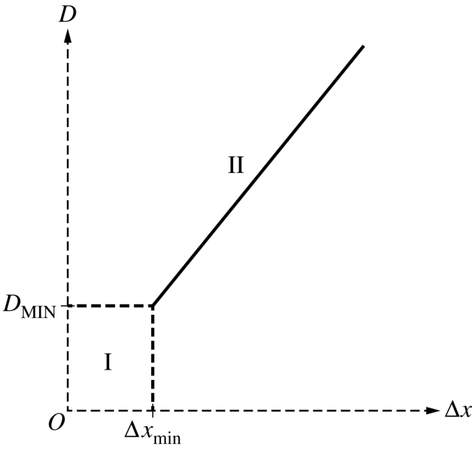
A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B.

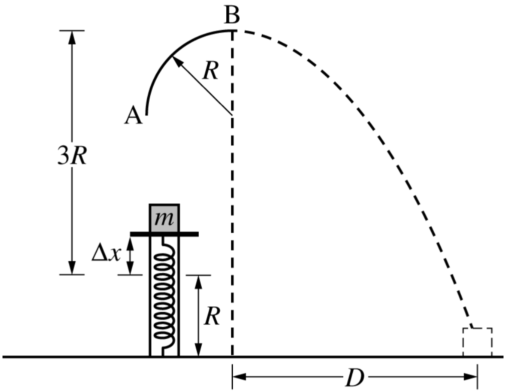
Question 3

Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.
[A single black dot is shown, representing the block at point B, with blank space around it for the student to draw force arrows.]
Justify your choice of vectors.

Question 3

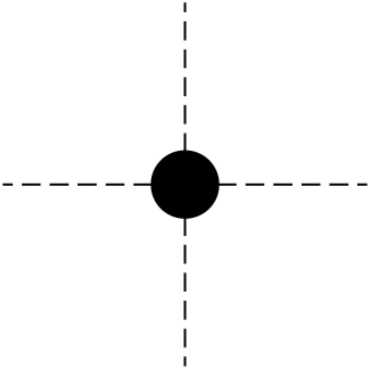


Question 3



Note: Figure not drawn to scale.

Question 3



Question 3

2021 Set 2 ▪ Q3

[Figure: A block of mass m sits on top of a vertical spring (spring constant k) which is compressed a distance Δx from its natural length, at the bottom of a vertical track. The track curves upward and to the right in a smooth arc, reaching a highest point labelled A at height $3R$ above the block's release point, where the track has constant radius of curvature R (shown with a radius arrow labelled R from a center point to point A). The track continues curving over the top and down to point B, at the same height $3R$, where the track becomes horizontal (radius R again, shown with a radius arrow labelled R from a center point to B) and the block exits horizontally moving to the right and slightly upward along a dashed projectile trajectory, landing a horizontal distance D from the end of the track (measured from directly below B). A note states "Figure not drawn to scale."]

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Question 3

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(b)(i) Derive an expression for the speed v of the block at point B.

(b)(ii) Derive an expression for the magnitude of the net force F on the block at point B.

Question 3

2021 Set 2 ▪ Q3

[Figure: A block of mass m sits on top of a vertical spring (spring constant k) which is compressed a distance Δx from its natural length, at the bottom of a vertical track. The track curves upward and to the right in a smooth arc, reaching a highest point labelled A at height $3R$ above the block's release point, where the track has constant radius of curvature R (shown with a radius arrow labelled R from a center point to point A). The track continues curving over the top and down to point B, at the same height $3R$, where the track becomes horizontal (radius R again, shown with a radius arrow labelled R from a center point to B) and the block exits horizontally moving to the right and slightly upward along a dashed projectile trajectory, landing a horizontal distance D from the end of the track (measured from directly below B). A note states "Figure not drawn to scale."]

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Question 3

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(c) Derive an expression for the minimum value of $\Delta x_{\min}$ required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{\min}$.

Question 3

2021 Set 2 ▪ Q3

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Question 3

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(d) Calculate the distance D that the block travels.

Question 3

2021 Set 2 ▪ Q3

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Question 3

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(e)(i) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.

[Graph: vertical axis D, horizontal axis Δx. A dashed horizontal line marks D_{MIN} on the D-axis, and a dashed vertical line marks Δx_{min} on the Δx-axis, meeting at the start of the data trend. Region "I" is labelled in the lower-left area (below/left of the dashed lines, near the origin O, where there is no plotted line). Region "II" is labelled

Question 3

along a straight line that starts at the point $(\Delta x_{\min}, D_{\mathsf{MIN}})$ and rises linearly up and to the right.]

Explain why there are no data for section I of the graph.

(e)(ii) Explain the reason for the shape and minimum value of section II on the graph.

Solution 3

(a) Mark scheme

- At point B, draw two forces on the dot: weight F_g pointing straight down (1 pt), and the normal force from the track F_N pointing toward the center of the circular path / perpendicular to the track surface (1 pt), with a justification consistent with the diagram (1 pt). Example: 'The force F_g represents the weight of the block and always points downward. The force F_N represents the force the track exerts on the block to keep it moving in a circular path and points perpendicular to the surface of the track.' Acceptable gravity labels: F_G, F_g, F_{grav}, W, mg, Mg, etc. (not G or g alone); acceptable normal-force labels: F_n, F_N, N, 'normal force'. Note: if extraneous forces are drawn, a maximum of 2 of the 3 points can be earned.

Solution 3

(b)(i) Mark scheme

- Use conservation of energy for the system, block starting at rest: $U_1 + K_1 = U_2 + K_2$, i.e. $U_1 + 0 = U_2 + K_2$ (1 pt). Relate the elastic PE at maximum spring compression to the gravitational PE at point B: $U_{s1} + 0 = U_{g2} + K_2$, so $K_2 = U_{s1} - U_{g2}$ (1 pt). Substitute: $\frac{1}{2}mv_B^2 = \frac{1}{2}k(\Delta x)^2 - mgh_2$, where $h_2 = 3R$ is the height of point B; so $v_B^2 = \frac{k}{m}(\Delta x)^2 - 2g(3R)$, giving $v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$ (1 pt). Final answer: $v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$.

Solution 3

(b)(ii) Mark scheme

- Relate the centripetal (net) force at point B to the speed found in part (b)(i): $F_C = \frac{mv^2}{r} = \frac{mv_B^2}{R}$ (1 pt). Substitute v_B from (b)(i) for an answer consistent with that result: $F_C = \frac{m}{R}\left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)^2 = \frac{k(\Delta x)^2}{R} - 6mg$ (1 pt). Final answer: $F_C = \frac{k(\Delta x)^2}{R} - 6mg$.

Solution 3

(c) Mark scheme

- At the minimum compression, the block barely maintains contact with the track at point B, so the normal force is zero and the net force equals gravity alone: relate the net-force expression from (a)/(b)(ii) to this condition (1 pt): $\dfrac{k(\Delta x)^2}{R} - 6mg = mg$. Substitute and solve (1 pt): $\dfrac{k(\Delta x)^2}{R} = 7mg \implies \Delta x = \sqrt{\dfrac{7mgR}{k}}$. Final answer: $\Delta x_{\mathrm{min}} = \sqrt{\dfrac{7mgR}{k}}$.

Solution 3

(d) Mark scheme

- Relate the height of fall ($4R$, from point B down to the ground) to the time of fall using projectile motion: $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$; with a fall height of $4R$: $t = \sqrt{\frac{2(4R)}{g}} = \sqrt{\frac{8R}{g}}$ (1 pt). Substitute into $D = v_x t$ using the (constant) horizontal velocity from part (b)(i): $D = v_x t = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR} \cdot \sqrt{\frac{8R}{g}}$ (1 pt, must be consistent with the expression for v_B from (b)(i)). Final answer: $D = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR} \cdot \sqrt{\frac{8R}{g}}$.

Solution 3

(e)(i)

Mark scheme

- Provide a correct justification for the general shape of the D-vs-Δx graph, i.e. why no data/motion occurs for compressions below the minimum (Region I). Example reasoning: 'The block needs a minimum speed to make it through point B on the track; thus, the flat/empty segment (Region I) represents compressions of the spring for which the block does not make it to point B.' (1 pt for a correct justification.)

Solution 3

(e)(ii) Mark scheme

- Explain both the shape and the minimum value of Region II: (1 pt) as the maximum compression Δx increases beyond Δx_{min}, the distance D increases — consistent with part (d), D grows with $\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$, so beyond the threshold the graph rises monotonically with Δx; (1 pt) the minimum value (onset of Region II, at Δx_{min}) occurs because that is exactly the minimum spring compression that gives the block the minimum speed needed to maintain contact with the track through point B (from part (c)).