Past-paper walk-through

Potential Energy ▪ 3.3

eXercise

Questions in this set

- 2025 Q2
- 2021 Set 1 Q2
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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = “or words to that effect” – the wording need not match.

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.
[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]
The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.
[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis

Question 2

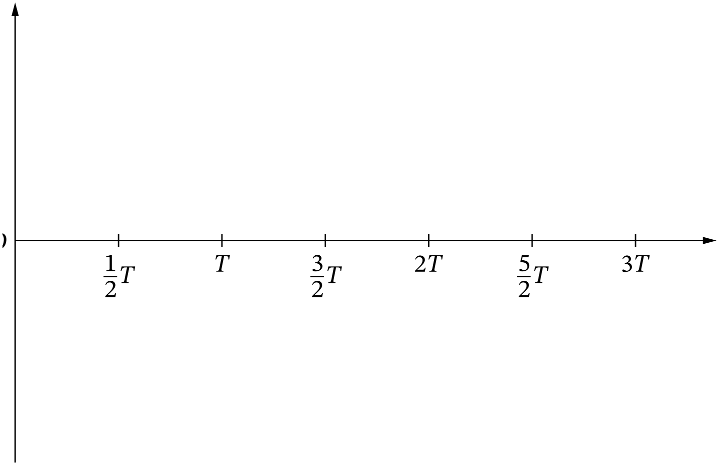
is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

(a) An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

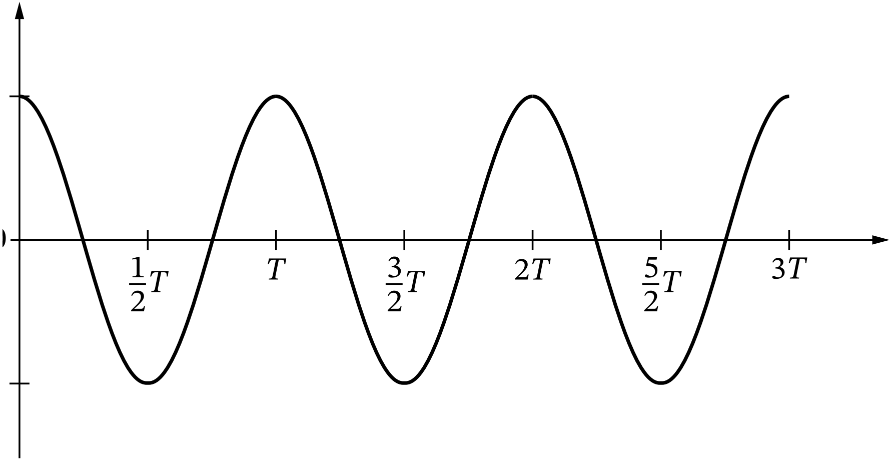
- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}. - Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

[Figure 3: A blank bar-chart grid titled "Energy" (vertical axis) with three labeled columns: U_A, U_B, K_{block}, and a horizontal "0" line partway up the grid, with gridlines above and below for plotting bar heights.]

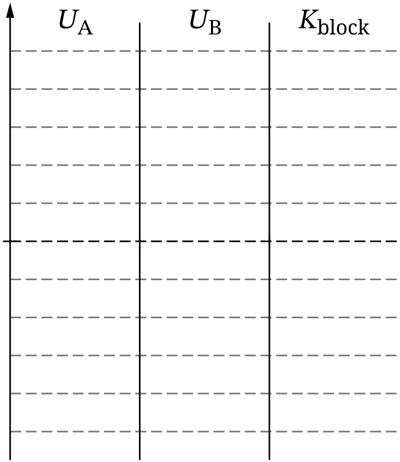
Question 2



Question 2



Question 2



Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(b) The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

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Question 2

(c)(i) In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.
[Figure 4: Graph titled "Scenario 1" of position x vs. time t. Vertical axis x with values $+x_1$ marked above 0 and $-x_1$ marked below 0; horizontal axis t with gridmarks at $\frac{1}{2}T$, T, $\frac{3}{2}T$, $2T$, $\frac{5}{2}T$, $3T$. The curve is a cosine-like oscillation: starts at $+x_1$ at $t = 0$, crosses zero at $\frac{1}{2}T$ going down to $-x_1$ at around $\frac{1}{2}T$ to T, back up through zero, reaching $+x_1$ again at T, and repeating this pattern periodically through $3T$.]
In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.
On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.

Question 2

[Figure 5: Blank axes titled "Scenario 2" with vertical axis K and horizontal axis t with gridmarks at $\frac{1}{2}T$, T, $\frac{3}{2}T$, $2T$, $\frac{5}{2}T$, $3T$, origin labeled O.]

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(d) In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

Solution 2

(a) Mark scheme

- Energy bar chart at $x = x_1$ (Figure 3), where the block is momentarily at rest (released from rest at x_1, so all mechanical energy is stored in the two springs): the K_{block} bar has zero height [Point A1]. Both U_A and U_B bars are drawn with positive height, since elastic PE stored in a spring is never negative [Point A2]. Spring B has spring constant $2k$ (twice Spring A's k) and both springs share displacement x_1: $U_A = \frac{1}{2}kx_1^2$ and $U_B = \frac{1}{2}(2k)x_1^2 = 2U_A$, so the U_B bar is drawn exactly twice the height of the U_A bar (this point is awarded on the height ratio regardless of which sign/direction the bars are drawn in) [Point A3].

Solution 2

(b) Mark scheme

- Derive v at $x = \frac{1}{2}x_1$ using energy conservation (an equivalent SHM method is also accepted) [Point B1: multistep derivation that includes energy conservation or simple harmonic motion]. The system behaves as a single effective spring combining both springs acting on the block together, $k_{eff} = k + 2k = 3k$ [Point B2: relates the presence of both springs to the system's behavior]. Initial state at $x = x_1$: $E_{tot,0} = U_A(x_1) + U_B(x_1) + K_{block}(x_1) = \frac{1}{2}kx_1^2 + \frac{1}{2}(2k)x_1^2 + 0 = \frac{3}{2}kx_1^2$. Final state at $x = \frac{1}{2}x_1$ [Point B3: relates positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block]: $E_{tot,f} = \frac{1}{2}k\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{block,\frac{1}{2}x_1} = \frac{3}{8}kx_1^2 + K_{block,\frac{1}{2}x_1}$. Setting $E_{tot,0} = E_{tot,f}$:

Solution 2

$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{block,\frac{1}{2}x_1} \Rightarrow K_{block,\frac{1}{2}x_1} = \frac{9}{8}kx_1^2 \Rightarrow \frac{1}{2}mv^2 = \frac{9}{8}kx_1^2 \Rightarrow v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$ [Point B4: correct final expression for v in terms of m, k, x1]. (Equivalent SHM route: $\omega = \sqrt{3k/m}$, $x(t) = x_1 \cos \omega t$, solve $\cos \omega t = \frac{1}{2} \Rightarrow \omega t = \pi/3$, then $v = x_1 \omega \sin(\pi/3) = \frac{3}{2}x_1\sqrt{k/m}$ – same result.)

Solution 2

(c)(i) Mark scheme

- Sketch K (kinetic energy of the block) vs. t for Scenario 2 (the block oscillating on the two-spring system WITH friction/damping), on axes marked at $\frac{1}{2}T, T, \frac{3}{2}T, 2T, \frac{5}{2}T, 3T$ (Figure 5), where T is the period from the frictionless Scenario 1. The block is released from rest at x_1, so $K = 0$ at $t = 0$, and the curve must start at zero and remain ≥ 0 for all t (kinetic energy cannot be negative) [Point C1]. Because the block passes through $x = 0$ (where K is maximum) twice per full oscillation period T and momentarily has $K = 0$ at each turning point (also twice per period T), the sketched curve should be periodic with zeros spaced $\frac{1}{2}T$ apart – i.e. humps peaking near $\frac{1}{4}T, \frac{3}{4}T, \frac{5}{4}T, \ldots$ and touching zero at $0, \frac{1}{2}T, T, \frac{3}{2}T, 2T, \ldots$ [Point C2]. Because friction dissipates

Solution 2

mechanical energy each cycle, the successive peak heights of K must decrease over time, i.e. the curve is a periodic wave of decreasing amplitude [Point C3].

Solution 2

(d) Mark scheme

- Scenario 3: the block is replaced with a new, LARGER-mass block (same coefficient of kinetic friction as Scenario 2). Compare the K vs. t graph of Scenario 3 to that of Scenario 2 (part C) and describe ONE feature that would differ, with a correct justification. Acceptable answers (state ONE, with matching reasoning): (1) 'The period increases' – because the period of the oscillator is $T = 2\pi\sqrt{m/k_{eff}}$, and increasing the mass m (with k_{eff} unchanged) increases T [Point D1 + Point D2]. (2) 'The rate at which the graph approaches zero amplitude increases' – because a larger mass under the same coefficient of friction experiences a larger friction force, so the block loses mechanical energy at a greater rate. (3) 'The maximum value of K over each period is less than the

Solution 2

graph drawn [in part C]' – a valid alternative feature with matching reasoning (more energy lost to friction per cycle). Award Point D1 for indicating any ONE of these three features, and Point D2 for a justification that correctly supports the specific feature chosen.

Question 2

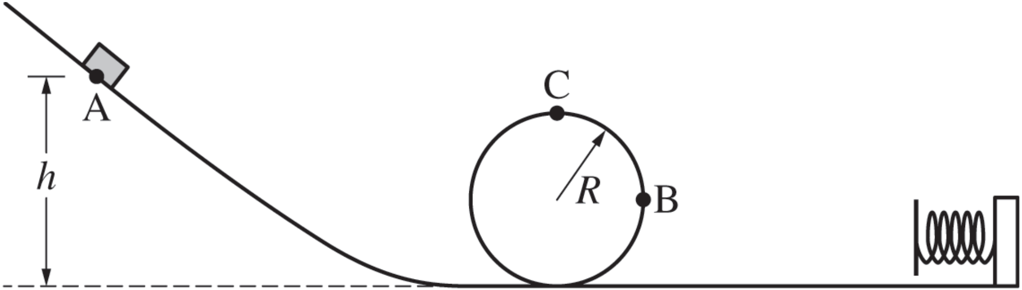
2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]
A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

Question 2

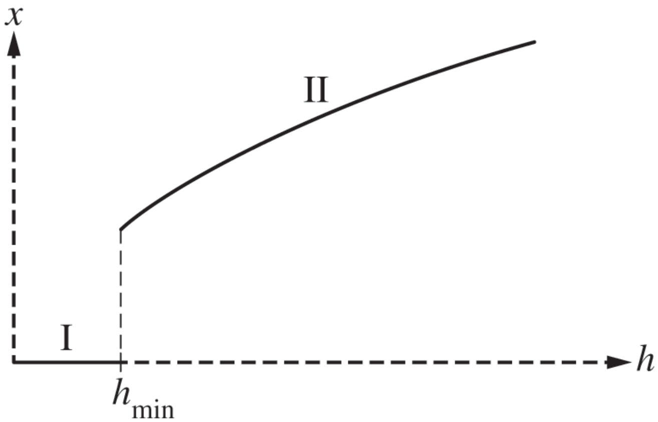
(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.
[Diagram: a single dot with a horizontal dashed line through it and a vertical dashed line through it, representing point B.]
Justify your answer.

Question 2



Note: Figure not drawn to scale.

Question 2



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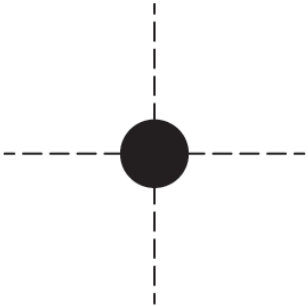


Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(b)(i) Derive an expression for the speed v of the block at point B.

Question 2

(b)(ii) Derive an expression for the magnitude of the net force F on the block at point B.

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

Question 2

(d) It is determined that $h = 0.30$ m and $R = 0.10$ m. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(e)(i) A graph of the maximum compression of the spring as a function of height is shown below. The height $h_{\min}$ is the height calculated in part (c).

Question 2

[Graph: axes x (vertical) vs. h (horizontal). For h from 0 to $h_{\min}$, labeled region I, x is a horizontal line segment at a constant low value along the horizontal axis. For $h > h_{\min}$, labeled region II, the curve rises smoothly and continues increasing with an upward-curving-then-leveling shape.]

Explain why section I appears as a horizontal line segment on the horizontal axis.

(e)(ii) Explain the reason for the shape of section II on the graph.

Solution 2

(a)

Mark scheme

- Draw a single acceleration vector from the dot at point B pointing down and to the left (1 point for correct direction with an attempted justification). Justification (1 point): at point B the block moves along a circular arc, so it has a centripetal acceleration component directed toward the center of the circle (horizontally, toward the left in this geometry) plus the usual downward gravitational acceleration; the vector sum of these two components points down and to the left.

Solution 2

(b)(i)

Mark scheme

- Apply conservation of energy between the release point A (height h, at rest) and point B (height $= R$, bottom of the loop):
  $K_A + U_{gA} = K_B + U_{gB} \Rightarrow \frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$ (1 point).
  Substitute $v_A = 0,\ h_A = h,\ h_B = R$ (1 point):
  $mgh = \frac{1}{2}mv_B^2 + mgR \Rightarrow v_B = \sqrt{2g(h-R)}$.

Solution 2

(b)(ii) Mark scheme

- Substitute v_B into the centripetal-force expression (1 point): $F_c = \frac{mv_B^2}{R} = \frac{m\left(\sqrt{2g(h-R)}\right)^2}{R} = \frac{2mg(h-R)}{R}$. Combine F_c (horizontal) with weight mg (vertical) via vector addition to get the net-force magnitude at B (1 point): $F_{net} = \sqrt{F_c^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h-R)\right)^2 + (mg)^2}$.

Solution 2

(c) Mark scheme

- Find the speed at point C (top of the loop, height $2R$) via conservation of energy (1 point): $0 + mgh = \frac{1}{2}mv_C^2 + mg(2R) \Rightarrow v_C = \sqrt{2g(h-2R)}$. Apply Newton's second law at C with the normal force set to zero (the minimum-speed / minimum-height condition, 1 point): $mg = \dfrac{mv_C^2}{R} \Rightarrow v_C = \sqrt{Rg}$. Combine the two expressions for v_C (1 point): $\sqrt{Rg} = \sqrt{2g(h-2R)} \Rightarrow R = 2(h-2R) \Rightarrow h_{\mathrm{min}} = 2.5R$.

Solution 2

(d) Mark scheme

- With $h = 0.30$ m, $R = 0.10$ m (above $h_{\min}$), apply conservation of energy from release to maximum spring compression, where all gravitational PE converts to elastic PE (1 point for using energy conservation): $mgh = \frac{1}{2}kx_{MAX}^2$ (1 point for the correctly substituted energy equation). Using the spring-constant relation $k = mg/(2R)$ (established earlier in the problem) and solving for x_{MAX} (1 point): $x_{MAX} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{4(0.30\text{ m})(0.10\text{ m})} = 0.35$ m.

Solution 2

(e)(i)

Mark scheme

- Justification (1 point) for why section I (for $h \leq h_{\min}$) is flat at $x \approx 0$: because the block cannot make it around the full loop at these heights (it loses contact with the track before reaching the top), it never reaches the spring, so there is no compression.

Solution 2

(e)(ii)

Mark scheme

- For section II ($h > h_{\mathrm{min}}$): as h increases, the compression x of the spring increases (1 point). From part (c)'s relation $x_{MAX} = \sqrt{4hR}$, x_{MAX} is proportional to $\sqrt{h}$, i.e. h is proportional to x^2 – so the curve has the shape of a sideways-opening (x-axis) parabola, rising from h_{min} and curving upward, rather than a straight line (1 point).

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

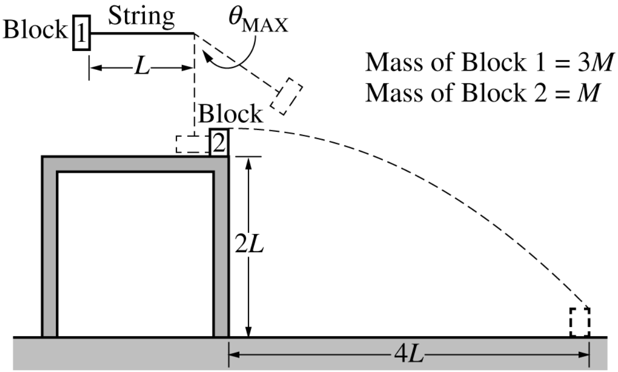
A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a

Question 2

distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

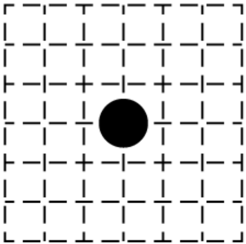
Question 2



Mass of Block 1 = $3M$
Mass of Block 2 = M

Note: Figure not drawn to scale.

Question 2



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2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 $= 3M$", "Mass of Block 2 $= M$". Note: Figure not drawn to scale.]

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.

[A dot on a dashed grid, representing block 1, for the student to draw force vectors on.]

Question 2

2019 Set 1 ▪ Q2

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A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

Question 2

2019 Set 1 ▪ Q2

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Question 2

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(e) Calculate the speed of block 2 as it leaves the table.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(f) Calculate the speed of block 1 just after it collides with block 2.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

Solution 2

(a)

Mark scheme

- Energy conservation for block 1's swing:
  $U_{g1} = K_2 \Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow gL = \frac{1}{2}v^2 \Rightarrow v = \sqrt{2gL}$. Full credit even if no work is shown.

Solution 2

(b)

Mark scheme

- Draw two vectors from the dot: the weight F_W pointing straight down, and the string tension F_T pointing straight up, with the tension arrow drawn noticeably LONGER than the weight arrow (net force must be upward/centripetal). 1 pt for correctly drawing and labeling both the weight and the tension; 1 pt for drawing the tension longer than the weight. Only a maximum of 1 point can be earned if any extraneous vectors are included.

Solution 2

(c)

Mark scheme

- Use Newton's second law along the string direction, tension minus weight equals centripetal force (1 pt):
  $F_T = mg + ma_c = mg + \frac{mv^2}{r} = 3Mg + \frac{(3M)(\sqrt{2gL})^2}{L}$. Correct final answer (1 pt): $F_T = 9Mg$.

Solution 2

(d) Mark scheme

- Use a correct kinematic equation for the vertical fall from table height $2L$ (1 pt): $\Delta y = v_i t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{4(0.75\ \text{m})}{9.8\ \text{m/s}^2}}$. Correct final answer (1 pt): $t = 0.55$ s.

Solution 2

(e)

Mark scheme

- Use a correct kinematic equation for the horizontal (projectile) motion (1 pt): $\Delta x = vt \Rightarrow v = \dfrac{\Delta x}{t} = \dfrac{4L}{t}$. Substitute the time from part (d) (1 pt): $v = \dfrac{(4)(0.75\ \mathrm{m})}{0.55\ \mathrm{s}} = 5.45\ \mathrm{m/s}$.

Solution 2

(f) Mark scheme

- Use conservation of momentum for the collision (1 pt): $p_i = p_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$. Correctly substitute the masses (1 pt): $(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$. Substitute the speeds found in parts (a) and (e) (1 pt): $v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{3M} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{3M}$, giving $v_{1f} = \dfrac{3\sqrt{2(9.8)(0.75)} - (1)(5.45)}{3} = 2.02$ m/s (or $v_{1f} = 2.06$ m/s using $g = 10$ m/s^2).

Solution 2

(g) Mark scheme

- Use conservation of energy for block 1's rise after the collision (1 pt): $K_1 = U_{g2} \Rightarrow \frac{1}{2}mv^2 = mgh$. Correctly substitute $h = L(1-\cos\theta)$ (1 pt): $\frac{1}{2}mv^2 = mgL(1-\cos\theta) \Rightarrow \frac{v^2}{2gL} = 1-\cos\theta$. Substitute the speed found in part (f) (1 pt): $\theta = \cos^{-1}\left(1 - \frac{v^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{(2.02)^2}{2(9.8)(0.75)}\right) = 43.5^\circ$ (or $\theta = 44.1^\circ$ using $g = 10$ m/s^2).

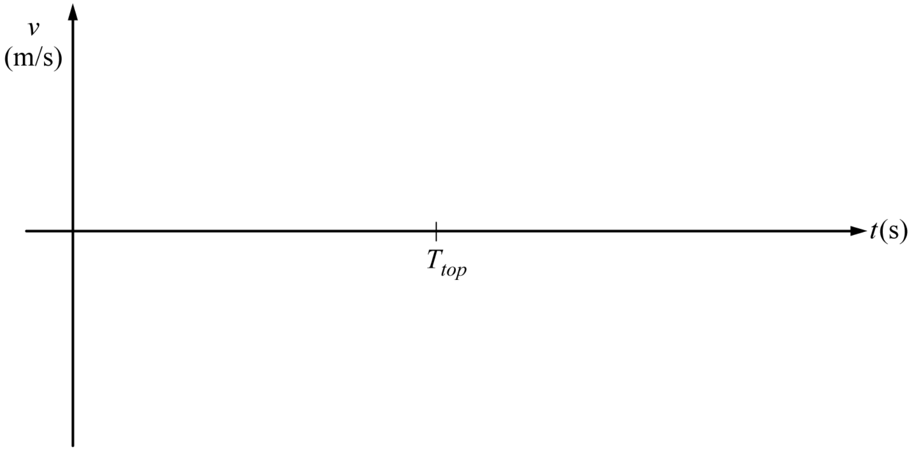
Question 2

2019 Set 2 ▪ Q2

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0$ m/s^2, $L = 0.25$ m/s^4, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

Question 2



Question 2

2019 Set 2 ▪ Q2

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(b) Calculate the speed of the rocket at $t = 6.0$ s.

Question 2

2019 Set 2 ▪ Q2

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(c)(i) Calculate the kinetic energy of the rocket at $t = 6.0$ s.

(c)(ii) Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

Question 2

2019 Set 2 ▪ Q2

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0$ m/s^2, $L = 0.25$ m/s^4, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(d) Calculate the maximum height reached by the rocket relative to its launching point.

Question 2

2019 Set 2 ▪ Q2

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \mathrm{m/s^2}$, $L = 0.25\ \mathrm{m/s^4}$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.

Question 2

[Graph axes: vertical axis labeled v (m/s), horizontal axis labeled t(s); the origin is at the intersection, with a tick mark labeled T_{top} on the horizontal axis to the right of the origin. The axes are otherwise unscaled and blank for the student to sketch on.]

Solution 2

(a)

Mark scheme

- $J = \int F(t)\,dt = \int ma(t)\,dt = (0.50)\int_0^6 (9.0 - 0.25t^2)\,dt$ (1 pt for the correct expression and substitution of $a(t)$ and m). Integrating with the correct limits: $J = (0.50)\left[9.0t - \frac{1}{12}t^3\right]_0^6 = (0.50)\left[(9.0)(6) - \frac{1}{12}(6)^3\right] = (0.50)(54 - 18)$ (1 pt for correct integration/limits). Final answer: $J = 18\ \mathrm{N \cdot s}$.

Solution 2

(b)

Mark scheme

- Relate impulse to the change in momentum: $J = \Delta p = m(v_2 - v_1)$ (1 pt for correctly relating impulse to speed). Substitute $J = 18$ N $\cdot$ s from part (a): $(18$ N $\cdot$ s$) = (0.50$ kg$)(v_2 - 0)$ (1 pt for correct substitution), giving $v_2 = 36$ m/s.

Solution 2

(c)(i)

Mark scheme

- $K = \frac{1}{2}mv^2$. Substitute the mass of the rocket (1 pt) and the speed $v = 36$ m/s found in part (b) (1 pt): $K = \frac{1}{2}(0.50 \text{ kg})(36 \text{ m/s})^2 = 324$ J.

Solution 2

(c)(ii)

Mark scheme

- Integrate the acceleration twice to obtain position. First: $v(t) = \int a(t)\,dt = \int (9.0 - 0.25t^2)\,dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$ (1 pt for integrating with correct limits/constant of integration, since $v_0 = 0$). Then integrate again: $\Delta y = \int_0^6 v(t)\,dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4\right]_0^6 = \frac{9}{2}(36) - \frac{1}{48}(1296) = 135\text{ m}$ (1 pt for the second integration with correct limits). Substitute into $\Delta U_g = mg\Delta y = (0.50\text{ kg})(9.8\text{ m/s}^2)(135\text{ m}) \approx 660\text{ J}$ (1 pt for substituting into the potential-energy equation).

Solution 2

(d) Mark scheme

- Use $v_2^2 = v_1^2 + 2a(y_2 - y_1)$ with $a = g = -9.8\ \mathrm{m/s^2}$ after burnout and $v_2 = 0$ at maximum height: $y_2 - y_1 = -\frac{v_1^2}{2a}$ (1 pt for using $a = g$ in a correct kinematics equation). Substitute the speed $v_1 = 36\ \mathrm{m/s}$ from part (b) (1 pt): $y_2 - y_1 = -\frac{(36\ \mathrm{m/s})^2}{2(-9.8\ \mathrm{m/s^2})} \approx 66\ \mathrm{m}$. Add the height already gained by $t = 6.0\ \mathrm{s}$, $\Delta y = 135\ \mathrm{m}$ from part (c)(ii) (1 pt for substituting the height from part (c)): $y_2 = 66\ \mathrm{m} + 135\ \mathrm{m} = 201\ \mathrm{m}$ (199.8 m if $g = 10\ \mathrm{m/s^2}$). [Equivalent alternate solutions using energy conservation, $mg\Delta y = \frac{1}{2}mv^2$, or $K_1 + U_1 = U_{top}$ with parts (c)(i)/(c)(ii): $324 + 660 = (0.5)(9.8)\Delta y$, also earn full credit.]

Solution 2

(e) Mark scheme

- Sketch: for $0 \leq t \leq 6.0$ s, an initial concave-down curve starting at the origin $(0,0)$ that rises to a maximum of $v = 36$ m/s at $t = 6.0$ s (1 pt, for an initial concave-down curve starting at the origin, consistent with the decreasing-but-positive acceleration $a = 9.0 - 0.25t^2$). The curve then transitions, before T_{top}, into a straight line with constant negative slope $-g$ (1 pt, for a transition before T_{top} into a straight line of negative slope). Label the maximum value 36 m/s (consistent with part (b)) with the line crossing the time axis exactly at $t = T_{top}$, continuing to negative v as the rocket falls back to the ground (1 pt, for labeling the maximum consistent with part (b) and crossing the axis at T_{top}).

Question 2

2018 ▪ Q2

Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200 \text{ N/s}^2\right) t^2 - (500 \text{ N/s})t$.

[Diagram: Cart 1 (mass $m = 0.500$ kg) with a Force Sensor attached at its right side, moving right with $v_0 = 3.00$ m/s toward a spring ($k = 130$ N/m) connected to Cart 2 (mass $m = 1.05$ kg, initially at rest, $v = 0$), all on a horizontal track.]

Question 2

[Graph: Force F (N) on the vertical axis (marked at 5, 0, -5, -10, -15, -20, -25) versus time t (s) on the horizontal axis (marked 0.02, 0.04, 0.06, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18). Data points start near $F = 0$ at small t, dip down to a minimum of about -20 N around $t = 0.08$–0.10 s, then rise back up toward $F = 0$ by about $t = 0.16$–0.18 s, tracing a roughly parabolic (U-shaped, inverted) curve consistent with the quadratic fit given.]

(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)(i) Calculate the speed of cart 1 after the collision.

(b)(ii) In which direction does cart 1 move after the collision?

______ Left ______ Right

______ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)(i) Calculate the speed of cart 2 after the collision.

Question 2

(c)(ii) Show that the collision between the two carts is elastic.
(d)(i) Calculate the speed of the center of mass of the two-cart-spring system.
(d)(ii) Calculate the maximum elastic potential energy stored in the spring.

Question 2

a spring constant of $k = 130\ \mathrm{N/m}$. The data for the force the spring exerts on cart 1 are sho

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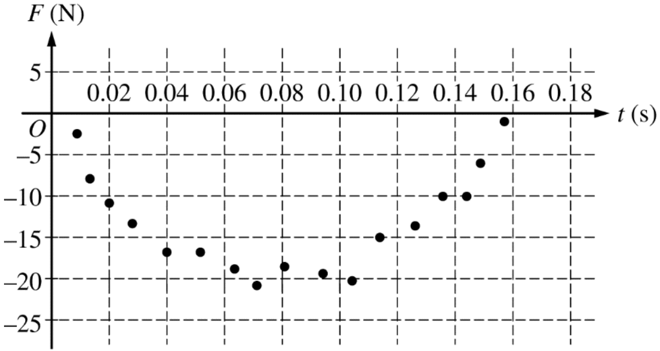
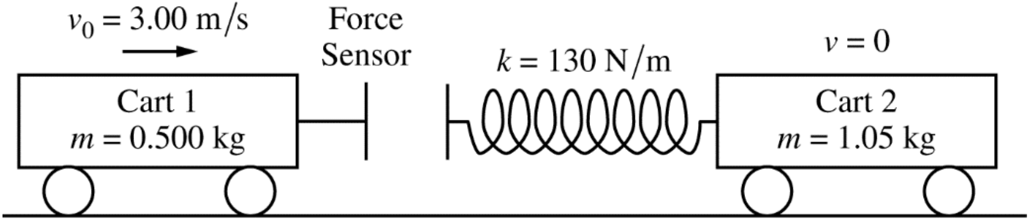


Question 2



Solution 2

(a)

Mark scheme

- $J = \int F\,dt = \int_0^{0.16}(3200t^2 - 500t)\,dt = \left[\frac{3200}{3}t^3 - 250t^2\right]_0^{0.16} = \frac{3200}{3}(0.16)^3 - 250(0.16)^2 = -2.03$ N·s. 1 point for using the given force equation in the impulse integral; 1 point for integrating with correct limits (or constant of integration); 1 point for the correct final numeric answer $J = -2.03$ N·s.

Solution 2

(b)(i)

Mark scheme

- $J = m_1(v_{1f} - v_{1i}) \;\Rightarrow\; v_{1f} = \dfrac{J}{m_1} + v_{1i} = \dfrac{-2.03\ \mathrm{N \cdot s}}{0.500\ \mathrm{kg}} + 3.00\ \mathrm{m/s} = -1.06\ \mathrm{m/s}.$

Solution 2

(b)(ii)

Mark scheme

- Left. Since $v_{1f} = -1.06$ m/s is negative (opposite the original direction of motion), cart 1 moves to the left after the collision.

Solution 2

(c)(i) Mark scheme

- By Newton's third law on the spring force, $-J = -m_1(v_{1f} - v_{1i}) = m_2 v_{2f} \;\Rightarrow\; v_{2f} = \dfrac{-J}{m_2} = \dfrac{2.03\ \mathrm{N\cdot s}}{1.05\ \mathrm{kg}} = 1.93\ \mathrm{m/s}$. (Equivalent alternate via momentum conservation: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \Rightarrow v_{2f} = \dfrac{m_1(v_{1i} - v_{1f})}{m_2} = \dfrac{(0.500)(3.00 - (-1.06))}{1.05} = 1.93$ m/s.) 1 point for a correct equation to find v_{2f}; 1 point for correct substitution.

Solution 2

(c)(ii)

Mark scheme

- An elastic collision requires initial KE $=$ final KE: $K_{ii} = K_{1f} + K_{2f}$. Check: $\frac{1}{2}(0.500)(3.00)^2 = \frac{1}{2}(0.500)(-1.06)^2 + \frac{1}{2}(1.05)(1.93)^2 \;\Rightarrow\; 2.25$ J ≈ 2.24 J — equal within rounding, so the collision is elastic. 1 point for indicating that initial and final kinetic energies must be equal; 1 point for correct substitution/numeric verification.

Solution 2

(d)(i)

Mark scheme

- $m_1 v_{1i} = (m_1 + m_2)v_{cm} \Rightarrow v_{cm} = \dfrac{m_1 v_{1i}}{m_1 + m_2} = \dfrac{(0.500\ \mathrm{kg})(3.00\ \mathrm{m/s})}{0.500\ \mathrm{kg} + 1.05\ \mathrm{kg}} = 0.97$ m/s. 1 point for the correct equation for the center-of-mass speed; 1 point for correct substitution.

Solution 2

(d)(ii) Mark scheme

- Using conservation of energy: $K_i = K_f + U_{Sf} \Rightarrow U_{Sf} = \frac{1}{2}m_1 v_{1i}^2 - \frac{1}{2}(m_1 + m_2)v_{cm}^2 = \frac{1}{2}(0.500)(3.00)^2 - \frac{1}{2}(1.55)(0.97)^2 = 1.52$ J. (Alternate: from $dF/dt = 0$, $t = 0.078$ s gives $|F_{max}| = 19.5$ N (accept 20-21 N from the graph); $x_{max} = F_{max}/k = 0.15$ m (accept 0.15-0.16 m); $U_S = \frac{1}{2}kx_{max}^2 = 1.46$ J, accept 1.46-1.70 J if estimated from the graph.) Scoring: 1 point for using conservation of energy to find the max elastic PE; 1 point for using the center-of-mass speed for the system's translational KE; 1 point for correct substitution/final numeric answer. Q2 also carries an overall 1-point "units" credit (correct units on all calculated answers through the question) — folded into this final part so Q2's 15 points are fully accounted for.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]

A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)(i) Derive an expression for the speed of the block just before it collides with the spring.

Question 2

(a)(ii) Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

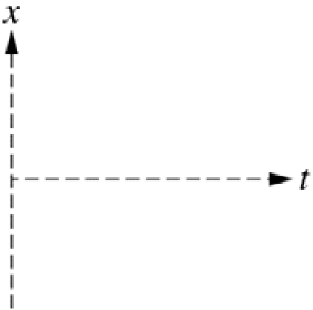
_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Question 2

Note: Figure not drawn to scale.

Question 2



Question 2

2017 ▪ Q2

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(b) Derive an expression for the maximum compression of the spring.

Question 2

2017 ▪ Q2

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A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

Question 2

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface.
The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]

A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

Question 2

(d)(i) Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

(d)(ii) Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]

A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

Question 2

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.
[Three blank axes side by side, each with a vertical axis and horizontal axis labeled t: the first labeled x, the second labeled v, the third labeled a — for sketching the respective graphs.]

Solution 2

(a)(i)

Mark scheme

- Using conservation of energy for the block sliding down the frictionless incline: loss in gravitational PE = gain in KE, $U_g = K$, i.e. $mgh = \frac{1}{2}mv^2$. Solving: $v = \sqrt{2gh}$. 1 point for correctly applying conservation of energy ($U_g = K$), 1 point for the correct final answer $v = \sqrt{2gh}$.

Solution 2

(a)(ii) Mark scheme

- Correct answer: Greater than. Since $v = \sqrt{2gh}$ is proportional to $\sqrt{h}$, reducing the height by a factor of 2 reduces the speed by a factor of $\sqrt{2} \approx 1.41$ (not by a factor of 2). So the speed halfway down the incline is $v/\sqrt{2} \approx 0.71v$, which is greater than half the bottom speed $(0.5v)$. 1 point for correctly selecting 'Greater than' AND giving this correct justification – an incorrect selection cannot earn credit even with valid-looking reasoning.

Solution 2

(b)

Mark scheme

- Using conservation of energy ($U_g = U_s$) for the block compressing the spring, consistent with part (a): $mgh = \frac{1}{2}kx_{max}^2$. Solving for the maximum compression: $x_{max} = \sqrt{\frac{2mgh}{k}}$. 1 point for this correct answer, consistent with the part (a) approach.

Solution 2

(c) Mark scheme

- Recognize that the block's contact with the spring, from first touching it to maximum compression, is one-quarter of a simple-harmonic-motion cycle, with period $T = 2\pi\sqrt{m/k}$. So $t = \frac{1}{4}T = \frac{1}{4}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$. 1 point for identifying the simple-harmonic-motion (quarter-period) approach, 1 point for the correct final expression.

Solution 2

(d)(i) Mark scheme

- Newton's second law for the block on the horizontal surface, with the resistive force opposing the motion: $F_{net} = ma$, and $F_{net} = -\beta v^2$ (negative because it decelerates the block), giving $-\beta v^2 = ma$. Writing acceleration as dv/dt gives the differential equation $-\beta v^2 = m\dfrac{dv}{dt}$. 1 point for correctly applying Newton's second law ($F_{net} = ma$), 1 point for correctly indicating that F_{net} is opposite the direction of motion, 1 point for expressing the result as a differential equation in dv/dt.

Solution 2

(d)(ii) Mark scheme

- Separate variables in $-\beta v^2 = m\, dv/dt$ to get $-\frac{\beta}{m} dt = \frac{1}{v^2} dv$. Integrating both sides: $\int -\frac{\beta}{m} dt = \int \frac{1}{v^2} dv$ gives $-\frac{\beta}{m}[t] = \left[-\frac{1}{v}\right]$. Applying the limits $t : 0 \to t$ and $v : v_0 \to v(t)$: $-\frac{\beta t}{m} = -\frac{1}{v(t)} - \left(-\frac{1}{v_0}\right) = -\frac{1}{v(t)} + \frac{1}{v_0}$. Rearranging gives $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, as required. 1 point for correctly separating variables, 1 point for correctly integrating, 1 point for correctly applying the limits/constant of integration to reach the given result.

Solution 2

(e) Mark scheme

- Since the resistive force continually decelerates the block (from the $1/v(t) = 1/v_0 + \beta t/m$ result, $v(t)$ decreases toward 0 but never reaches it in finite time, and the deceleration itself decreases), the three sketches are: position $x(t)$ – concave down, increasing and leveling off toward (approaching but not reaching) a horizontal asymptote; velocity $v(t)$ – concave up, decreasing from v_0 toward the horizontal (time) axis as an asymptote; acceleration $a(t)$ – concave down (magnitude decreasing, stays negative), also approaching the horizontal axis as an asymptote. (Full credit is also earned if all three curves are reflected vertically; if a plausible but different nonlinear $v(t)$ shape is drawn, $x(t)$ and $a(t)$ consistent with that shape still earn credit.) 1 point per correct curve (x, v, a).

Question 1

2014 ▪ Q1

[Figure: A horizontal track marked in centimeters from 0 to 120 cm. A cart with a spring attached to the left end of the track sits near the 0 cm mark, with two wheels shown. Five vertical photogates labelled 1, 2, 3, 4, 5 are mounted on the track at positions 20 cm, 40 cm, 60 cm, 80 cm, and 100 cm respectively, labelled "Photogates" above.]

In an experiment, a student wishes to use a spring to accelerate a cart along a horizontal, level track. The spring is attached to the left end of the track, as shown in the figure above, and produces a nonlinear restoring force of magnitude $F_s = As^2 + Bs$, where s is the distance the spring is compressed, in meters. A measuring tape, marked in centimeters, is attached to the side of the track. The student places five photogates on the track at the locations shown.

Question 1

(a) Derive an expression for the potential energy U as a function of the compression s. Express your answer in terms of A, B, s, and fundamental constants, as appropriate.

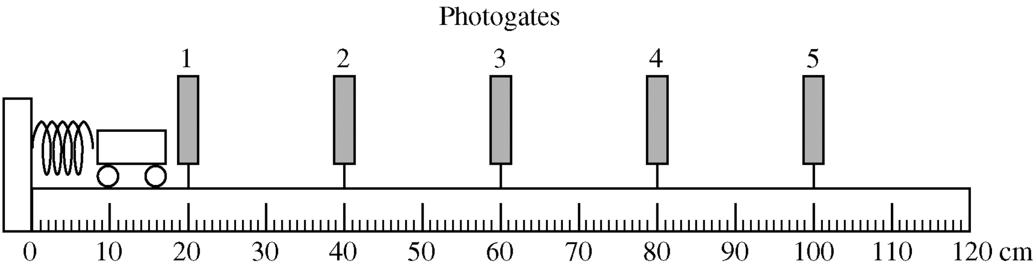
Question 1

Question 1

the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of $\pm 0.002\ \mathrm{m/s}$. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

Question 1



Question 1

2014 ▪ Q1

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(b)(i) In a preliminary experiment, the student pushes the cart of mass 0.30 kg into the spring, compressing the spring 0.040 m. For this spring, $A = 200\ \mathrm{N/m^2}$ and

Question 1

$B = 150$ N/m. The cart is released from rest. Assume friction and air resistance are negligible only during the short time interval when the spring is accelerating the cart. Calculate the following: The speed of the cart immediately after it loses contact with the spring

(b)(ii) Calculate the following: The impulse given to the cart by the spring

Question 1

2014 ▪ Q1

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(c) In a second experiment, the student collects data using the photogates. Each photogate measures the speed of the cart as it passes through the gate. The student calculates a spring compression that should give the cart a speed of 0.320 m/s after

Question 1

the cart loses contact with the spring. The student runs the experiment by pushing the cart into the spring, compressing the spring the calculated distance, and releasing the cart. The speeds are measured with a precision of ± 0.002 m/s. The positions are measured with a precision of ± 0.005 m.

Photogate	1	2	3	4	5
Cart speed (m/s)	0.412	0.407	0.399	0.374	0.338
Photogate position (m)	0.20	0.40	0.60	0.80	1.00

On the axes below, plot the data points for the speed *v* of the cart as a function of position *x*. Clearly scale and label all axes, as appropriate.

[Blank grid of axes for plotting speed *v* vs. position *x*, ruled in a 4x4 arrangement of major squares each subdivided into a 5x5 fine grid, no pre-printed scale numbers or labels.]

Question 1

2014 ▪ Q1

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(d)(i) Compare the speed of the cart measured by photogate 1 to the predicted value of the speed of the cart just after it loses contact with the spring. List a physical source of error that could account for the difference.

Question 1

(d)(ii) From the measured speed values of the cart as it rolls down the track, give a physical explanation for any trend you observe.