Past-paper walk-through

Resistive Forces ▪ 2.9

eXercise

Questions in this set

- 2026 Q1
- 2024 Set 1 Q2
- 2024 Set 2 Q2
- 2019 Set 1 Q1
- 2017 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2026 ▪ Q1

A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.
[Figure 1: Side view diagram. A box labeled "Box, M" slides to the right (shown by motion lines and arrows on the left). Inside the box, a small shaded cube labeled "Cube" and "M" rests against the inside of the right wall of the box, above the bottom of the box, labeled with μ_s, μ_k at the contact surface. An arrow labeled $\vec{F}_R$ points left, into the box, from the right side, representing the resistive force on the box.]
The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_R$ is modeled by $\vec{F}_R = -b\vec{v}$, where b is a positive constant

Question 1

and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.
Consider the motion of the box and cube for the following times.
- At time $t = 0$, the box-cube system slides to the right with speed v_0. - For $0 \leq t < t_{\text{crit}}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box. - At $t = t_{\text{crit}}$, the cube begins to slide downward along the wall of the box.
(a)(i) The magnitude of the normal force exerted on the cube by the wall of the box is F_N.
Determine an expression for the magnitude of the acceleration of the cube. Express your answer in terms of M, F_N, and physical constants, as appropriate.
(a)(ii) Derive an expression for F_N as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box. Express your answer in terms of M, b,

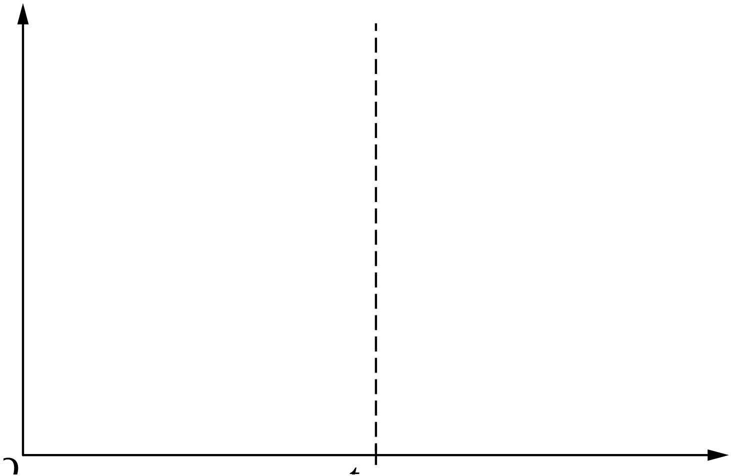
Question 1

v_0, t, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

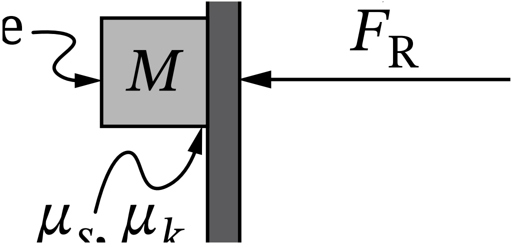
(a)(iii) On the axes shown in Figure 2, **sketch** a graph of the magnitude F_f of the frictional force exerted on the cube by the wall of the box as a function of t from $t = 0$ until immediately before the cube reaches the bottom of the box.

[Figure 2: A blank graph with vertical axis labeled F_f and horizontal axis labeled t, origin labeled O. A vertical dashed line on the horizontal axis is labeled t_{crit}.]

Question 1



Question 1



Question 1

Question 1

2026 ▪ Q1

A box of mass M slides to the right on a horizontal surface. A small cube, also of mass M, is inside the box. The cube is against the right wall of the box and above the bottom of the box, as shown in Figure 1. The coefficients of static and kinetic friction between the cube and the wall of the box are μ_s and μ_k, respectively, where $\mu_s > \mu_k$.

[Figure 1: Side view diagram. A box labeled "Box, M" slides to the right (shown by motion lines and arrows on the left). Inside the box, a small shaded cube labeled "Cube" and "M" rests against the inside of the right wall of the box, above the bottom of the box, labeled with μ_s, μ_k at the contact surface. An arrow labeled $\vec{F}_R$ points left, into the box, from the right side, representing the resistive force on the box.]

The speed of the box is decreasing as a result of a resistive force that is exerted on the box. The resistive force $\vec{F}_R$ is modeled by $\vec{F}_R = -b\vec{v}$, where b is a positive constant and $\vec{v}$ is the velocity of the box. Friction between the box and horizontal surface is negligible.

Consider the motion of the box and cube for the following times.

Question 1

- At time $t = 0$, the box-cube system slides to the right with speed v_0. - For $0 \leq t < t_{\text{crit}}$, the cube remains in contact with the wall of the box at a constant height above the bottom of the box. - At $t = t_{\text{crit}}$, the cube begins to slide downward along the wall of the box.

(b) **Derive** an expression for t_{crit} in terms of M, b, μ_s, v_0, and physical constants, as appropriate. Begin your derivation with a fundamental physics principle or an equation from the reference information.

Question 2

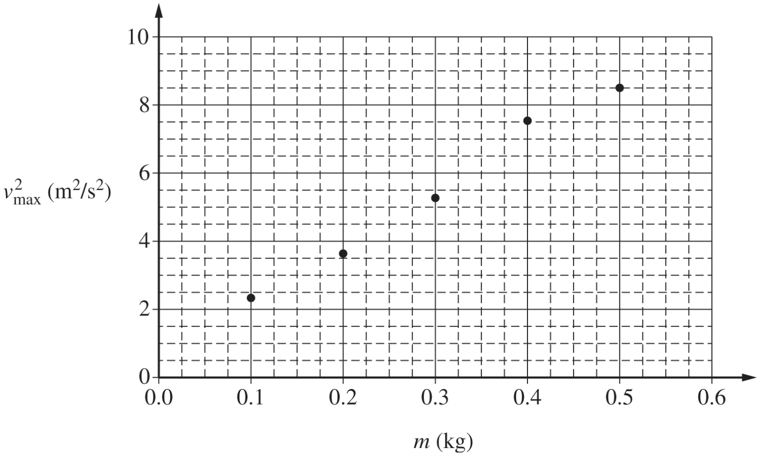
2024 Set 1 ▪ Q2

[Figure 1: A horizontal cylinder is shown falling, with several downward arrows above it indicating the direction it was dropped from (air resistance direction shown), and an upward arrow labeled F_{drag} acting on the cylinder.]

A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(a) Derive, but do NOT solve, a differential equation that could be used to determine the speed v of the cylinder as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

Question 2



Question 2

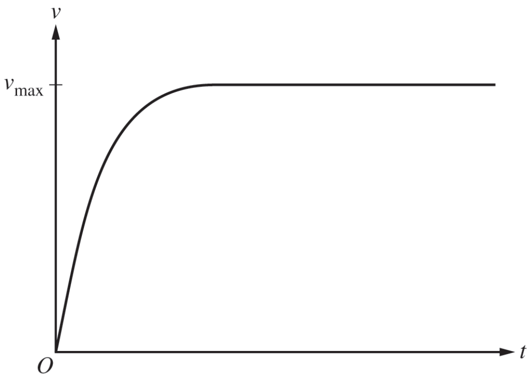


Figure 2

Question 2

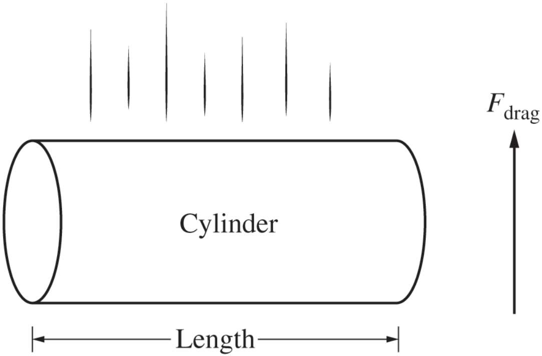


Figure 3

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A horizontal cylinder is shown falling, with several downward arrows above it indicating the direction it was dropped from (air resistance direction shown), and an upward arrow labeled F_{drag} acting on the cylinder.]

A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(b)(i) [Figure 2: Graph of speed v (vertical axis) versus time t (horizontal axis), starting at origin O. The curve rises steeply at first, curving over and leveling off asymptotically at a horizontal line labeled v_{max}.]

The student correctly sketches the speed v of the cylinder as a function of time t, as shown in Figure 2.

Question 2

Draw a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} on the cylinder is equal to the magnitude of the weight of the cylinder. Label this time as t_1 on the time axis.

(b)(ii) Justify the location of t_1. Explicitly reference appropriate features of the sketch in Figure 2.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A horizontal cylinder is shown falling, with several downward arrows above it indicating the direction it was dropped from (air resistance direction shown), and an upward arrow labeled F_{drag} acting on the cylinder.]

A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(c) Rather than dropping the cylinder from rest, the student throws the cylinder upward with a nonzero initial speed. The cylinder is in the same orientation as when the cylinder was previously dropped. The student allows the cylinder to fall toward the ground.

Question 2

Indicate whether the magnitude of the cylinder's maximum downward speed after being thrown upward would be greater than, less than, or equal to the maximum speed v_{max} in Figure 2.

_______ Greater than _______ Less than _______ Equal to

Briefly justify your answer.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A horizontal cylinder is shown falling, with several downward arrows above it indicating the direction it was dropped from (air resistance direction shown), and an upward arrow labeled F_{drag} acting on the cylinder.]

A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(d)(i) The student conducts an experiment to better understand the relationship between maximum speed v_{max} and mass. The student collects data to determine the maximum speed for cylinders dropped from rest, each with the same physical size and shape but a different mass m. The student then graphs v_{max}^2 as a function of mass.

Question 2

[Scatter plot: vertical axis v_{max}^2 ($\mathrm{m}^2/\mathrm{s}^2$) from 0 to 10 in increments of 2; horizontal axis m (kg) from 0.0 to 0.6 in increments of 0.1. Data points approximately at: $(0.1, 2.6)$, $(0.15, 3.3)$, $(0.2, 4.0)$, $(0.3, 5.6)$, $(0.35, 6.0)$, $(0.45, 7.6)$, $(0.5, 7.5)$, $(0.55, 8.6)$. Gridlines shown at every 0.05 kg horizontally and 0.5 $\mathrm{m}^2/\mathrm{s}^2$ vertically.]

Draw the best-fit line for the data.

(d)(ii) Use the best-fit line to calculate an experimental value for b.

Question 2

2024 Set 1 ▪ Q2

[Figure 1: A horizontal cylinder is shown falling, with several downward arrows above it indicating the direction it was dropped from (air resistance direction shown), and an upward arrow labeled F_{drag} acting on the cylinder.]

A student drops a cylinder of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the cylinder, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv^2$, where v is the speed of the cylinder and b is a positive constant with appropriate units.

(e)(i) [Figure 3: A horizontal cylinder shown with several downward arrows above it (direction from which it was dropped) and an upward arrow labeled F_{drag}; a labeled dimension "Length" with an arrow spanning the cylinder's length.]

A student claims that the magnitude of the maximum speed of a cylinder dropped from rest depends on the length of the cylinder. The student designs an experiment to

Question 2

collect data that can be used to provide evidence to support the claim. The student drops cylinders with the orientation shown in Figure 3.

The student has access to but does not have to use all of the following equipment.

- Cylinder Set 1: cylinders of the same known length with different known masses
- Cylinder Set 2: cylinders of the same known mass with different known lengths
- A motion detector that can measure velocity as a function of time

Indicate two quantities that when graphed could be used to determine whether the length of the cylinder affects the maximum speed.

Vertical axis: ______________________ Horizontal axis: ______________________

(e)(ii) Briefly describe how the quantities graphed could be used to determine the relationship between cylinder length and maximum speed.

Solution 2

(a) Mark scheme

- Apply Newton's second law to the falling cylinder, where the only forces are gravity and drag: $\Sigma F = ma \Rightarrow F_g - F_{drag} = ma_y \Rightarrow mg - bv^2 = ma_y$ (using the given quadratic drag force). Writing $a_y = dv/dt$: $m\dfrac{dv}{dt} = mg - bv^2$. (Variables do not need to be separated to earn full credit.) Points: (1) a multi-step derivation that includes Newton's second law; (2) indicating the net force includes only the gravitational force and a drag force; (3) a correct differential equation in terms of the given quantities.

Solution 2

(b)(i)

Mark scheme

- Draw a vertical line labeled t_1 at the approximate location on the v-t graph where the curve first becomes (essentially) horizontal, i.e., where v first reaches v_{max} — this is the earliest time the drag force equals the cylinder's weight in magnitude. Point: correct approximate placement of the vertical line at that location.

Solution 2

(b)(ii)

Mark scheme

- Because the sketched line is horizontal after t_1, the velocity is constant. If the velocity is constant, the acceleration is zero. Therefore the net force is zero, which means the gravitational and drag forces are equal in magnitude at t_1 (terminal velocity reached). Points: (1) relating t_1 to the time the v-t graph becomes constant / has zero slope; (2) indicating that constant velocity means the net force (and hence acceleration) is zero, so drag equals weight.

Solution 2

(c)

Mark scheme

- Answer: "Equal to." Justification: at its peak (after being thrown upward), the cylinder momentarily has a speed of 0 m/s — the same initial condition as when it was dropped from rest. Because the cylinder then falls from that height under the same forces (gravity and the same v^2 drag law) as in the original drop-from-rest scenario, it reaches the same v_{max}. Points: (1) selecting "Equal to" with an attempt at a relevant justification; (2) a correct justification (recognizing zero speed at the peak makes the subsequent fall identical to a drop from rest).

Solution 2

(d)(i)

Mark scheme

- Draw a straight line of best fit through the origin (approximately) that follows the trend of the plotted v_{max}^2 vs. m data points, e.g. passing near $(0.25\ \text{kg}, 4.5\ \text{m}^2/\text{s}^2)$ and $(0.5\ \text{kg}, 9\ \text{m}^2/\text{s}^2)$. Point: an appropriate straight line that reasonably approximates the data (not simply connecting dots).

Solution 2

(d)(ii) Mark scheme

- Using two points on the best-fit line, e.g. $(0.25 \text{ kg}, 4.5 \text{ m}^2/\text{s}^2)$ and $(0.5 \text{ kg}, 9 \text{ m}^2/\text{s}^2)$: slope $= \dfrac{9 - 4.5}{0.5 - 0.25} = 18\ (\text{m}^2/\text{s}^2)/\text{kg}$. At terminal velocity $mg - bv_{max}^2 = 0 \Rightarrow v_{max}^2 = (g/b)m$, so slope $= g/b \Rightarrow b = g/\text{slope} = \dfrac{9.8}{18} \approx 0.544 \text{ kg/m}$. Points: (1) calculating a slope value using two points on the best-fit line (points that fall on the line are acceptable); (2) using the correct relationship slope $= g/b$; (3) a calculated value of b in the range $0.45 \text{ kg/m} \leq b \leq 0.75 \text{ kg/m}$.

Solution 2

(e)(i)

Mark scheme

- The student should graph the length of the cylinder (independent variable) against the maximum (terminal) speed v_{max} of the cylinder (dependent variable), since the claim under test is that cylinder length affects maximum speed. Points: (1) indicating that the length of the cylinder should be graphed; (2) indicating that the maximum velocity of the cylinder should be graphed.

Solution 2

(e)(ii)

Mark scheme

- The slope of the length vs. maximum-velocity graph can be used to determine whether length affects terminal velocity (e.g., a nonzero slope indicates a relationship, and its sign/magnitude describes how v_{max} depends on length). Point: a correct description linking the graphed quantities to the experiment's conclusion.

Question 2

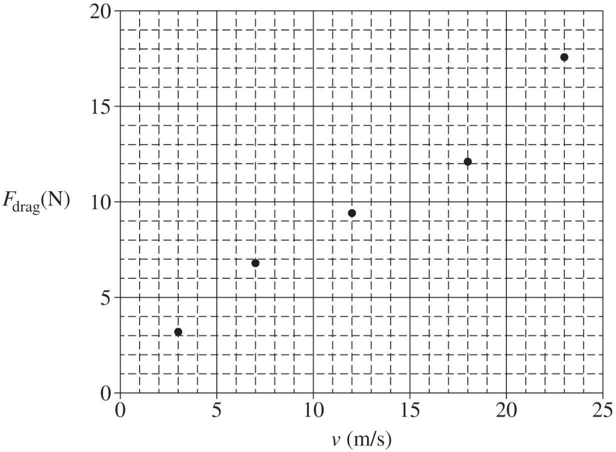
2024 Set 2 ▪ Q2

[Figure 1: A sphere of diameter D falls downward through the air; arrows above the sphere point downward representing airflow/motion, and an arrow labeled F_{drag} points upward from the top of the sphere, opposing the motion. The sphere is labeled "Sphere" with its diameter D marked across it.]

A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

(a) Derive, but do NOT solve, a differential equation that could be used to determine the speed v of the sphere as a function of time t. Express your answer in terms of given quantities and physical constants, as appropriate.

Question 2



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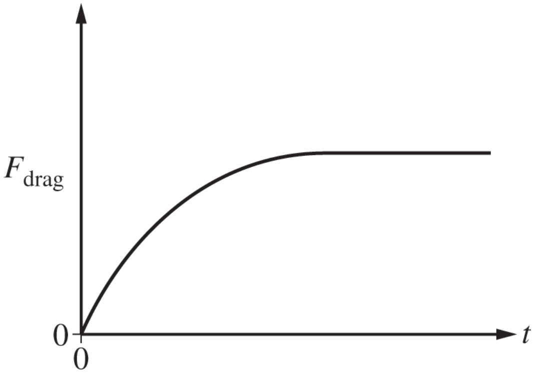


Figure 2

Question 2

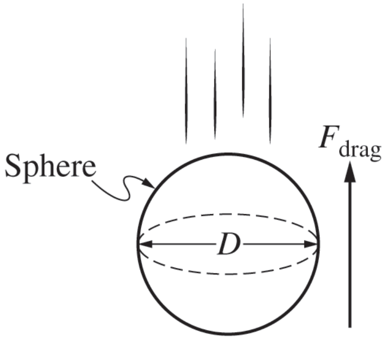


Figure 1

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A sphere of diameter D falls downward through the air; arrows above the sphere point downward representing airflow/motion, and an arrow labeled F_{drag} points upward from the top of the sphere, opposing the motion. The sphere is labeled "Sphere" with its diameter D marked across it.]

A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

(b)(i) [Figure 2: A graph of drag force F_{drag} (vertical axis) versus time t (horizontal axis), both axes starting at 0. The curve starts at the origin, rises steeply, then curves over and levels off to a constant (horizontal) value at large t.]

Question 2

The student sketches the drag force F_{drag} exerted on the sphere as a function of time t, as shown in Figure 2.
Draw a vertical line on the sketch in Figure 2 to indicate the earliest time at which F_{drag} is equal to the magnitude of the weight of the sphere, which occurs when the sphere reaches terminal speed. Label this time as t_T on the time axis.
(b)(ii) Justify the location of t_T. Explicitly reference appropriate features of the sketch in Figure 2.

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A sphere of diameter D falls downward through the air; arrows above the sphere point downward representing airflow/motion, and an arrow labeled F_{drag} points upward from the top of the sphere, opposing the motion. The sphere is labeled "Sphere" with its diameter D marked across it.]

A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

(c) Suppose the student throws the same sphere downward with a nonzero initial speed. The magnitude of the new drag force at terminal speed after being thrown downward is F_{new}.

Question 2

Indicate whether F_{new} would be greater than, less than, or equal to the magnitude of F_{drag} at terminal speed represented in Figure 2.

______ Greater than ______ Less than ______ Equal to

Briefly justify your answer.

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A sphere of diameter D falls downward through the air; arrows above the sphere point downward representing airflow/motion, and an arrow labeled F_{drag} points upward from the top of the sphere, opposing the motion. The sphere is labeled "Sphere" with its diameter D marked across it.]

A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

(d)(i) The student conducts an experiment to better understand the relationship between F_{drag} and v. The student makes measurements to calculate and graph the magnitude of F_{drag} as a function of v for the falling sphere.

Question 2

[Graph: scatter plot of F_{drag} (N) on the vertical axis, from 0 to 20 in increments of 5, versus v (m/s) on the horizontal axis, from 0 to 25 in increments of 5, on a fine grid. Data points (approximate positions read from the grid): $(3, 3)$, $(6, 6)$, $(9, 6.5)$, $(12, 9.5)$, $(15, 13.5)$, $(18, 17.5)$, $(20, 18)$, $(21, 18.5)$. The points show a roughly linear increasing trend with some scatter.]

Draw the best-fit line for the data.

(d)(ii) Use the best-fit line to calculate an experimental value for b.

Question 2

2024 Set 2 ▪ Q2

[Figure 1: A sphere of diameter D falls downward through the air; arrows above the sphere point downward representing airflow/motion, and an arrow labeled F_{drag} points upward from the top of the sphere, opposing the motion. The sphere is labeled "Sphere" with its diameter D marked across it.]

A student drops a sphere of mass m from rest. The air exerts a drag force of magnitude F_{drag} on the sphere, as shown in Figure 1. The student models the magnitude of the drag force as $F_{drag} = bv$, where v is the speed of the sphere and b is a positive constant with appropriate units.

(e)(i) A student claims that the terminal speed v_T of the sphere depends on the diameter D of the sphere. The student designs an experiment to collect data that can be used to provide evidence to support the claim.

The student has access to but does not have to use all of the following equipment.

Question 2

- Sphere Set 1: spheres of the same known mass with different known diameters
- Sphere Set 2: spheres of the same known diameter with different known masses
- A motion detector that can measure velocity as a function of time

Indicate two quantities that when graphed could be used to determine whether the diameter of the sphere affects the terminal speed.

Vertical axis: ____________________ Horizontal axis: ____________________

(e)(ii) Briefly describe how the quantities graphed could be used to determine the relationship between sphere diameter and terminal speed.

Solution 2

(a)

Mark scheme

- Newton's second law on the falling sphere, with only gravity and (linear) drag acting: $F_{net} = F_g - F_{drag}$, i.e. $ma = mg - bv$. Writing $a = dv/dt$:

$$m\frac{dv}{dt} = mg - bv.$$

Points: 1 for a multi-step derivation using Newton's second law, 1 for indicating the net force on the sphere includes only the gravitational force and a drag force, 1 for a correct differential equation in terms of the given variables (need not be separated).

Solution 2

(b)(i)

Mark scheme

- Draw a vertical line on the F_{drag}-vs-t sketch at time t_T, located at the approximate point where the curve transitions from rising/concave-down to flat/horizontal – i.e. the earliest time the curve becomes (approximately) horizontal, marking where F_{drag} first equals the magnitude of the sphere's weight.

Solution 2

(b)(ii) Mark scheme

- Justification referencing the graph's shape: for $t < t_T$ the slope of the F_{drag}-vs-t curve is positive, meaning the magnitude of the drag force is still increasing; at and after t_T the slope is (and stays) zero, meaning the drag force is constant and equal to the downward gravitational force. A constant drag force equal to gravity means the net force on the sphere is zero, so the sphere has reached a constant terminal velocity – that is why t_T is placed where the curve flattens. Points: 1 for a response that references the slope of the graph or the rate at which the slope changes, 1 for correctly relating that feature of the graph to the forces exerted on the sphere as it reaches terminal speed.

Solution 2

(c)

Mark scheme

- 'Equal to' – F_{new} equals the magnitude of F_{drag} at terminal speed shown in Figure 2. Justification: at terminal speed the net force on the sphere is zero, so the drag force must equal the sphere's weight mg regardless of the initial speed or the direction the sphere was thrown; since the sphere's mass is unchanged, the drag force at terminal speed is the same as before.

Solution 2

(d)(i)

Mark scheme

- Draw a straight best-fit line through the scattered (v, F_{drag}) data points on the graph, balancing points above and below the line (e.g. a line passing near/through $(5\ \text{m/s}, 5\ \text{N})$ and $(21\ \text{m/s}, 15\ \text{N})$).

Solution 2

(d)(ii) Mark scheme

- Using two points that lie on the best-fit line, e.g. $(5\text{ m/s}, 5\text{ N})$ and $(21\text{ m/s}, 15\text{ N})$: slope $= \dfrac{15\text{ N} - 5\text{ N}}{21\text{ m/s} - 5\text{ m/s}}$. Since $|F_{drag}| = bv$, the slope of the F_{drag}-vs-v line equals b: slope $= |F_{drag}|/v = b$. So $b = \dfrac{10\text{ N}}{16\text{ m/s}} = 0.625\text{ kg/s}$ (accepted range $0.6\text{ kg/s} < b < 0.8\text{ kg/s}$). Points: 1 for calculating a value for the slope using two points on the best-fit line, 1 for using the correct relationship slope $= b$, 1 for a calculated value of b in the accepted range (example: $b = 0.625\text{ kg/s}$).

Solution 2

(e)(i)

Mark scheme

- Using Sphere Set 1 (spheres of the same known mass but different known diameters) and the motion detector to measure each sphere's terminal velocity, the student should graph the terminal velocity v_T of the sphere (dependent variable, vertical axis) versus the diameter D of the sphere (independent variable, horizontal axis), holding mass constant across trials so diameter is the only varied quantity. Points: 1 for indicating the diameter of the sphere should be graphed, 1 for indicating the terminal velocity of the sphere should be graphed.

Solution 2

(e)(ii)

Mark scheme

- The slope (or overall trend/shape) of the terminal-velocity-vs-diameter graph can be used to determine whether sphere diameter affects terminal velocity: a nonzero slope (a clear trend of v_T changing with D) supports the claim that terminal speed depends on diameter, while a slope of approximately zero (no trend) would not support it.

Question 1

2019 Set 1 ▪ Q1

In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.
[Figure above the question: a cylinder of fluid on a table, with hands releasing an object labeled "Object, Mass = 12 g" at the top of the "Cylinder" containing "Fluid", resting on a "Table".]
[Graph: v (m/s) vs. t (s); y-axis from 0.0 to 1.0 in increments of 0.2, x-axis from 0.00 to 0.35 in increments of 0.05. Data points (dots) roughly follow a dashed best-fit curve that rises steeply from the origin, e.g. passing near (0.00, 0.0), (0.025, 0.17), (0.05, 0.25), (0.075, 0.42), (0.10, 0.47), (0.125, 0.58), (0.15, 0.66), (0.175, 0.65),

Question 1

(0.20, 0.76), (0.225, 0.78), (0.25, 0.80), (0.275, 0.90), (0.30, 0.93), (0.325, 0.92); the curve rises quickly at first then levels off, approaching an asymptote near $v \approx 1.0$ m/s.]

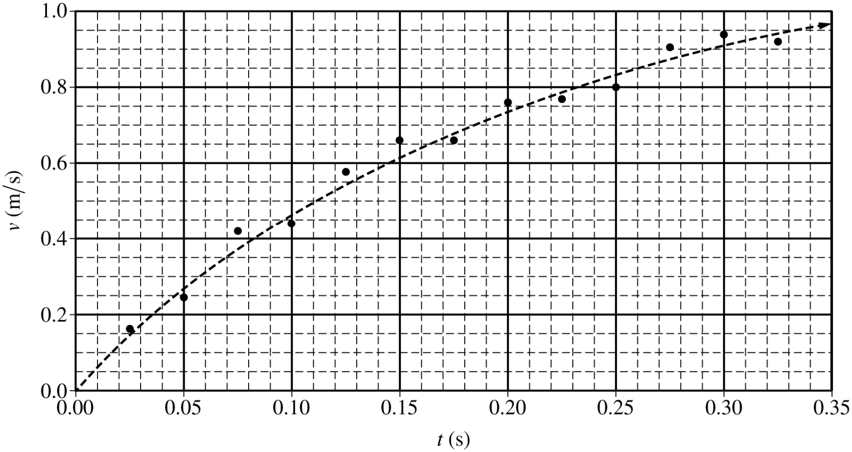
(a)(i) Does the speed of the object increase, decrease, or remain the same?

______ Increase   ______ Decrease   ______ Remain the same

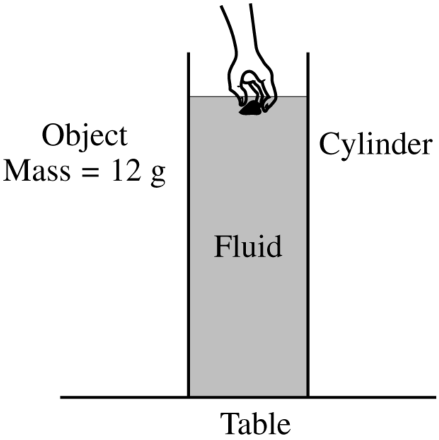
(a)(ii) In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

(a)(iii) Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

Question 1



Question 1



Question 1

2019 Set 1 ▪ Q1

In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.

[Figure above the question: a cylinder of fluid on a table, with hands releasing an object labeled "Object, Mass = 12 g" at the top of the "Cylinder" containing "Fluid", resting on a "Table".]

[Graph: v (m/s) vs. t (s); y-axis from 0.0 to 1.0 in increments of 0.2, x-axis from 0.00 to 0.35 in increments of 0.05. Data points (dots) roughly follow a dashed best-fit curve that rises steeply from the origin, e.g. passing near (0.00, 0.0), (0.025, 0.17), (0.05, 0.25), (0.075, 0.42), (0.10, 0.47), (0.125, 0.58), (0.15, 0.66), (0.175, 0.65), (0.20, 0.76), (0.225, 0.78), (0.25,

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0.80), (0.275, 0.90), (0.30, 0.93), (0.325, 0.92); the curve rises quickly at first then levels off, approaching an asymptote near $v \approx 1.0$ m/s.]

(b)(i) The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5$ s^{-1}. Use the above equation to derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

(b)(ii) Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

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2019 Set 1 ▪ Q1

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(c)(i) The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed. Determine the constant speed of the object.

Justify your answer.

(c)(ii) Determine the force exerted by the fluid on the object at this time.

Justify your answer.

Solution 1

(a)(i)

Mark scheme

- The speed **Increases** (full credit for selecting 'Increase'). The object is falling and speeding up as it accelerates through the fluid.

Solution 1

(a)(ii)

Mark scheme

- The acceleration points downward (1 pt), and its magnitude decreases as the object falls (1 pt). Example: Because the object is moving downward and speeding up, the acceleration must be downward. Because the slope of the graph of speed vs. time is decreasing, the magnitude of the acceleration must be decreasing.

Solution 1

(a)(iii)

Mark scheme

- Draw a tangent line to the curve at $t = 0.20$ s and compute its slope using two points on that tangent line (1 pt): $a = \dfrac{\Delta v}{\Delta t} = \dfrac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$. Correct answer using points from the tangent line (1 pt): $a \approx 2.22\ \text{m/s}^2$ (accept values close to this from a reasonable tangent line).

Solution 1

(b)(i) Mark scheme

- Recognize that vertical displacement is the integral of velocity (1 pt): $\Delta y = \int v\,dt = \int A(1-e^{-Bt})\,dt$. Evaluate with appropriate limits or constant of integration (1 pt): $\Delta y = \int_0^t A(1-e^{-Bt'})\,dt' = A\left[\left(t+\frac{1}{B}e^{-Bt}\right)-\left(0+\frac{1}{B}e^{0}\right)\right]$. Correct final answer (1 pt): $\Delta y = A\left(t+\frac{1}{B}\left(e^{-Bt}-1\right)\right) = 1.18\left(t+\frac{1}{5}\left(e^{-5t}-1\right)\right)$. Credit is given for leaving the answer in terms of A and B or for plugging in the given numeric values.

Solution 1

(b)(ii)

Mark scheme

- Attempt the derivative of the speed equation (1 pt): $a = \frac{dv}{dt} = \frac{d}{dt}\left[A(1 - e^{-Bt})\right]$. Correct expression for acceleration (1 pt): $a = ABe^{-Bt}$. Multiply by the mass to get net force (1 pt): $F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$, i.e. $F = (0.012)(1.18)(5)e^{-5t} = 0.071\, e^{-5t}$ N. Credit is given for leaving the answer in terms of A, B, m or for plugging in the given numeric values.

Solution 1

(c)(i)

Mark scheme

- State that the constant (terminal) speed is $v = A$ (1 pt), found by letting $t \to \infty$ in the fit equation (1 pt). After a long time the falling object reaches a terminal constant speed in the fluid; setting $t \to \infty$ in $v = A(1 - e^{-Bt})$ gives the constant speed $v = A = 1.18$ m/s.

Solution 1

(c)(ii)

Mark scheme

- Indicate that the net force is zero at constant speed (1 pt), and that the resistive (fluid) force therefore equals the weight of the object (1 pt). Since the object moves at constant speed, the net force is zero; the only vertical forces are gravity and the fluid's resistive force, so the resistive force equals the weight: $F = mg = (0.012\ \text{kg})(9.8\ \text{m/s}^2) = 0.12\ \text{N}$.

Question 2

2017 ▪ Q2

[Figure: A block of mass m rests at the top of an inclined plane of height h, which descends smoothly onto a horizontal surface. Farther along the horizontal surface, a coiled spring is fixed against a wall. Note: Figure not drawn to scale.]

A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)(i) Derive an expression for the speed of the block just before it collides with the spring.

Question 2

(a)(ii) Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

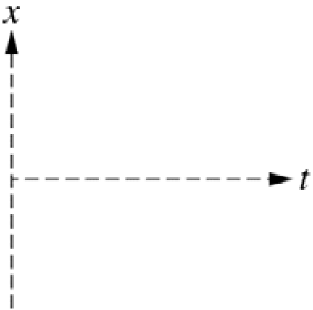
_____ Greater than _____ Less than _____ Equal to

Justify your answer.

Question 2

Note: Figure not drawn to scale.

Question 2



Question 2

2017 ▪ Q2

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(b) Derive an expression for the maximum compression of the spring.

Question 2

2017 ▪ Q2

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(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

Question 2

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface.
The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

Question 2

2017 ▪ Q2

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Question 2

(d)(i) Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

(d)(ii) Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

Question 2

2017 ▪ Q2

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Question 2

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.
[Three blank axes side by side, each with a vertical axis and horizontal axis labeled t: the first labeled x, the second labeled v, the third labeled a — for sketching the respective graphs.]

Solution 2

(a)(i)

Mark scheme

- Using conservation of energy for the block sliding down the frictionless incline: loss in gravitational PE = gain in KE, $U_g = K$, i.e. $mgh = \frac{1}{2}mv^2$. Solving: $v = \sqrt{2gh}$. 1 point for correctly applying conservation of energy ($U_g = K$), 1 point for the correct final answer $v = \sqrt{2gh}$.

Solution 2

(a)(ii) Mark scheme

- Correct answer: Greater than. Since $v = \sqrt{2gh}$ is proportional to $\sqrt{h}$, reducing the height by a factor of 2 reduces the speed by a factor of $\sqrt{2} \approx 1.41$ (not by a factor of 2). So the speed halfway down the incline is $v/\sqrt{2} \approx 0.71v$, which is greater than half the bottom speed $(0.5v)$. 1 point for correctly selecting 'Greater than' AND giving this correct justification – an incorrect selection cannot earn credit even with valid-looking reasoning.

Solution 2

(b)

Mark scheme

- Using conservation of energy ($U_g = U_s$) for the block compressing the spring, consistent with part (a): $mgh = \frac{1}{2}kx_{max}^2$. Solving for the maximum compression: $x_{max} = \sqrt{\frac{2mgh}{k}}$. 1 point for this correct answer, consistent with the part (a) approach.

Solution 2

(c) Mark scheme

- Recognize that the block's contact with the spring, from first touching it to maximum compression, is one-quarter of a simple-harmonic-motion cycle, with period $T = 2\pi\sqrt{m/k}$. So $t = \frac{1}{4}T = \frac{1}{4}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{\pi}{2}\sqrt{\frac{m}{k}}$. 1 point for identifying the simple-harmonic-motion (quarter-period) approach, 1 point for the correct final expression.

Solution 2

(d)(i) Mark scheme

- Newton's second law for the block on the horizontal surface, with the resistive force opposing the motion: $F_{net} = ma$, and $F_{net} = -\beta v^2$ (negative because it decelerates the block), giving $-\beta v^2 = ma$. Writing acceleration as dv/dt gives the differential equation $-\beta v^2 = m\dfrac{dv}{dt}$. 1 point for correctly applying Newton's second law ($F_{net} = ma$), 1 point for correctly indicating that F_{net} is opposite the direction of motion, 1 point for expressing the result as a differential equation in dv/dt.

Solution 2

(d)(ii) Mark scheme

- Separate variables in $-\beta v^2 = m\,dv/dt$ to get $-\dfrac{\beta}{m}dt = \dfrac{1}{v^2}dv$. Integrating both sides: $\int -\dfrac{\beta}{m}dt = \int \dfrac{1}{v^2}dv$ gives $-\dfrac{\beta}{m}[t] = \left[-\dfrac{1}{v}\right]$. Applying the limits $t : 0 \to t$ and $v : v_0 \to v(t)$: $-\dfrac{\beta t}{m} = -\dfrac{1}{v(t)} - \left(-\dfrac{1}{v_0}\right) = -\dfrac{1}{v(t)} + \dfrac{1}{v_0}$. Rearranging gives $\dfrac{1}{v(t)} = \dfrac{1}{v_0} + \dfrac{\beta t}{m}$, as required. 1 point for correctly separating variables, 1 point for correctly integrating, 1 point for correctly applying the limits/constant of integration to reach the given result.

Solution 2

(e) Mark scheme

- Since the resistive force continually decelerates the block (from the $1/v(t) = 1/v_0 + \beta t/m$ result, $v(t)$ decreases toward 0 but never reaches it in finite time, and the deceleration itself decreases), the three sketches are: position $x(t)$ – concave down, increasing and leveling off toward (approaching but not reaching) a horizontal asymptote; velocity $v(t)$ – concave up, decreasing from v_0 toward the horizontal (time) axis as an asymptote; acceleration $a(t)$ – concave down (magnitude decreasing, stays negative), also approaching the horizontal axis as an asymptote. (Full credit is also earned if all three curves are reflected vertically; if a plausible but different nonlinear $v(t)$ shape is drawn, $x(t)$ and $a(t)$ consistent with that shape still earn credit.) 1 point per correct curve (x, v, a).