Past-paper walk-through

Spring Forces ▪ 2.8

eXercise

Questions in this set

- 2026 Q3
- 2025 Q2
- 2024 Set 1 Q1
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- 2023 Set 1 Q1
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- 2016 Q3

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2026 ▪ Q3

The following information applies to parts A and B.

A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

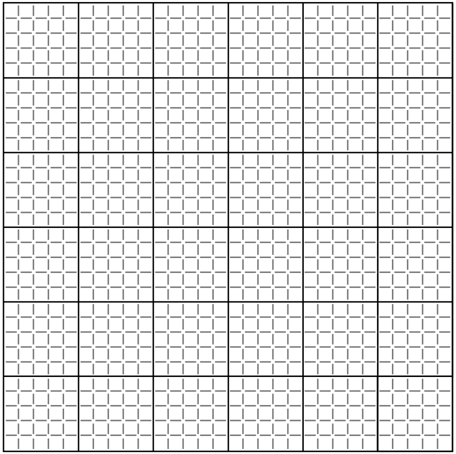
[Figure 1: A horizontal diagram with a $+x$ axis arrow pointing right at the top. A spring with spring constant k is attached to a wall on the left and connects to a block labeled m. To the right of the block (for $x > 0$), the surface is marked with a squiggly friction symbol and labeled μ_k. Below the block is the label $x = 0$.]

A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

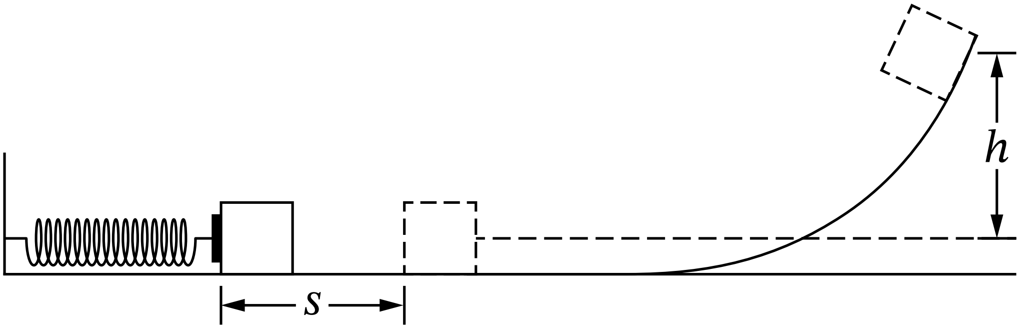
Question 3

(a)(i) Indicate quantities that could be measured by the student that would allow them to determine μ_k using a graph.
(a)(ii) Briefly **describe** a method to reduce experimental uncertainty for the measured quantities.

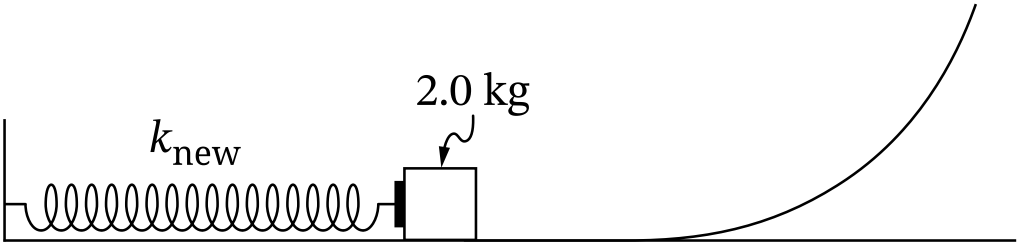
Question 3



Question 3



Question 3



Question 3

2026 ▪ Q3

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A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Question 3

(b)(i) Indicate what quantities the student could graph on the horizontal and vertical axes to create a graph that can be used to determine μ_k.
(b)(ii) Briefly **describe** the relationship between μ_k and a feature of the graph from part B (i). Your answer may include an equation that relates μ_k and the chosen feature of the graph.

Question 3

2026 ▪ Q3

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A horizontal spring of known spring constant k is attached to a wall. A block of known mass m is placed on a horizontal surface next to but not attached to the spring. The spring is at its relaxed length when the block is at position $x = 0$, as shown in Figure 1. There is friction between the block and the surface only for positions $x > 0$.

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A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

(c)(i)(n)(t)(r)(o) The following information applies to parts C and D.

Question 3

In a different experiment, the student is asked to determine the spring constant k_{new} of a new spring that is attached to a wall. A block of mass 2.0 kg is placed next to the spring on a surface where frictional forces are negligible. The surface is horizontal near the spring and slopes upward into a ramp to the right of the initial position of the block, as shown in Figure 2.
[Figure 2: A diagram showing a spring labeled k_{new} attached to a wall on the left, connected to a block labeled "2.0 kg" with an arrow "f" curving above it. The horizontal surface to the right of the block slopes upward into a curved ramp rising to the upper right.]
The student pushes the block, compressing the spring a distance s, as shown in Figure 3. The block is released from rest.
[Figure 3: A diagram showing the spring compressed, attached to the wall on the left and to the block, with a distance s marked by a double-arrow between the wall-side

Question 3

spring end and the block's initial (uncompressed) dashed outline position. To the right, a dashed outline of the block is shown partway up the curved ramp at height h above the horizontal surface level, with h marked by a vertical double-arrow.]

The student uses a meterstick to measure the maximum height h of the block on the ramp. The experiment is repeated several times with different compression distances s. The student's measurements of s and h are shown in Table 1.

Table 1

| s (m) | h (m) | |—|—| | 0.020 | 0.02 | | 0.040 | 0.04 | | 0.060 | 0.13 | | 0.080 | 0.18 | | 0.100 | 0.33 |

(c)(i) **Label** the axes of the grid provided with measured or calculated quantities. Include units, as appropriate. The graphed quantities should yield a linear graph that can be used to determine k_{new}.

(c)(ii) On the grid provided, create a graph of the quantities indicated in part C (i).

Question 3

- Clearly **label** the vertical and horizontal axes with numerical scales.
- **Plot** the corresponding data points on the grid.
- Table 2 is provided in your booklet for scratch work and will not be scored.

[Figure: Two blank labeled-axis lines reading "Quantity (units, if appropriate)" each with two blank underlines for axis labels, flanking a blank square grid for plotting, printed twice (before and after the grid) for the vertical and horizontal axes.]

(c)(iii) Draw a best-fit line for the data graphed in part C (ii).

Question 3

2026 ▪ Q3

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A student is asked to experimentally determine the coefficient of kinetic friction μ_k between the block and the surface using a graph. The student is permitted to use only measurements from a meterstick.

Question 3

(d) Using the best-fit line that you drew in part C (iii), **calculate** an experimental value for k_{new}.

Question 2

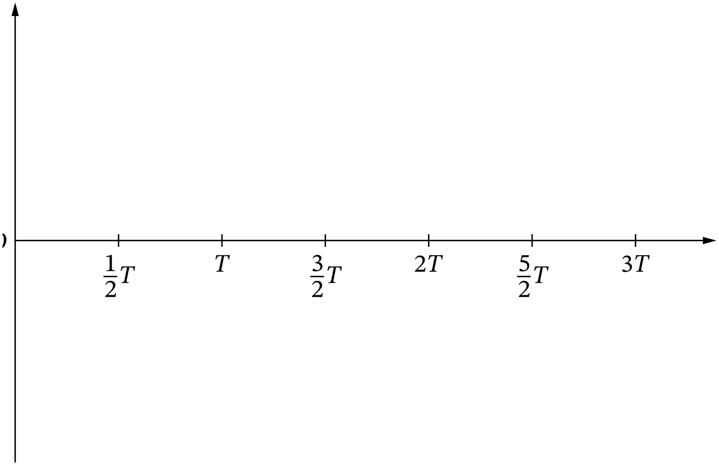
2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.
[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]
The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.
[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis

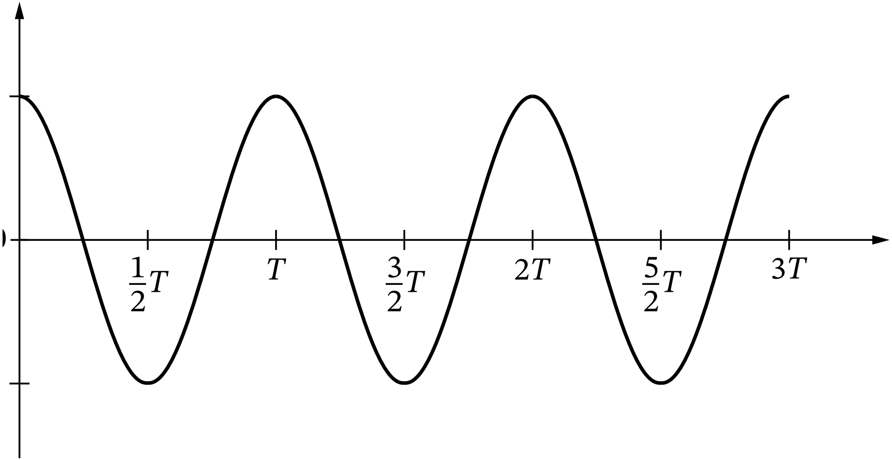
Question 2

is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]
(a) An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.
- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}. - Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.
[Figure 3: A blank bar-chart grid titled "Energy" (vertical axis) with three labeled columns: U_A, U_B, K_{block}, and a horizontal "0" line partway up the grid, with gridlines above and below for plotting bar heights.]

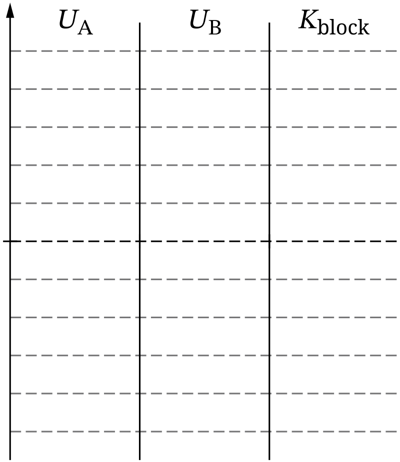
Question 2



Question 2



Question 2



Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(b) The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.
[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]
The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.
[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(c)(i) In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.
[Figure 4: Graph titled "Scenario 1" of position x vs. time t. Vertical axis x with values $+x_1$ marked above 0 and $-x_1$ marked below 0; horizontal axis t with gridmarks at $\frac{1}{2}T$, T, $\frac{3}{2}T$, $2T$, $\frac{5}{2}T$, $3T$. The curve is a cosine-like oscillation: starts at $+x_1$ at $t=0$, crosses zero at $\frac{1}{2}T$ going down to $-x_1$ at around $\frac{1}{2}T$ to T, back up through zero, reaching $+x_1$ again at T, and repeating this pattern periodically through $3T$.]
In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.
On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.

Question 2

[Figure 5: Blank axes titled "Scenario 2" with vertical axis K and horizontal axis t with gridmarks at $\frac{1}{2}T$, T, $\frac{3}{2}T$, $2T$, $\frac{5}{2}T$, $3T$, origin labeled O.]

Question 2

2025 ▪ Q2

In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

[Figure 1: Spring A (spring constant k) attached to a wall on the left, connected to a block; Spring B (spring constant $2k$) attached to the block on the right, connected to a wall on the right. The block's position is marked $x = 0$.]

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

[Figure 2: Same spring-block system as Figure 1, now with the block displaced to the right to position $x = x_1$ (Spring A stretched, Spring B compressed). A coordinate axis is shown at top right with $+y$ (up) and $+x$ (right) directions labeled. The positions $x = 0$ and $x = x_1$ are marked on the horizontal surface.]

Question 2

(d) In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.
The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.
Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.
Briefly **justify** your answer.

Solution 2

(a) Mark scheme

- Energy bar chart at $x = x_1$ (Figure 3), where the block is momentarily at rest (released from rest at x_1, so all mechanical energy is stored in the two springs): the K_{block} bar has zero height [Point A1]. Both U_A and U_B bars are drawn with positive height, since elastic PE stored in a spring is never negative [Point A2]. Spring B has spring constant $2k$ (twice Spring A's k) and both springs share displacement x_1: $U_A = \frac{1}{2}kx_1^2$ and $U_B = \frac{1}{2}(2k)x_1^2 = 2U_A$, so the U_B bar is drawn exactly twice the height of the U_A bar (this point is awarded on the height ratio regardless of which sign/direction the bars are drawn in) [Point A3].

Solution 2

(b) Mark scheme

- Derive v at $x = \frac{1}{2}x_1$ using energy conservation (an equivalent SHM method is also accepted) [Point B1: multistep derivation that includes energy conservation or simple harmonic motion]. The system behaves as a single effective spring combining both springs acting on the block together, $k_{eff} = k + 2k = 3k$ [Point B2: relates the presence of both springs to the system's behavior]. Initial state at $x = x_1$: $E_{tot,0} = U_A(x_1) + U_B(x_1) + K_{block}(x_1) = \frac{1}{2}kx_1^2 + \frac{1}{2}(2k)x_1^2 + 0 = \frac{3}{2}kx_1^2$. Final state at $x = \frac{1}{2}x_1$ [Point B3: relates positions $x = x_1$ and $x = \frac{1}{2}x_1$ to the oscillation of the block]: $E_{tot,f} = \frac{1}{2}k\left(\frac{1}{2}x_1\right)^2 + \frac{1}{2}(2k)\left(\frac{1}{2}x_1\right)^2 + K_{block,\frac{1}{2}x_1} = \frac{3}{8}kx_1^2 + K_{block,\frac{1}{2}x_1}$. Setting $E_{tot,0} = E_{tot,f}$:

Solution 2

$\frac{3}{2}kx_1^2 = \frac{3}{8}kx_1^2 + K_{block,\frac{1}{2}x_1} \Rightarrow K_{block,\frac{1}{2}x_1} = \frac{9}{8}kx_1^2 \Rightarrow \frac{1}{2}mv^2 = \frac{9}{8}kx_1^2 \Rightarrow v = \frac{3}{2}x_1\sqrt{\frac{k}{m}}$ [Point B4: correct final expression for v in terms of m, k, x1]. (Equivalent SHM route: $\omega = \sqrt{3k/m}$, $x(t) = x_1 \cos \omega t$, solve $\cos \omega t = \frac{1}{2} \Rightarrow \omega t = \pi/3$, then $v = x_1 \omega \sin(\pi/3) = \frac{3}{2}x_1\sqrt{k/m}$ – same result.)

Solution 2

(c)(i) Mark scheme

- Sketch K (kinetic energy of the block) vs. t for Scenario 2 (the block oscillating on the two-spring system WITH friction/damping), on axes marked at $\frac{1}{2}T, T, \frac{3}{2}T, 2T, \frac{5}{2}T, 3T$ (Figure 5), where T is the period from the frictionless Scenario 1. The block is released from rest at x_1, so $K = 0$ at $t = 0$, and the curve must start at zero and remain ≥ 0 for all t (kinetic energy cannot be negative) [Point C1]. Because the block passes through $x = 0$ (where K is maximum) twice per full oscillation period T and momentarily has $K = 0$ at each turning point (also twice per period T), the sketched curve should be periodic with zeros spaced $\frac{1}{2}T$ apart – i.e. humps peaking near $\frac{1}{4}T, \frac{3}{4}T, \frac{5}{4}T, \ldots$ and touching zero at $0, \frac{1}{2}T, T, \frac{3}{2}T, 2T, \ldots$ [Point C2]. Because friction dissipates

Solution 2

mechanical energy each cycle, the successive peak heights of K must decrease over time, i.e. the curve is a periodic wave of decreasing amplitude [Point C3].

Solution 2

(d) Mark scheme

- Scenario 3: the block is replaced with a new, LARGER-mass block (same coefficient of kinetic friction as Scenario 2). Compare the K vs. t graph of Scenario 3 to that of Scenario 2 (part C) and describe ONE feature that would differ, with a correct justification. Acceptable answers (state ONE, with matching reasoning): (1) 'The period increases' – because the period of the oscillator is $T = 2\pi\sqrt{m/k_{eff}}$, and increasing the mass m (with k_{eff} unchanged) increases T [Point D1 + Point D2]. (2) 'The rate at which the graph approaches zero amplitude increases' – because a larger mass under the same coefficient of friction experiences a larger friction force, so the block loses mechanical energy at a greater rate. (3) 'The maximum value of K over each period is less than the

Solution 2

graph drawn [in part C]' – a valid alternative feature with matching reasoning (more energy lost to friction per cycle). Award Point D1 for indicating any ONE of these three features, and Point D2 for a justification that correctly supports the specific feature chosen.

Question 1

2024 Set 1 ▪ Q1

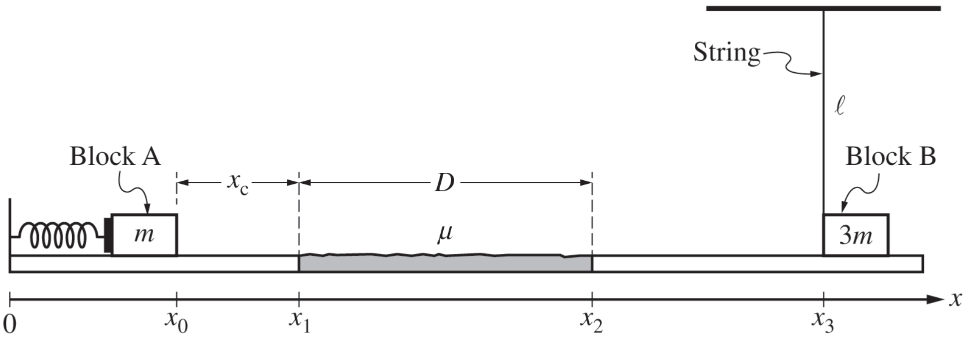
Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]

Question 1

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.
- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.
(a)(i) Derive an expression for the speed v of Block A at time t_1.
(a)(ii) Derive an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

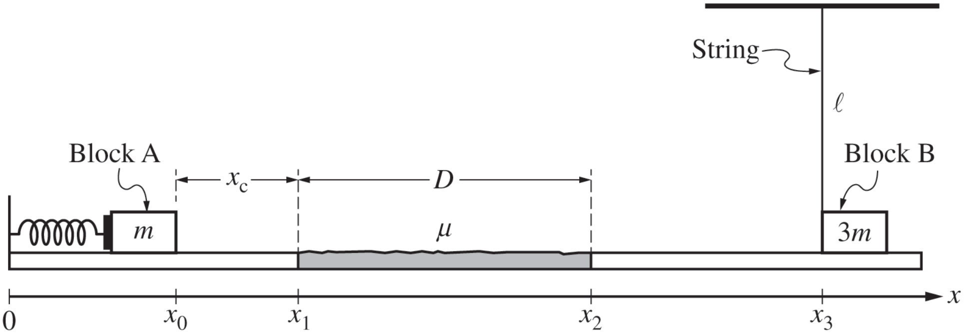
Question 1



Note: Figure not drawn to scale.

Figure 1

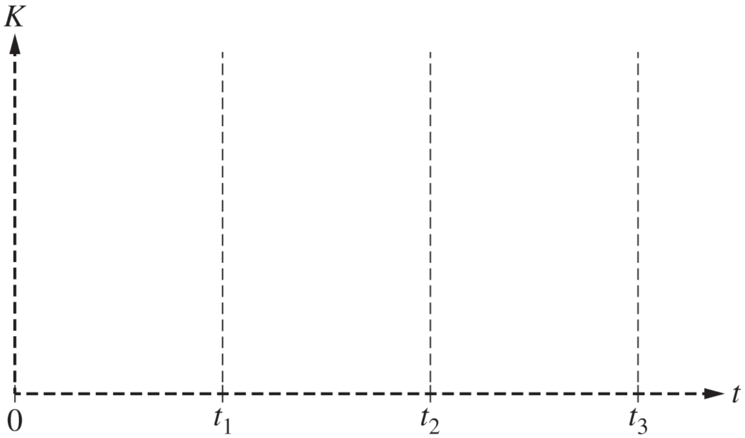
Question 1



Note: Figure not drawn to scale.

Figure 1

Question 1



Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.
[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]
At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure 1 repeated: same horizontal setup with Block A (mass m) against the spring, distance x_c, friction region of length D with coefficient μ between x_1 and x_2, and Block B (mass $3m$) hanging by a string of length ℓ at x_3. Positions marked $0, x_0, x_1, x_2, x_3$ along axis x. Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to t_3.

Question 1

[Blank graph: vertical axis K, horizontal axis t with gridlines/tick marks at t_1, t_2, t_3 (origin at 0); axes given with no curve drawn — student is to sketch the curve.]

(b)(ii) Use principles of work and energy to justify the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 1 ▪ Q1

Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.
[Figure 1: A horizontal setup on surface with position axis x from 0 to x_3. Block A (mass m) sits at the left against a spring, with a labelled distance x_c from Block A to a dashed reference line. A region of length D with friction coefficient μ (shaded) lies between positions x_1 and x_2. Block B (mass $3m$) hangs from a string of length ℓ attached to the ceiling, positioned at x_3 on the right. Marked positions along the x-axis: $0, x_0, x_1, x_2, x_3$. Note: Figure not drawn to scale.]
At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

Question 1

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position. - At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction. - At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

(c) [Figure 2: The two-block system (combined mass $m + 3m$) hangs from a string of length ℓ attached to a fixed support at the top. The string makes an angle θ_{max} with the vertical (dashed vertical reference line shown), with the block system displaced to one side at time t_4, having swung from its position at time t_3 (shown hanging straight down). Note: Figure not drawn to scale.]

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in

Question 1

Figure 2. For times $t > t_4$, the system oscillates with frequency f_i. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ. Indicate how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_i from the original procedure.

______ $f_{2\ell} > f_i$ ______ $f_{2\ell} < f_i$ ______ $f_{2\ell} = f_i$

Briefly justify your answer.

Solution 1

(a)(i)

Mark scheme

- Apply conservation of mechanical energy: all of the initial energy is stored as spring potential energy U_s, so $E_{initial} = E_{final}$ gives $\frac{1}{2}kx_c^2 = \frac{1}{2}mv^2$. Solving for v: $v = x_c\sqrt{k/m}$. Point 1: multi-step energy derivation that identifies the initial energy as entirely spring PE U_s. Point 2: correct algebraic solution for v.

Solution 1

(a)(ii) Mark scheme

- Step 1 (energy or kinematics to get the speed v_2 at x_2, after friction acts over distance D): Energy method — $K_{x_1} - \Delta E_{friction} = K_{before\ collision}$: $\frac{1}{2}m\left(x_c\sqrt{k/m}\right)^2 - \mu mgD = \frac{1}{2}mv_2^2 \Rightarrow v_2 = \sqrt{\frac{kx_c^2}{m} - 2\mu gD}$. (Equivalently by kinematics: $a = -\mu g$ from $-\mu mg = ma$, then $v_2^2 = v^2 + 2aD = \left(x_c\sqrt{k/m}\right)^2 - 2\mu gD$, same result.) Step 2 (perfectly inelastic collision, conservation of momentum): $\Sigma p_{before} = \Sigma p_{after}$: $mv_2 = (m+3m)v_{A,B} = 4m\,v_{A,B}$. Solving: $v_{A,B} = \dfrac{m\sqrt{\frac{kx_c^2}{m} - 2\mu gD}}{4m} = \dfrac{1}{4}\sqrt{\dfrac{kx_c^2}{m} - 2\mu gD}$. Points: (1) energy or kinematics

Solution 1

application to find speed at x_2; (2) correct expression for friction energy loss or deceleration ($\Delta E_{friction} = \mu mgD$ or $a = -\mu g$); (3) attempt at conservation of momentum $m_{initial}v_{initial} = m_{A,B}v_{A,B}$; (4) correctly substituting the consistent speed and masses into the momentum equation.

Solution 1

(b)(i) Mark scheme

- Sketch: $K = 0$ at $t = 0$; the curve rises nonlinearly and increases throughout $0 \leq t \leq t_1$, reaching a maximum at t_1 (when Block A leaves the spring). From t_1 to t_2 (the friction region) the curve decreases but does NOT reach zero, and is concave up (a constant friction deceleration makes v decrease linearly in time, so $K = \frac{1}{2}mv^2$ decreases at a decreasing rate). From t_2 to t_3 (after leaving the friction region, coasting to the collision) the curve is a horizontal line at a constant value greater than zero, and the whole curve from t_1 to t_3 is continuous (no jump at t_2). Points: (1) nonlinear sketch beginning at zero and increasing throughout $0 \leq t \leq t_1$; (2) decreasing throughout $t_1 \leq t \leq t_2$ without reaching

Solution 1

zero; (3) concave up on $t_1 \leq t \leq t_2$; (4) a continuous function on $t_1 \leq t \leq t_3$ that is a horizontal line greater than zero on $t_2 \leq t \leq t_3$.

Solution 1

(b)(ii) Mark scheme

- From $0 < t < t_1$, the kinetic energy of Block A increases. The force exerted on the block by the compressed spring transfers the elastic potential energy in the block-spring system to the kinetic energy of the block. Because the force exerted by the spring is not applied at a constant rate (it decreases as the spring relaxes), the kinetic energy of the block does not increase at a constant rate — hence the nonlinear shape. Points: (1) a statement about the change in KE consistent with the graph (KE increasing on this interval); (2) a correct explanation for why KE is increasing (positive work done on the block / mechanical energy conserved as spring PE converts to KE); (3) a correct explanation for why the graph is

Solution 1

nonlinear (the spring force — and hence the rate of work done — changes as the block moves, so KE does not increase at a constant rate).

Solution 1

(c)

Mark scheme

- Select $f_{2\ell} < f_\ell$ (doubling the pendulum length decreases its frequency). Justification: the period of a pendulum is $T = 2\pi\sqrt{\ell/g}$. As the length increases, the period increases. Because frequency and period are inversely related ($f = 1/T$), an increase in period results in a decrease in frequency, so $f_{2\ell} < f_\ell$. Points: (1) selecting $f_{2\ell} < f_\ell$ with an attempt at a relevant justification; (2) correctly applying the pendulum period/frequency-length relationship $T = 2\pi\sqrt{\ell/g}$.

Question 1

2024 Set 2 ▪ Q1

Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.
[Figure: A ramp inclined at angle θ to a horizontal table. Block A (mass $2m$) sits on the ramp at a distance D up the incline, with the region of length D on the ramp labeled with friction coefficient μ. The ramp meets the table at the origin 0. On the horizontal table, Block B (mass m) rests at position x_1. Further right along the table, a spring of constant k is fixed to a wall, with its uncompressed (equilibrium) position at x_3; a distance x_c is marked from x_3 back toward the wall/spring, near the wall. The

Question 1

horizontal axis is labeled with positions 0, x_1, x_3 increasing to the right toward the wall.]
Note: Figure not drawn to scale.
Continue your response to QUESTION 1 on this page.
At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.
- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

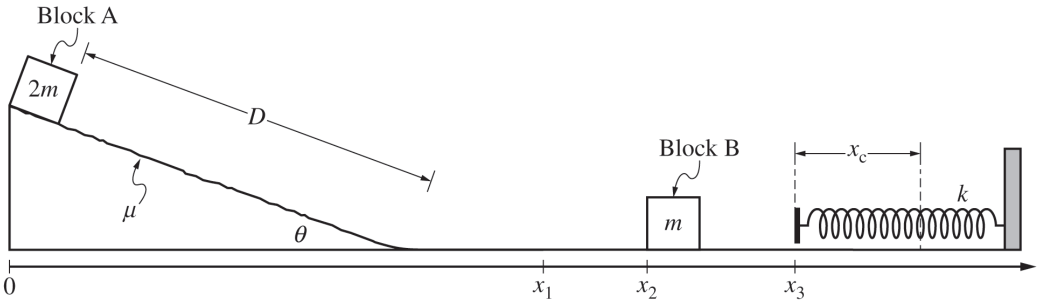
Question 1

For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

(a)(i) Derive an expression for the speed v of Block A at time t_1.

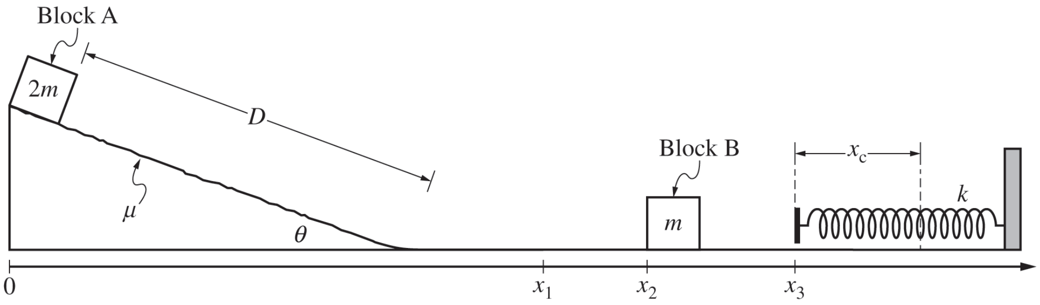
(a)(ii) Derive an expression for the spring constant k of the spring.

Question 1



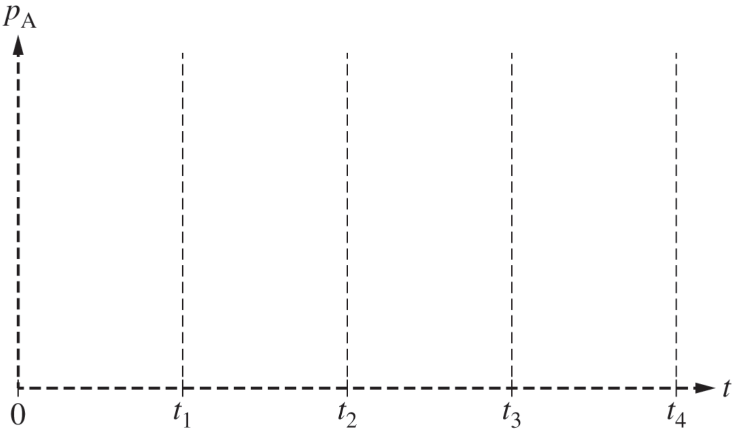
Note: Figure not drawn to scale.

Question 1



Note: Figure not drawn to scale.

Question 1



Question 1

2024 Set 2 ▪ Q1

Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

[Figure: A ramp inclined at angle θ to a horizontal table. Block A (mass $2m$) sits on the ramp at a distance D up the incline, with the region of length D on the ramp labeled with friction coefficient μ. The ramp meets the table at the origin 0. On the horizontal table, Block B (mass m) rests at position x_1. Further right along the table, a spring of constant k is fixed to a wall, with its uncompressed (equilibrium) position at x_3; a distance x_c is marked from x_3 back toward the wall/spring, near the wall. The horizontal axis is labeled with positions 0, x_1, x_3 increasing to the right toward the wall.]

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Note: Figure not drawn to scale.

Continue your response to QUESTION 1 on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

(b)(i) [Figure repeated: the same ramp-table-spring setup with Block A (mass $2m$) on the ramp, Block B (mass m) on the table, and the spring of constant k near the wall;

Question 1

axis positions 0, x_1, x_2, x_3 marked left to right, with x_c marked near the spring. Note: Figure not drawn to scale.]

On the following axes, sketch a graph of the magnitude of the momentum p_A of Block A as a function of time t from $t = 0$ to t_4.

[Graph: vertical axis labeled p_A (unlabeled scale), horizontal axis labeled t with tick marks at 0, t_1, t_2, t_3, t_4 (unlabeled scale/blank grid for the student to draw on).]

(b)(ii) Use principles of forces to justify the graph drawn in part (b)(i) for the time interval $t = t_3$ to $t = t_4$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

Question 1

2024 Set 2 ▪ Q1

Blocks A and B of masses $2m$ and m, respectively, are arranged in a setup consisting of a ramp that makes an angle θ with a smooth horizontal table and an ideal spring of spring constant k fixed to a wall, as shown. Block A is held at rest a distance D up the ramp, and Block B is at rest on the horizontal table. The coefficient of kinetic friction between Block A and the rough ramp is μ in the region of length D, and there is negligible friction between the blocks and the smooth table.

[Figure: A ramp inclined at angle θ to a horizontal table. Block A (mass $2m$) sits on the ramp at a distance D up the incline, with the region of length D on the ramp labeled with friction coefficient μ. The ramp meets the table at the origin 0. On the horizontal table, Block B (mass m) rests at position x_1. Further right along the table, a spring of constant k is fixed to a wall, with its uncompressed (equilibrium) position at x_3; a distance x_c is marked from x_3 back toward the wall/spring, near the wall. The horizontal axis is labeled with positions 0, x_1, x_3 increasing to the right toward the wall.]

Question 1

Note: Figure not drawn to scale.

Continue your response to QUESTION 1 on this page.

At time $t = 0$, Block A is located at horizontal position $x = 0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A has traveled a distance D down the ramp, has transitioned to the table, and is moving with speed v at $x = x_1$. - At time $t = t_2$, Block A is at $x = x_2$ when it collides with and sticks to Block B. - At time $t = t_3$, the combined blocks A and B are at $x = x_3$ when they collide with and stick to the spring in its equilibrium position. - At time $t = t_4$, the combined blocks A and B are instantaneously at rest and the spring is compressed a distance x_c from its equilibrium position.

For parts (a)(i) and (a)(ii), express your answer in terms of m, θ, D, μ, x_c, and physical constants, as appropriate.

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(c) For times $t > t_4$, the two-block-spring system oscillates with period T_O. The procedure is then repeated using a new ramp, where there is negligible friction between Block A and the ramp.

Indicate how the new period of oscillation T_N in the procedure that uses the new ramp compares with the period of oscillation T_O from the original procedure.

_____ $T_N > T_O$ _____ $T_N < T_O$ _____ $T_N = T_O$

Briefly justify your answer.

Solution 1

(a)(i) Mark scheme

- Using energy conservation for Block A sliding down the ramp (mass $2m$, ramp length D, angle θ, coefficient of friction μ): $U_g - \Delta E_{friction} = K$, i.e. $m_A g D \sin\theta - \mu m_A g D \cos\theta = \frac{1}{2} m_A v^2$, giving $v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$. (Equivalently via kinematics: Newton's second law gives $a = g\sin\theta - \mu g\cos\theta$, then $v^2 = v_0^2 + 2a\Delta x = 2(g\sin\theta - \mu g\cos\theta)D$.) Points: 1 for a correct multi-step derivation start (energy-conservation statement with U_g, or Newton's second law including the signed friction force), 1 for a consistent next step (friction-energy expression with correct sign, or substituting acceleration into a kinematics equation), 1 for the final correct expression $v = \sqrt{2gD(\sin\theta - \mu\cos\theta)}$.

Solution 1

(a)(ii) Mark scheme

- Momentum conservation for the collision between Block A (mass $2m$, speed v from part (a)(i)) and Block B (mass m, at rest): $2mv = (2m+m)v_{A,B}$, so $v_{A,B} = \frac{2}{3}v = \frac{2}{3}\sqrt{2gD(\sin\theta - \mu\cos\theta)}$. Then energy conservation from just after the collision to maximum spring compression: $K_{after\ collision} = U_{s,max}$, i.e. $\frac{1}{2}(3m)v_{A,B}^2 = \frac{1}{2}kx_c^2$ (using that the combined blocks' speed just before contacting the spring equals $v_{A,B}$). Solving: $k = \frac{3m}{x_c^2}v_{A,B}^2 = \frac{8}{3}\frac{mgD(\sin\theta - \mu\cos\theta)}{x_c^2}$. Points: 1 for using momentum conservation to find $v_{A,B}$, 1 for equating post-collision kinetic energy to the maximum elastic potential energy of the spring

Solution 1

$(K_{after\ collision} = U_{s,max})$, 1 for indicating the speed just before the spring contact equals $v_{A,B}$.

Solution 1

(b)(i) Mark scheme

- Sketch of p_A (magnitude of Block A's momentum) vs. t from $t = 0$ to t_4, with four required features: (1) a straight line increasing from 0 at $t = 0$ to a value $p_1 = m_A v$ at $t = t_1$ (Block A accelerates down the ramp under the net gravity-minus-friction force); (2) a horizontal line from t_1 to t_2 at the same level p_1, continuous at t_1 (Block A crosses the frictionless table at constant velocity before the collision); (3) a horizontal line from t_2 to t_3 at a smaller constant level $p_2 = m_A v_{A,B} < p_1$ (the perfectly inelastic collision with Block B at t_2 reduces Block A's speed to $v_{A,B} = \frac{2}{3}v$, then A+B move together at constant velocity toward the spring); (4) a concave-down curve from t_3 to t_4, continuous with level p_2 at t_3, decreasing to 0 at t_4 (the spring's restoring force grows as compression

Solution 1

increases, decelerating the blocks at an increasing rate until they momentarily stop at maximum compression).

Solution 1

(b)(ii) Mark scheme

- For $t_3 \leq t \leq t_4$: the momentum of Block A decreases because the spring exerts a force on the blocks directed opposite to their velocity, decelerating them toward rest. As the spring compresses further, the spring force increases in magnitude, so the rate of decrease of momentum (the magnitude of the slope of the p_A-vs-t graph) increases with time – this is why the curve is concave down, becoming steeper as $t \to t_4$, consistent with the sketch in part (b)(i). Points: 1 for a statement about the change in momentum consistent with the (b)(i) graph, 1 for identifying that a decreasing graph means the net force on Block A is opposite its motion, 1 for explicitly relating the changing steepness of the curve to the increasing magnitude of the spring force.

Solution 1

(c)

Mark scheme

- $T_N = T_O$ (the period is unchanged). Justification: the period of a spring-block oscillator depends only on the oscillating mass and the spring constant ($T = 2\pi\sqrt{m/k}$), not on friction, amplitude, velocity, or compression distance. Removing friction on the new ramp changes only the compression distance/amplitude of the oscillation, not m or k, so the period stays the same.

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[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\mathrm{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\mathrm{max,P}}$ for Spring P with the different height $h_{\mathrm{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in

Question 1

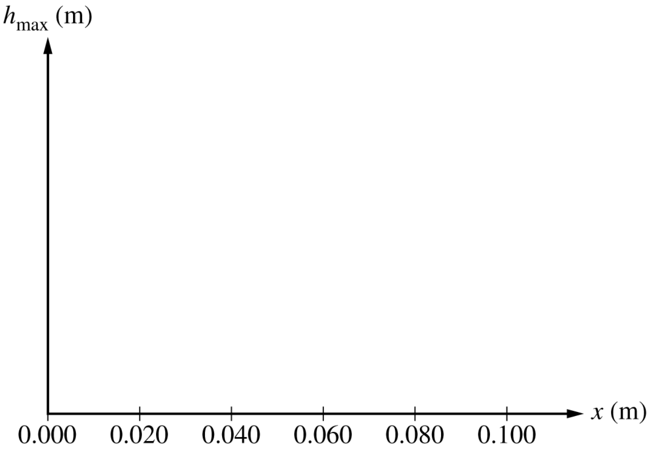
Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]
(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010$ m. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following representations of the block in Trials 1 and 2, draw and label the forces (non-components) that are exerted on the block at the instant the block is released. Each force must be

Question 1

represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.

[Two grid diagrams are shown, each with a dot at the center representing the block: "Trial 1 (Spring P)" on the left and "Trial 2 (Spring Q)" on the right. Each grid is for the student to draw labeled force vectors.]

Question 1



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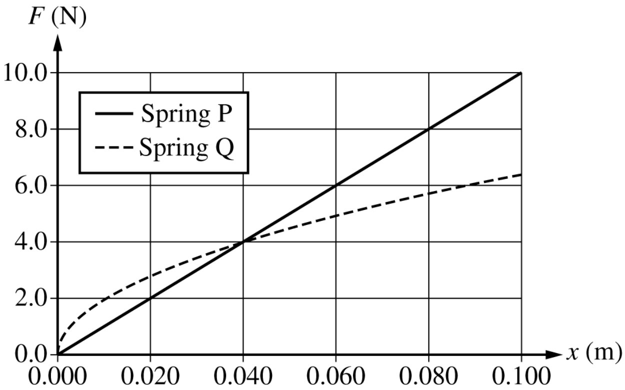
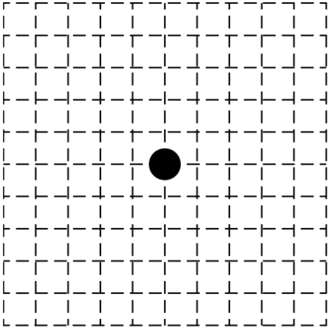


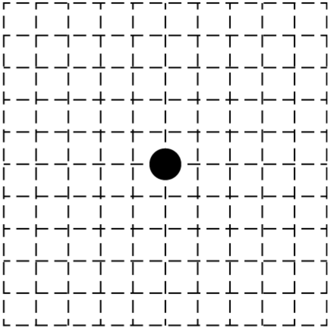
Figure 2

Question 1

Trial 1
(Spring P)



Trial 2
(Spring Q)



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[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.
A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

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straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(b)(i) What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

(b)(ii) There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

(b)(iii) On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance x each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.

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[Blank axes provided: y-axis labeled h_{max} (m), x-axis labeled x (m) from 0.000 to 0.100 in increments of 0.020, for the student to sketch two curves.]

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2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.
A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

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straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(c)(i) [Figure 3: Block A rests at the top of an inclined ramp of height H, connected by a string over the top of the ramp; Block B rests on the horizontal surface at the bottom of the ramp, connected to Spring Q which is attached to a wall, with $x = 0$ marked at the natural length position of the spring.]

Spring Q is attached to the wall and at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp on the horizontal surface. The change in vertical height of Block A is $H = 0.75$ m. The student releases Block A, and it moves down the ramp and collides with Block B. After the collision, the blocks stick together and move to the right,

Question 1

compressing the spring. Frictional forces between the blocks and all surfaces are negligible.

Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.

(c)(ii) Calculate the maximum compression of the spring.

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[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
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A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

Question 1

straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D?

______ $C < D$ ______ $C > D$ ______ $C = D$

Briefly justify your answer.

Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]
Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the

Question 2

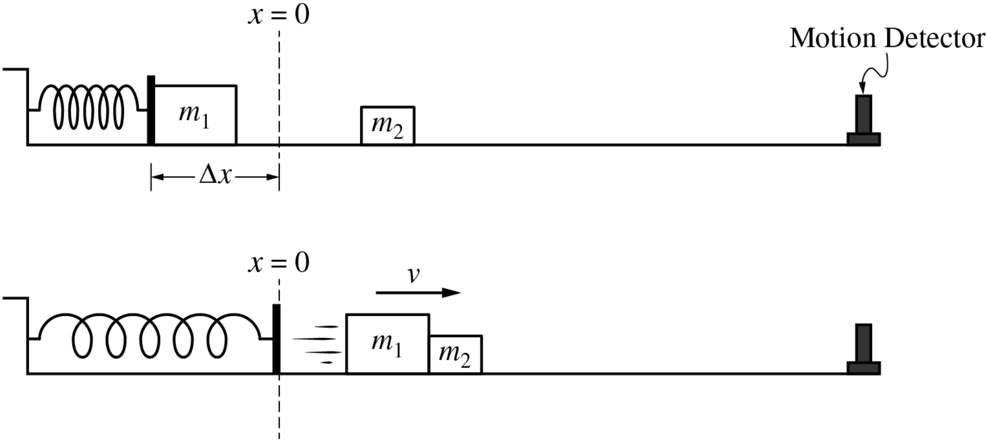
two-block system moves with constant speed v, as shown. Frictional effects are negligible.

(a) The impulse on Block 1 from the spring during the time interval $0 < t < t_C$ is J_S. The impulse on Block 1 from Block 2 during the collision is J_2. Which of the following expressions correctly compares the magnitudes of J_S and J_2 ?

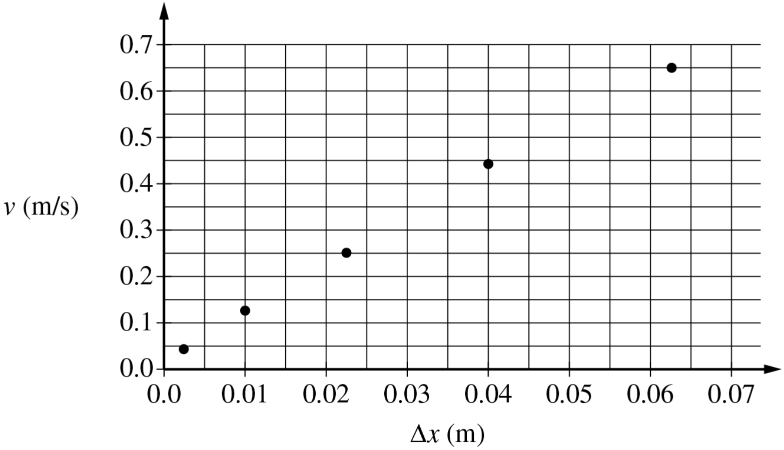
____ $J_S > J_2$ ____ $J_S < J_2$ ____ $J_S = J_2$

Justify your answer.

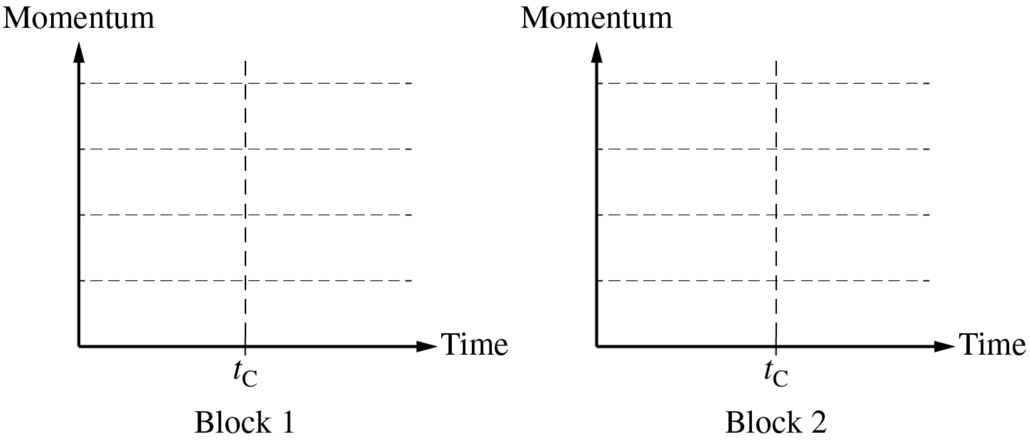
Question 2



Question 2



Question 2



Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]
Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(b) On the following axes, draw graphs of the magnitude of the momentum of each block as a function of time, before and after t_C. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.
[Two blank graphs labeled "Momentum" on the vertical axis and "Time" on the horizontal axis, each with a marked point t_C on the time axis and dashed gridlines; left graph labeled "Block 1", right graph labeled "Block 2", for the student to sketch momentum vs. time before and after t_C.]

Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]

Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(c) Show that the velocity v of the two-block system after the collision is given by the equation $v = \dfrac{\sqrt{km_1}}{m_1 + m_2}\Delta x$.

Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]

[Figure (bottom): After release, the spring is no longer compressed (uncompressed length shown), and the combined Block 1 + Block 2 system moves to the right with speed v, starting from $x = 0$.]

Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(d)(i) A group of students use the setup to perform an experiment. They measure the mass of Block 1 to be $m_1 = 0.500$ kg, and the spring constant *k* of the spring to be 150 N/m. The mass of Block 2 is unknown. They perform several trials and in each trial the spring is compressed a different distance Δx and the final velocity *v* of the two-block system is measured. They graph *v* as a function of Δx, as shown below. Draw a line that represents the best fit to the data points shown.
[Scatter plot: vertical axis *v* (m/s) from 0.0 to 0.7 in increments of 0.1; horizontal axis Δx (m) from 0.0 to 0.07 in increments of 0.01. Data points approximately at $(0.01, 0.08)$, $(0.02, 0.16)$, $(0.035, 0.28)$, $(0.05, 0.45)$, $(0.06, 0.58)$, $(0.065, 0.62)$, for the student to draw a best-fit line through.]
(d)(ii) Use the best-fit line to calculate the mass of Block 2.

Question 2

2022 Set 2 ▪ Q2

[Figure (top): A spring with spring constant k is attached to a wall on the left, compressed by an amount Δx from its natural length, holding Block 1 (mass m_1). Block 2 (mass m_2) sits at rest at position $x = 0$, to the right of Block 1, in front of a Motion Detector mounted on a wall at the far right.]
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Block 1 of mass m_1 is held at rest while compressing an ideal spring an amount Δx. The spring constant of the spring is k. Block 2 has mass m_2, where $m_2 < m_1$. At time $t = 0$, Block 1 is released. At time t_C, the spring is no longer compressed and Block 1 immediately collides with and sticks to Block 2. The blocks stick together and the two-block system moves with constant speed v, as shown. Frictional effects are negligible.

Question 2

(e) After the experiment, the students use a balance to measure the mass of Block 2 and find it to be greater than what was determined in part (d). To explain this discrepancy, one of the students proposes that the spring constant was incorrectly measured at the beginning of the experiment. The students measure the spring constant again and record a new value, k'.

Should the students expect that k' be greater than 150 N/m, less than 150 N/m, or equal to 150 N/m ?

k' $>$ 150 N/m k' $<$ 150 N/m k' $=$ 150 N/m

Justify your answer.

Solution 2

(a) Mark scheme

- Answer: $J_S > J_2$ — select this choice with an attempt at a relevant justification (1 point), plus a fully correct justification (1 point). Example justification: By Newton's third law, the impulse Block 2 exerts on Block 1 during the collision has the same magnitude as the impulse Block 1 exerts on Block 2. Since Block 1 is still moving forward (nonzero momentum) after the collision, the impulse from the spring J_S (which accelerated Block 1 from rest up to its pre-collision speed) must be greater in magnitude than the impulse J_2 from the collision (which only reduces, not zeroes, Block 1's momentum).

Solution 2

(b) Mark scheme

- Full-credit sketch, 1 point per criterion: (1) For $0 < t < t_C$: Block 1's momentum rises along a concave-down (decreasing-slope) curve up to its maximum at $t = t_C$, while Block 2's momentum stays at zero (flat on the time axis) throughout. (2) For $t > t_C$: Block 1's momentum is a horizontal line at a value smaller in magnitude than its momentum at $t = t_C$ (it lost momentum in the collision). (3) For $t > t_C$: Block 2's momentum is a horizontal line at a value smaller in magnitude than Block 1's momentum at $t = t_C$ (Block 2 now moves, but with less momentum than Block 1 had just before the collision). (4) Blocks 1 and 2 have an equal-magnitude change in momentum at t_C — Block 1's drop equals Block 2's rise (momentum conservation in the collision).

Solution 2

(c) Mark scheme

- Use conservation of energy (spring PE converts to KE of Block 1) to find Block 1's speed at $x = 0$ — 1 point: $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m_1 v_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m_1}}\Delta x$. Use conservation of momentum for the perfectly inelastic collision to find the speed of the combined two-block system — 1 point: $m_1 v_1 = (m_1 + m_2)v \Rightarrow v = \frac{m_1 v_1}{m_1 + m_2}$. Combine the two equations — 1 point: $v = \frac{\Delta x\sqrt{km_1}}{m_1 + m_2}$, as required to show.

Solution 2

(d)(i)

Mark scheme

- Draw a straight best-fit line through the plotted v vs. Δx data, with approximately the same number of data points above the line as below it (need not pass exactly through any single point). 1 point for a reasonable best-fit line satisfying this balance criterion.

Solution 2

(d)(ii) Mark scheme

- Calculate the slope of the best-fit line using two well-separated points on it — 1 point, e.g. slope $= \dfrac{(0.60-0.02)\ \text{m/s}}{(0.055-0.0175)\ \text{m}} \approx 10.67\ \text{s}^{-1}$. Correctly relate the slope to m_2 using the part (c) result $v = \dfrac{\sqrt{km_1}}{m_1+m_2}\Delta x$ — 1 point: slope $= \dfrac{\sqrt{km_1}}{m_1+m_2} \Rightarrow m_2 = \dfrac{\sqrt{km_1}}{\text{slope}} - m_1$. Compute a correct numeric mass — 1 point: $m_2 = \dfrac{\sqrt{(150\ \text{N/m})(0.500\ \text{kg})}}{10.67\ \text{s}^{-1}} - 0.500\ \text{kg} \approx 0.31\ \text{kg}$ (acceptable range 0.27 kg to 0.37 kg, since it depends on the student's own graph read).

Solution 2

(e) Mark scheme

- Answer: $k' > 150$ N/m — select this choice with an attempt at relevant justification (1 point), plus a fully correct justification (1 point). Example justification: A balance shows the true mass of Block 2 is greater than the value determined in part (d). From slope $= \dfrac{\sqrt{k' m_1}}{m_1 + m_2}$, the measured slope of the graph (an experimental result) stays the same, so for a larger true m_2 the spring constant k' needed to keep that same slope must be greater than the originally assumed 150 N/m.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

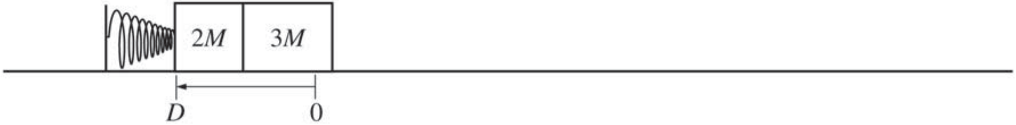
A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Question 2

[Two dots are shown side by side, labeled "Block of Mass $2M$" (left dot) and "Block of Mass $3M$" (right dot), for the student to draw force vectors on.]
[Figure: The spring (coil) anchored at the wall on the left is now attached to a combined block labeled "2M" next to "3M" (the two blocks shown adjacent, having moved together). A distance D is marked from the wall/spring's compressed end to a reference position "0".]
The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

Question 2



Question 2

d pointing away from, the appropriate dot.

Question 2

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

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(b) Derive an expression for the speed of the blocks immediately after the collision.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(d) Derive an expression for the maximum distance D that the spring is compressed.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(e)(i) In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

_____ Left _____ Right _____ The net force on the block of mass $2M$ is zero.

Question 2

Justify your answer.

(e)(ii) Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$. ____ The magnitude of the net force is greater on the block of mass $3M$. ____ The magnitude of the net force on each block has the same nonzero value. ____ The magnitude of the net force on each block is zero.

Justify your answer.

Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity v_0 (arrow pointing left labeled v_0).]

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

______ Yes ______ No

Justify your answer.

Solution 2

(a)

Mark scheme

- Draw two force diagrams (blocks not yet touching the spring, frictionless surface): on the $2M$ block, normal force F_{N1} up and weight $2Mg$ down; on the $3M$ block, normal force F_{N2} up and weight $3Mg$ down (distinct symbols since the two blocks have different masses/normal forces). 1 pt for correctly drawing/labeling the vectors on the $2M$ block; 1 pt for correctly drawing/labeling the vectors on the $3M$ block, using different, physically correct symbols. Note: a maximum of 1 of the 2 points can be earned if any extraneous vectors are drawn.

Solution 2

(b)

Mark scheme

- The $3M$ block moving at v_0 collides and sticks to the stationary $2M$ block (perfectly inelastic). Conservation of momentum: $p_1 = p_2 \Rightarrow 3Mv_0 = (3M + 2M)v_f$. 1 pt for correctly substituting into the momentum equation; 1 pt for the correct final answer $v_f = \frac{3}{5}v_0$.

Solution 2

(c)

Mark scheme

- Kinetic energy of the combined system: $K = \frac{1}{2}mv^2 = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2 = \frac{9}{10}Mv_0^2$. 1 pt for an answer consistent with the speed found in part (b).

Solution 2

(d) Mark scheme

- By conservation of energy, $\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$, so $K_0 = U_{\text{final}}$: the kinetic energy right after the collision converts entirely to spring potential energy at maximum compression. With nonlinear spring force $F = Bx^3$: $\frac{9}{10} M v_0^2 = \int_0^D Bx^3\,dx = \left[\frac{Bx^4}{4}\right]_0^D = \frac{BD^4}{4}$. Solving: $D = \sqrt[4]{\frac{18Mv_0^2}{5B}}$. 1 pt for a correct conservation-of-energy expression ($\Delta K + \Delta U = 0$, $K_0 = U_{\text{final}}$); 1 pt for attempting to integrate the spring-force equation; 1 pt for using correct limits of integration (0 to D) or an appropriate constant of integration; 1 pt for a final answer consistent with the speed from (b) or the kinetic energy from (c). Correct answer: $D = \sqrt[4]{18Mv_0^2/(5B)}$.

Solution 2

(e)(i)

Mark scheme

- Answer: Right. At maximum compression the two-block system is instantaneously at rest, but the (still-compressed) spring continues to exert a rightward force on the system, so the block of mass $2M$ has a net force/acceleration to the right at that instant. 1 pt for selecting 'Right' with a reasonable justification attempt (no credit for the justification if the wrong choice is made); 1 pt for indicating that at maximum compression the $2M$ block has a rightward acceleration, due either to the forces acting on it directly or to the external spring force on the system.

Solution 2

(e)(ii)

Mark scheme

- Answer: The magnitude of the net force is greater on the block of mass $3M$. Because the blocks are stuck together they share the same acceleration; since $F = ma$, the block with more mass ($3M$) experiences the greater net force at that same acceleration. 1 pt for indicating both blocks have the same acceleration; 1 pt for a correct justification that the net force is greater on the $3M$ block (no credit if the wrong block is selected).

Solution 2

(f) Mark scheme

- Answer: No. Although the spring exerts a restoring force on the stuck-together blocks, the spring is nonlinear ($F = -Bx^3$, not $F = -kx$), so the force is not proportional to displacement from equilibrium. SHM requires a linear restoring force ($a = -\frac{k}{m}\Delta x$), so the blocks do not exhibit simple harmonic motion. 1 pt for selecting 'No' with a reasonable justification attempt (the selection point is not earned if 'No' is chosen with no justification; no credit for the justification if the wrong answer is selected); 1 pt for indicating the spring's force is not linear and that SHM specifically requires a linear restoring force.

Question 3

2016 ▪ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.]
[Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]
A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in

Question 3

frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

Question 3

(a) Derive an expression for the spring constant of the spring.

Question 3

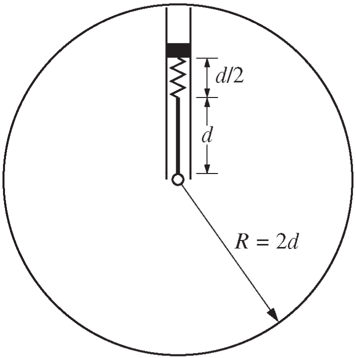

Figure 1

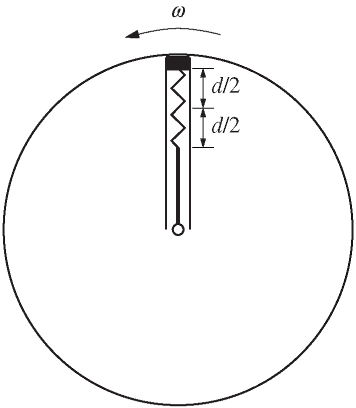

Figure 2

Question 3

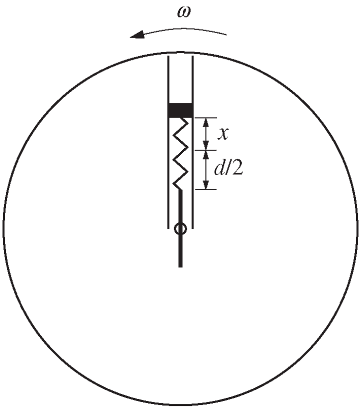

Figure 3

Question 3

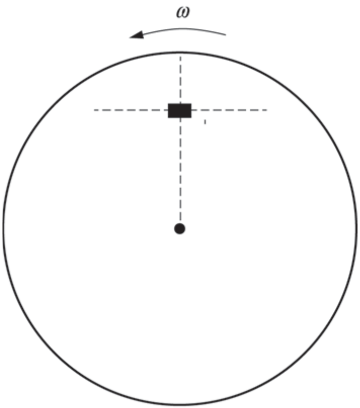

Figure 4

Question 3

2016 ▪ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Question 3

length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(b)(i) Determine an expression for the rotational inertia of the block around the axis of the platform.

Question 3

(b)(ii) Derive an expression for the rotational inertia of the entire system about the axis of the platform.

Question 3

2016 ▪ Q3

[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]
A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium

Question 3

length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\dfrac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\dfrac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

Question 3

[Figure 3: The same circular platform, rotating counterclockwise at ω. The rod's block position is shown partway along the rod, with the spring stretched by amount x and a remaining segment labeled $d/2$ below it.]
While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

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[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

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Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

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For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

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[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]
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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing   _____ Decreasing   _____ Staying the same

Question 3

Justify your answer.

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[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

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Question 3

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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive   _____ Negative   _____ No work

Question 3

Justify your answer.

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[Figure 1: A circular platform of radius $R = 2d$ viewed from above, with a vertical rod of length d fixed at the central axis (a small circle marking the pivot at the center). A spring of natural (unstretched) length $d/2$ sits along the rod, connected to a block (shown as a small dark rectangle) at distance $d/2$ from the outer end of the rod.] [Figure 2: The same circular platform now rotating counterclockwise (as viewed from above) at angular speed ω (curved arrow at top indicating rotation direction). The block has moved outward; the spring is now stretched, shown with two segments each labeled $d/2$ (i.e., the spring plus the gap between block and rod end each measure $d/2$).]

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Question 3

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A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

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[Figure 4: The circular platform rotating counterclockwise at ω, with the block shown at the center on the axis (a small dark square marker), a dashed horizontal line through it, and a dot below the platform on the vertical rod line, for the student to draw an acceleration vector on the block.]

Solution 3

(a)

Mark scheme

- Radial form of Newton's second law: spring force equals centripetal force, $F_S = F_C = ma_C$, i.e. $kx = mr\omega^2$. With spring stretch $x = d/2$ and orbit radius $r = 2d$ (per the given geometry): $k\left(\frac{d}{2}\right) = m(2d)\omega^2$. Solving: $k = 4m\omega^2$. 1 pt for indicating $F_S = F_C = ma_C$; 1 pt for a correct substitution for x; 1 pt for a correct substitution for r that lets you solve for k. Correct answer: $k = 4m\omega^2$.

Solution 3

(b)(i)

Mark scheme

- Treating the block as a point mass at radius $r = 2d$:
  $I = \sum mr^2 = m(2d)^2 = 4md^2$. 1 pt for a correct answer, or an answer consistent with the radius used in part (a).

Solution 3

(b)(ii) Mark scheme

- Total rotational inertia is the sum of the platform's, rod's, and block's: $I = I_P + I_R + I_O = \frac{m_P R^2}{2} + \frac{m_R d^2}{3} + mr^2$. Substituting $m_P = 5m$, $R = 2d$ (platform as a disk), $m_R = 3m$ (rod), and $mr^2 = 4md^2$ from (b)i: $I = \frac{5m(2d)^2}{2} + \frac{3md^2}{3} + 4md^2 = 10md^2 + md^2 + 4md^2 = 15md^2$. 1 pt for indicating the system's rotational inertia must include the platform, rod, and object; 1 pt for correctly substituting each piece consistent with (b)i. Correct answer: $I = 15md^2$.

Solution 3

(c)

Mark scheme

- $L = I\omega$. Using $I = 15md^2$ from part (b)ii: $L = 15md^2\omega$. 1 pt for an answer consistent with the answer from part (b)ii.

Solution 3

(d)

Mark scheme

- Same radial equation as part (a): $F_S = F_C = ma_C \Rightarrow kx = mr\omega^2$. Using $k = 4m\omega^2$ (from part (a), or the student's own consistent value) and the new radius $r = d + x$ (rod's center now at the axis, spring stretched by x): $(4m\omega^2)x = m(d+x)\omega^2 \Rightarrow 4x = d + x \Rightarrow 3x = d \Rightarrow x = d/3$. 1 pt for indicating the spring force equals mass times centripetal acceleration; 1 pt for a correct substitution for k (or one consistent with part (a)); 1 pt for a correct substitution for r that lets you solve for x. Correct answer: $x = d/3$.

Solution 3

(e)

Mark scheme

- Answer: Decreasing. As the block moves inward toward the axis, the system's rotational inertia I decreases while ω is held constant; since $L = I\omega$, a decreasing I with constant ω means L decreases. 1 pt for selecting 'Decreasing' (no credit for the justification if the wrong selection is made); 1 pt for a correct justification (I decreases while ω stays the same, so $L = I\omega$ decreases).

Solution 3

(f)

Mark scheme

- Answer: Negative. With ω held constant while I decreases, the rotational kinetic energy $K = \frac{1}{2}I\omega^2$ decreases; by the work-energy theorem the net work on the rotating system is negative, so the motor is doing negative work (removing energy to keep ω constant as I shrinks). 1 pt for selecting 'Negative' (no credit for the justification if the wrong selection is made); 1 pt for a correct justification (ω is constant, I has decreased, so rotational KE has decreased, so the work done must be negative).

Solution 3

(g)

Mark scheme

- The scoring guidelines show the correct acceleration direction as a single arrow starting on the block and pointing down and to the right — i.e. combining a centripetal (inward) component with a tangential component consistent with the block's inward radial motion while the platform rotates counterclockwise. 1 pt for an arrow starting on the box and pointing down and to the right.