Past-paper walk-through

Newton's First Law ▪ 2.4

eXercise

Questions in this set

- 2019 Set 1 Q1
- 2019 Set 2 Q1

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 1

2019 Set 1 ▪ Q1

In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.
[Figure above the question: a cylinder of fluid on a table, with hands releasing an object labeled "Object, Mass = 12 g" at the top of the "Cylinder" containing "Fluid", resting on a "Table".]
[Graph: v (m/s) vs. t (s); y-axis from 0.0 to 1.0 in increments of 0.2, x-axis from 0.00 to 0.35 in increments of 0.05. Data points (dots) roughly follow a dashed best-fit curve that rises steeply from the origin, e.g. passing near (0.00, 0.0), (0.025, 0.17), (0.05, 0.25), (0.075, 0.42), (0.10, 0.47), (0.125, 0.58), (0.15, 0.66), (0.175, 0.65),

Question 1

(0.20, 0.76), (0.225, 0.78), (0.25, 0.80), (0.275, 0.90), (0.30, 0.93), (0.325, 0.92); the curve rises quickly at first then levels off, approaching an asymptote near $v \approx 1.0$ m/s.]

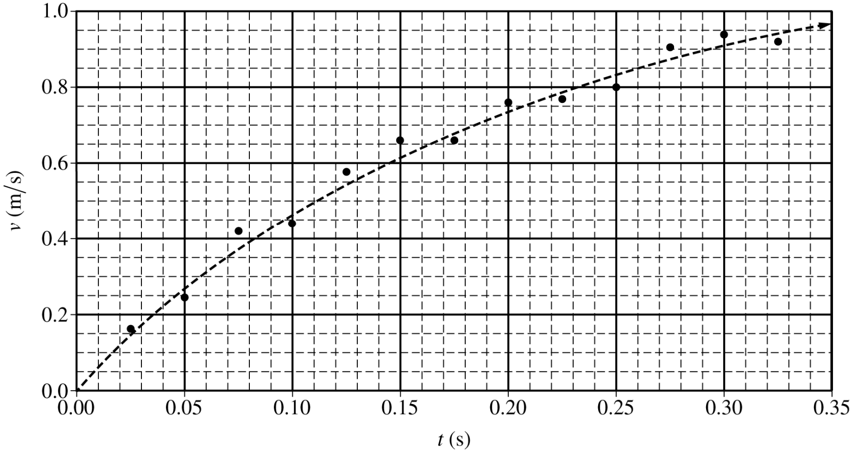
(a)(i) Does the speed of the object increase, decrease, or remain the same?

______ Increase   ______ Decrease   ______ Remain the same

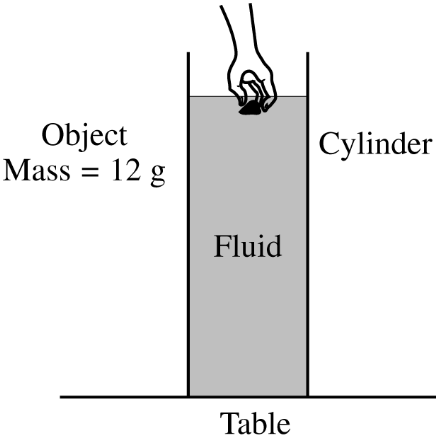
(a)(ii) In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

(a)(iii) Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

Question 1



Question 1



Question 1

2019 Set 1 ▪ Q1

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Question 1

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(b)(i) The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5$ s^{-1}. Use the above equation to derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t.

(b)(ii) Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t.

Question 1

2019 Set 1 ▪ Q1

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Question 1

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(c)(i) The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed. Determine the constant speed of the object.
Justify your answer.

(c)(ii) Determine the force exerted by the fluid on the object at this time.
Justify your answer.

Solution 1

(a)(i)

Mark scheme

- The speed **Increases** (full credit for selecting 'Increase'). The object is falling and speeding up as it accelerates through the fluid.

Solution 1

(a)(ii)

Mark scheme

- The acceleration points downward (1 pt), and its magnitude decreases as the object falls (1 pt). Example: Because the object is moving downward and speeding up, the acceleration must be downward. Because the slope of the graph of speed vs. time is decreasing, the magnitude of the acceleration must be decreasing.

Solution 1

(a)(iii)

Mark scheme

- Draw a tangent line to the curve at $t = 0.20$ s and compute its slope using two points on that tangent line (1 pt): $a = \frac{\Delta v}{\Delta t} = \frac{(0.8 - 0.6)\ \text{m/s}}{(0.226 - 0.136)\ \text{s}}$. Correct answer using points from the tangent line (1 pt): $a \approx 2.22\ \text{m/s}^2$ (accept values close to this from a reasonable tangent line).

Solution 1

(b)(i) Mark scheme

- Recognize that vertical displacement is the integral of velocity (1 pt): $\Delta y = \int v\,dt = \int A(1 - e^{-Bt})\,dt$. Evaluate with appropriate limits or constant of integration (1 pt): $\Delta y = \int_0^t A(1 - e^{-Bt'})\,dt' = A\left[\left(t + \frac{1}{B}e^{-Bt}\right) - \left(0 + \frac{1}{B}e^{0}\right)\right]$. Correct final answer (1 pt): $\Delta y = A\left(t + \frac{1}{B}\left(e^{-Bt} - 1\right)\right) = 1.18\left(t + \frac{1}{5}\left(e^{-5t} - 1\right)\right)$. Credit is given for leaving the answer in terms of A and B or for plugging in the given numeric values.

Solution 1

(b)(ii)

Mark scheme

- Attempt the derivative of the speed equation (1 pt): $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\left[A(1 - e^{-Bt})\right]$. Correct expression for acceleration (1 pt): $a = ABe^{-Bt}$. Multiply by the mass to get net force (1 pt): $F = ma = m\left(ABe^{-Bt}\right) = mABe^{-Bt}$, i.e. $F = (0.012)(1.18)(5)e^{-5t} = 0.071\, e^{-5t}$ N. Credit is given for leaving the answer in terms of A, B, m or for plugging in the given numeric values.

Solution 1

(c)(i)

Mark scheme

- State that the constant (terminal) speed is $v = A$ (1 pt), found by letting $t \to \infty$ in the fit equation (1 pt). After a long time the falling object reaches a terminal constant speed in the fluid; setting $t \to \infty$ in $v = A(1 - e^{-Bt})$ gives the constant speed $v = A = 1.18$ m/s.

Solution 1

(c)(ii)

Mark scheme

- Indicate that the net force is zero at constant speed (1 pt), and that the resistive (fluid) force therefore equals the weight of the object (1 pt). Since the object moves at constant speed, the net force is zero; the only vertical forces are gravity and the fluid's resistive force, so the resistive force equals the weight: $F = mg = (0.012\ \text{kg})(9.8\ \text{m/s}^2) = 0.12\ \text{N}$.

Question 1

2019 Set 2 ▪ Q1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.
[Figure 1: An inclined plane rising to the right at angle θ from the horizontal, with a vertical wall/support on the right holding a pulley at the top. On the incline, two blocks are shown in contact, connected by a light string: a lower block labeled $2m$ and, distance d up the slope from the bottom of the incline to the lower block, an upper block labeled m connected to the lower block by tension T_1. A second string from block m, with tension T_2, runs up the incline and over the pulley at the top, then down vertically to a hanging sphere of mass M.]

Question 1

(a) On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.
[Two dots are shown for the student to draw free-body diagrams on: one labeled m, positioned on a dashed line representing the incline surface (inclined at angle θ), and one labeled M, positioned in open space to represent the hanging sphere.]

Question 1

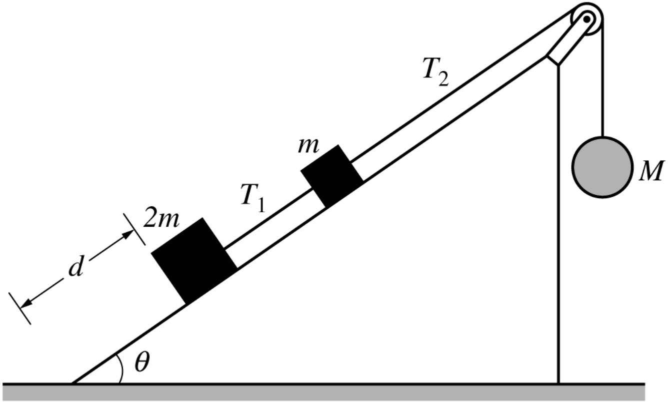

Figure 1

Question 1

Question 1

2019 Set 2 ▪ Q1

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Question 1

(b)(i) Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).
The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ, and physical constants, as appropriate.
(b)(ii) The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ, and physical constants, as appropriate.

Question 1

2019 Set 2 ▪ Q1

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Question 1

(c)(i) Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

_____ Increase _____ Decrease to a nonzero value

_____ Decrease to zero _____ Stay the same

(c)(ii) Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

(c)(iii) Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

Question 1

2019 Set 2 ▪ Q1

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(d) Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is

Question 1

μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

Question 1

2019 Set 2 ▪ Q1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium. [Figure 1: An inclined plane rising to the right at angle θ from the horizontal, with a vertical wall/support on the right holding a pulley at the top. On the incline, two blocks are shown in contact, connected by a light string: a lower block labeled $2m$ and, distance d up the slope from the bottom of the incline to the lower block, an upper block labeled m connected to the lower block by tension T_1. A second string from block m, with tension T_2, runs up the incline and over the pulley at the top, then down vertically to a hanging sphere of mass M.]

(e) The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d

Question 1

from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

Solution 1

(a)

Mark scheme

- Free-body diagrams (3 points total, max 2 if extraneous vectors are drawn). On block m: draw F_N (normal force, perpendicular to and away from the incline surface) and mg (weight, straight down) — 1 point; draw T_1 (tension from the string to block $2m$, directed down the incline) and T_2 (tension from the string over the pulley, directed up the incline) — 1 point. On sphere M: draw T_2 (tension, upward) and Mg (weight, downward) — 1 point. Each force must be a single arrow starting on and pointing away from the dot.

Solution 1

(b)(i)

Mark scheme

- Treat blocks m and $2m$ as one system of total mass $3m$ in equilibrium ($a = 0$): $F_{net} = (m + 2m)a = 0 \therefore T_2 - (m + 2m)g \sin\theta = 0$ (1 pt for an attempt at a correct Newton's-second-law statement for the two blocks). Solving gives the final answer $T_2 = 3mg \sin\theta$ (1 pt for the correct answer with supporting work).

Solution 1

(b)(ii)

Mark scheme

- Apply Newton's second law to the whole system (all three masses, $a = 0$): $F_{net} = m_{tot}a \therefore Mg - 2mg\sin\theta - mg\sin\theta = 0$, giving $M = 3m\sin\theta$ (1 pt). [Equivalent alternate solution: from part (b)(i), $T_2 - Mg = 0 \therefore Mg = T_2 \therefore M = T_2/g = 3m\sin\theta$ — also earns the 1 point.]

Solution 1

(c)(i)

Mark scheme

- Correct choice: tension T_1 decreases to zero (1 pt). Once the sphere M lands, the string over the pulley (carrying T_2) goes slack because it can no longer pull the blocks. With no tension pulling them and the incline frictionless, both blocks m and $2m$ independently experience the same gravitational acceleration component $g \sin \theta$ down the incline, so no tension is needed to keep them moving together, and $T_1 \to 0$.

Solution 1

(c)(ii)

Mark scheme

- Correct choice: up the ramp (1 pt). Velocity cannot change instantaneously (only acceleration can). Immediately after the sphere lands, the blocks are still moving in the same direction as the instant before — up the incline, since they had been dragged upward as M descended on the other side of the pulley.

Solution 1

(c)(iii)

Mark scheme

- Correct choice: down the ramp (1 pt). With $T_2 = 0$ once the sphere lands, the only net force along the frictionless incline is the gravity component, which points down the incline, so the acceleration is directed down the ramp — this decelerates the upward motion and will eventually reverse it.

Solution 1

(d)

Mark scheme

- Newton's second law for the 3-mass system on the verge of sliding down, with static friction at its maximum value acting up the incline on each block: $F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{s2} + mg\sin\theta - f_{s1} - Mg = 0$ (1 pt, attempt at a correct statement for the system). Substitute $f_{s1} = \mu_s F_{N1} = \mu_s mg\cos\theta$ and $f_{s2} = \mu_s F_{N2} = \mu_s(2mg\cos\theta)$: $3mg\sin\theta - \mu_s(2mg\cos\theta) - \mu_s(mg\cos\theta) = Mg$ (1 pt, attempting to substitute the friction forces). Final answer: $M = 3m(\sin\theta - \mu_s\cos\theta)$ (1 pt, correct answer with supporting work).

Solution 1

(e)

Mark scheme

- Newton's second law for the two blocks (total mass $3m$) sliding down with kinetic friction: $F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{k2} + mg\sin\theta - f_{k1} = 3ma$ (1 pt, attempt at a correct statement), which gives $a = g(\sin\theta - \mu_k\cos\theta)$. Use kinematics with $v_1 = 0$ over distance d: $v_2^2 = v_1^2 + 2ad$ (1 pt, correct kinematics equation) $= 2g(\sin\theta - \mu_k\cos\theta)d$. Final answer: $v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$ (1 pt, correct answer with supporting work). [Equivalent alternate solution via conservation of energy, $U_1 + K_1 = U_2 + K_2 + E_{lost}$, gives the same result and earns full credit.]