# Past-paper walk-through

## Newton's Third Law ▪ 2.3

eXercise

# Questions in this set

- 2022 Set 1 Q2
- 2016 Q2

# Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

# Question 2

2022 Set 1 ▪ Q2

[Diagram, Figure setup: Cart 1 of mass $m_1$ sits at the top of an incline (marked "Cart 1"), held at rest at a height $H$ above the bottom of the incline. The incline descends and transitions smoothly into a horizontal section. Cart 2, of mass $m_2$, sits at rest on the horizontal section further along, marked "Cart 2". A Motion Detector is mounted vertically at the far right end of the horizontal track, facing left toward the carts. A second diagram below shows the two-cart system, now combined, moving to the right with velocity $v$ (arrow labeled "$v$" pointing right) toward the Motion Detector.]
Cart 1 of mass $m_1$ is held at rest above the bottom of the incline. Cart 2 has mass $m_2$, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance $H$, as shown. At time $t_C$, Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2.

# Question 2

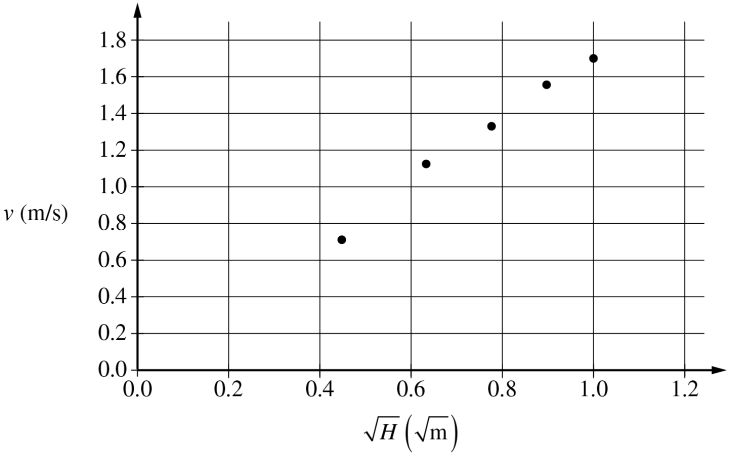
After the collision, the two-cart system moves with constant speed $v$. Frictional and rotational effects are negligible.

**(a)** During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

______ Greater than      ______ Less than      ______ Equal to

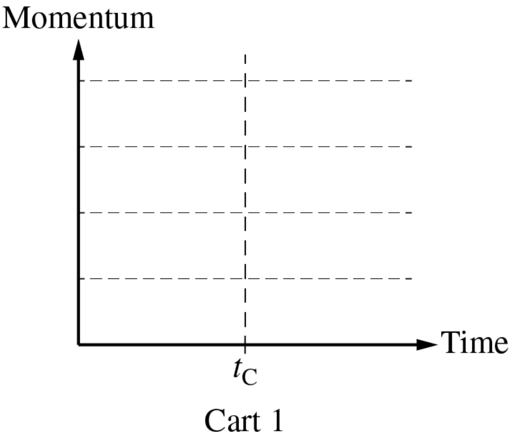
Justify your answer.

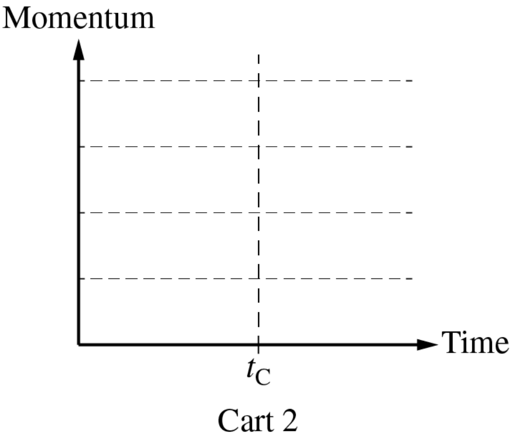
# Question 2



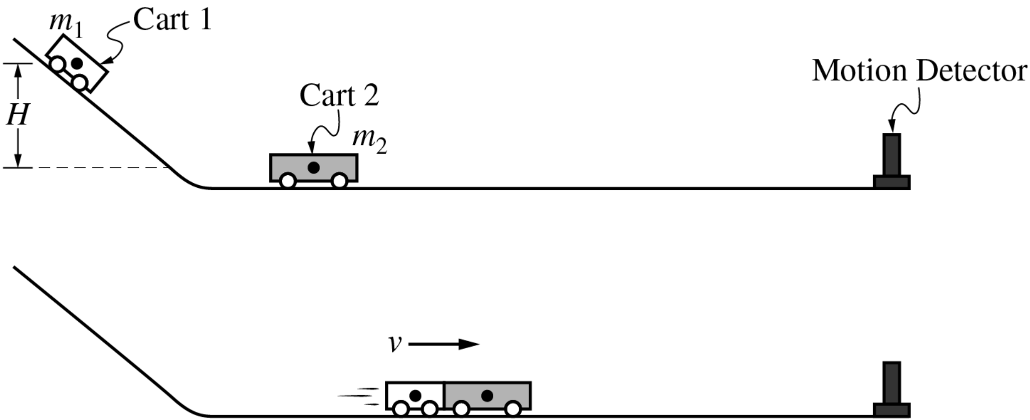
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
v (m/s)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
$\sqrt{H}\left(\sqrt{\mathrm{m}}\right)$

# Question 2



# Question 2



# Question 2

2022 Set 1 ▪ Q2

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Cart 1 of mass $m_1$ is held at rest above the bottom of the incline. Cart 2 has mass $m_2$, where $m_2 > m_1$, and is at rest at the bottom of the incline. At time $t = 0$, Cart 1 is released and then travels down the incline and transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance $H$, as shown. At time $t_C$, Cart 1 reaches the bottom of the

# Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed $v$. Frictional and rotational effects are negligible.

**(b)** On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time $t$, before and after $t_C$. The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

[Graph 1, titled "Cart 1": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at $t_C$ on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]

[Graph 2, titled "Cart 2": axes labeled "Momentum" (vertical) vs. "Time" (horizontal); a dashed vertical gridline is marked at $t_C$ on the time axis; horizontal dashed gridlines are evenly spaced on the momentum axis; no data plotted, blank axes for the student to draw on.]

# Question 2

2022 Set 1 ▪ Q2

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incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed $v$. Frictional and rotational effects are negligible.

**(c)** Show that the velocity $v$ of the two-cart system after the collision is given by the equation

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}.$$

# Question 2

2022 Set 1 ▪ Q2

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incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed $v$. Frictional and rotational effects are negligible.

**(d)(i)** A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be $m_1 = 0.250$ kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height $H$ and the final velocity of the two-cart system is measured. The students graph $v$ as a function of $\sqrt{H}$, as shown below.

[Graph: scatter plot; vertical axis "$v$ (m/s)" ranges from 0.0 to 1.8 in increments of 0.2; horizontal axis "$\sqrt{H}\,(\sqrt{\text{m}})$" ranges from 0.0 to 1.2 in increments of 0.2; plotted points approximately at $(0.45, 0.75)$, $(0.65, 1.05)$, $(0.65, 1.25)$, $(0.85, 1.35)$, $(1.0, 1.55)$, $(1.0, 1.65)$, showing a roughly linear increasing trend; no line drawn yet.]

Draw a line that represents the best fit to the data points shown.

**(d)(ii)** Use the best-fit line to calculate the mass of Cart 2.

# Question 2

2022 Set 1 ▪ Q2

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# Question 2

incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed $v$. Frictional and rotational effects are negligible.

**(e)** After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, $m_1'$.

Should the students expect that $m_1'$ will be greater than $0.250$ kg, less than $0.250$ kg, or equal to $0.250$ kg?

$\mathrm{m}_1' > 0.250$ kg  $\mathrm{m}_1' < 0.250$ kg  $\mathrm{m}_1' = 0.250$ kg

Justify your answer.

# Solution 2

## (a)

**Mark scheme**

- Answer: Equal to (1 point for selecting this with an attempted justification). Justification: By Newton's third law, the force Cart 2 exerts on Cart 1 and the force Cart 1 exerts on Cart 2 are equal in magnitude (and opposite in direction) at every instant during the collision; since both carts experience the collision over the same time interval, the impulses ($\int F\,dt$) on each cart must be equal in magnitude (1 point).

# Solution 2

## (b) Mark scheme

- Momentum ($p$) vs. time ($t$) graphs, before/after $t_C$ (collision assumed instantaneous). Cart 1: for $0 < t < t_C$, a straight line increasing linearly from zero (Cart 1 accelerates down the incline); at $t_C$ its momentum drops abruptly to a smaller constant (horizontal) value for $t > t_C$ (1 point for the $0 < t < t_C$ portion: linear increasing for Cart 1 and zero for Cart 2; 1 point for Cart 1's post-collision horizontal line being smaller in magnitude than its momentum at $t = t_C$). Cart 2: zero (flat on the axis) for $0 < t < t_C$, then jumps at $t_C$ to a constant (horizontal) value for $t > t_C$ that is GREATER in magnitude than Cart 1's post-collision momentum (1 point). The magnitude of Cart 1's momentum loss at the collision

## Solution 2

must equal the magnitude of Cart 2's momentum gain (equal and opposite changes), consistent with the equal-impulse answer in part (a) (1 point).

# Solution 2

## (c) Mark scheme

- Derivation of $v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$. Conservation of energy (or equivalently kinematics) gives Cart 1's speed at the bottom of the incline: $m_1 g H = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gH}$ (1 point). Conservation of momentum for the perfectly inelastic collision: $m_1 v_1 = (m_1 + m_2)v \Rightarrow v = \frac{m_1}{m_1 + m_2} v_1$ (1 point). Combining: $v = \frac{m_1}{m_1 + m_2}\sqrt{2gH} = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$, as required (1 point).

# Solution 2

**(d)(i)**

**Mark scheme**

- Draw a straight best-fit line through the $v$ vs. $\sqrt{H}$ scatter plot with approximately equal numbers of data points above and below the line (through or near the origin).

# Solution 2

## (d)(ii) Mark scheme

- Use the best-fit line to find $m_2$. Calculate the slope using two well-separated points on the best-fit line, e.g. slope $= \dfrac{(1.20 - 0.60)\ \text{m/s}}{(0.70 - 0.35)\sqrt{\text{m}}} = 1.72\ \sqrt{\text{m}}/\text{s}$ (1 point). Relate the slope to the masses via the part (c) result, $\mathit{slope} = \dfrac{m_1}{m_1 + m_2}\sqrt{2g}$, and solve for $m_2$: $m_2 = \dfrac{m_1\sqrt{2g}}{\mathit{slope}} - m_1$ (1 point). Substituting $m_1 = 0.250$ kg and $g = 9.8\ \text{m/s}^2$: $m_2 = \dfrac{(0.25\ \text{kg})\sqrt{2(9.8)}}{1.72} - 0.25\ \text{kg} \approx 0.39\ \text{kg}$ (1 point). Any value in the range 0.30–0.60 kg is acceptable given a valid best-fit slope.

# Solution 2

## (e)

**Mark scheme**

- Answer: $m_1' < 0.250$ kg (1 point for selecting this with an attempted justification). Justification: The directly-measured mass of Cart 2 came out smaller than the value calculated in part (d). A smaller true $m_2$ corresponds to a smaller initial energy/momentum for the same drop height $H$; since the best-fit slope (and hence the computed relationship) stays the same, this discrepancy is explained if the actual mass of Cart 1 is smaller than the recorded $0.250$ kg — i.e., $m_1' < 0.250$ kg (1 point).

# Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity $v_0$ (arrow pointing left labeled $v_0$).]

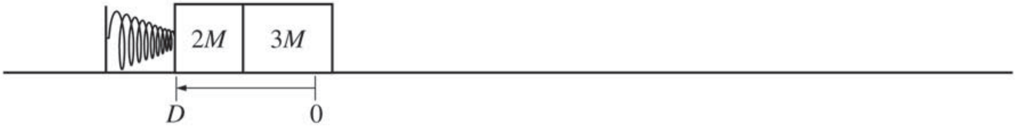
A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where $B$ is a positive constant and $x$ is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed $v_0$ is moving to the left as shown.

**(a)** On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

# Question 2

[Two dots are shown side by side, labeled "Block of Mass $2M$" (left dot) and "Block of Mass $3M$" (right dot), for the student to draw force vectors on.]
[Figure: The spring (coil) anchored at the wall on the left is now attached to a combined block labeled "2M" next to "3M" (the two blocks shown adjacent, having moved together). A distance $D$ is marked from the wall/spring's compressed end to a reference position "0".]
The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance $D$, as shown above. Express your answers to parts (b), (c), and (d) in terms of $M$, $B$, $v_0$, and physical constants, as appropriate.

# Question 2



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# Question 2

# Question 2

2016 ▪ Q2

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**(b)** Derive an expression for the speed of the blocks immediately after the collision.

# Question 2

2016 ▪ Q2

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**(c)** Determine an expression for the kinetic energy of the two-block system immediately after the collision.

# Question 2

2016 ▪ Q2

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**(d)** Derive an expression for the maximum distance $D$ that the spring is compressed.

# Question 2

2016 ▪ Q2

[Figure: A horizontal table. On the left, a spring (drawn as a coil) is anchored to a wall and attached to a block labeled "2M" at position marked "0". On the right, separated by open space, a block labeled "3M" moves to the left with initial velocity $v_0$ (arrow pointing left labeled $v_0$).]

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**(e)(i)** In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

_____ Left   _____ Right   _____ The net force on the block of mass $2M$ is zero.

# Question 2

Justify your answer.

**(e)(ii)** Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?
_____ The magnitude of the net force is greater on the block of mass $2M$. _____ The magnitude of the net force is greater on the block of mass $3M$. _____ The magnitude of the net force on each block has the same nonzero value. _____ The magnitude of the net force on each block is zero.
Justify your answer.

# Question 2

2016 ▪ Q2

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**(f)** Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

______ Yes        ______ No

Justify your answer.

# Solution 2

## (a)

### Mark scheme

- Draw two force diagrams (blocks not yet touching the spring, frictionless surface): on the $2M$ block, normal force $F_{N1}$ up and weight $2Mg$ down; on the $3M$ block, normal force $F_{N2}$ up and weight $3Mg$ down (distinct symbols since the two blocks have different masses/normal forces). 1 pt for correctly drawing/labeling the vectors on the $2M$ block; 1 pt for correctly drawing/labeling the vectors on the $3M$ block, using different, physically correct symbols. Note: a maximum of 1 of the 2 points can be earned if any extraneous vectors are drawn.

# Solution 2

## (b)

### Mark scheme

- The $3M$ block moving at $v_0$ collides and sticks to the stationary $2M$ block (perfectly inelastic). Conservation of momentum: $p_1 = p_2 \Rightarrow 3Mv_0 = (3M + 2M)v_f$. 1 pt for correctly substituting into the momentum equation; 1 pt for the correct final answer $v_f = \frac{3}{5}v_0$.

# Solution 2

## (c)

**Mark scheme**

- Kinetic energy of the combined system: $K = \frac{1}{2}mv^2 = \frac{1}{2}(5M)\left(\frac{3}{5}v_0\right)^2 = \frac{9}{10}Mv_0^2$.
  1 pt for an answer consistent with the speed found in part (b).

# Solution 2

## (d) Mark scheme

- By conservation of energy, $\Delta K_{\text{system}} + \Delta U_{\text{system}} = 0$, so $K_0 = U_{\text{final}}$: the kinetic energy right after the collision converts entirely to spring potential energy at maximum compression. With nonlinear spring force $F = Bx^3$: $\frac{9}{10} M v_0^2 = \int_0^D Bx^3\, dx = \left[\frac{Bx^4}{4}\right]_0^D = \frac{BD^4}{4}$. Solving: $D = \sqrt[4]{\frac{18 M v_0^2}{5B}}$. 1 pt for a correct conservation-of-energy expression ($\Delta K + \Delta U = 0$, $K_0 = U_{\text{final}}$); 1 pt for attempting to integrate the spring-force equation; 1 pt for using correct limits of integration (0 to $D$) or an appropriate constant of integration; 1 pt for a final answer consistent with the speed from (b) or the kinetic energy from (c). Correct answer: $D = \sqrt[4]{18 M v_0^2/(5B)}$.

# Solution 2

## (e)(i)

**Mark scheme**

- Answer: Right. At maximum compression the two-block system is instantaneously at rest, but the (still-compressed) spring continues to exert a rightward force on the system, so the block of mass $2M$ has a net force/acceleration to the right at that instant. 1 pt for selecting 'Right' with a reasonable justification attempt (no credit for the justification if the wrong choice is made); 1 pt for indicating that at maximum compression the $2M$ block has a rightward acceleration, due either to the forces acting on it directly or to the external spring force on the system.

# Solution 2

## (e)(ii)

**Mark scheme**

- Answer: The magnitude of the net force is greater on the block of mass $3M$. Because the blocks are stuck together they share the same acceleration; since $F = ma$, the block with more mass ($3M$) experiences the greater net force at that same acceleration. 1 pt for indicating both blocks have the same acceleration; 1 pt for a correct justification that the net force is greater on the $3M$ block (no credit if the wrong block is selected).

# Solution 2

## (f) Mark scheme

- Answer: No. Although the spring exerts a restoring force on the stuck-together blocks, the spring is nonlinear ($F = -Bx^3$, not $F = -kx$), so the force is not proportional to displacement from equilibrium. SHM requires a linear restoring force ($a = -\frac{k}{m}\Delta x$), so the blocks do not exhibit simple harmonic motion. 1 pt for selecting 'No' with a reasonable justification attempt (the selection point is not earned if 'No' is chosen with no justification; no credit for the justification if the wrong answer is selected); 1 pt for indicating the spring's force is not linear and that SHM specifically requires a linear restoring force.