Past-paper walk-through

Forces and Free-Body Diagrams ▪ 2.2

eXercise

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Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

Question 3

(a) [Figure: A schematic representation of the rod alone (no pole, no string, no block), extending diagonally with a pivot circle at the lower-left end. Points marked along the rod: P, C, and Q (in that order from the pivot outward).]
On the following representation of the rod, draw and label the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.

Question 3

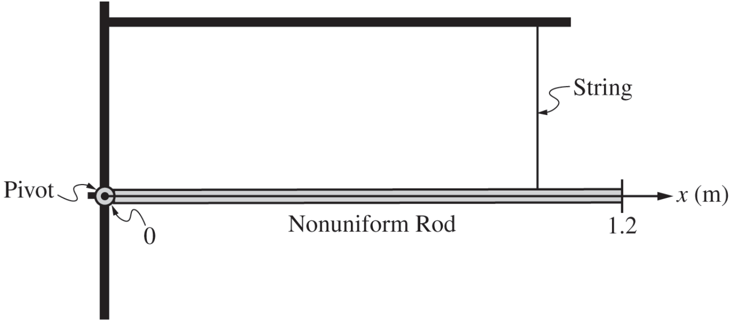

Figure 3

Note: Figure not drawn to scale.

Question 3

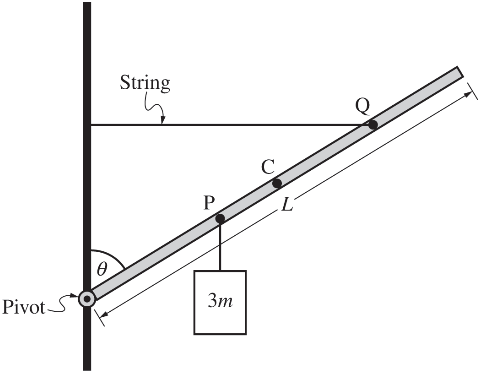

Figure 1

Note: Figure not drawn to scale.

Question 3

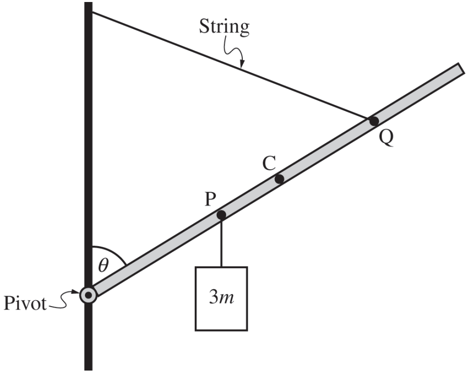

Figure 2

Note: Figure not drawn to scale.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot.

Question 3

Derive an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

Question 3

2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(c) [Figure 2: Same setup as Figure 1, but the string now connects from Point Q on the rod to a higher point on the vertical pole (the string attachment point is above the

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top of the pole shown in Figure 1), making a steeper angle with the pole. The rod still makes angle θ with the pole, with Points P, C, Q marked along the rod, and the block of mass $3m$ hanging from Point P. Pivot labeled "Pivot-S" at the base. Note: Figure not drawn to scale.]

The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\ new}$ compare with the original tension F_T from part (b)? Justify your reasoning.

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2024 Set 1 ▪ Q3

[Figure 1: A uniform rod of length L is attached at a pivot on a vertical pole. The rod extends up and to the right at angle θ from the pole. A horizontal string connects a point labeled Q on the rod (near its far end) to the top of the vertical pole. Points along the rod from the pivot outward: P (closer to pivot), C (center of mass, further along), and Q (near the far end). A block of mass $3m$ hangs from the rod at point P via a short vertical line. The pivot is labeled "Pivot-S" at the base of the vertical pole. Note: Figure not drawn to scale.]

A uniform rod of length L and mass m is attached to a pivot on a vertical pole. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(d)(i) [Figure 3: A nonuniform rod attached horizontally to a pivot on a vertical pole, extending to the right along a horizontal x (m) axis from 0 at the pivot to 1.2 at the

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far end labeled "Nonuniform Rod". A string labeled "String" connects from a point partway along the rod up to the top of the vertical pole, forming a right-triangle-like support. Pivot labeled "Pivot-S" at the base of the pole. Note: Figure not drawn to scale.]

A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0$ kg/m, and $B = 10.0$ kg/m^2.

Calculate the mass of the rod.

(d)(ii) Calculate the rotational inertia of the rod about the pivot.

Solution 3

(a) Mark scheme

- Draw and label: a downward force $F_{T,block}$ (or $3mg$) at point P (tension from the string holding the hanging block); a downward gravitational force mg at point C (the rod's own weight, acting at its center of mass); a leftward force $F_{T,string}$ at point Q (tension from the string to the pole); and a force directed up and to the right at the pivot, F_{pivot} (the pivot's reaction force), with no other (extraneous) forces drawn. Points: (1) correctly drawn/labeled downward forces at P and C; (2) correctly drawn/labeled leftward force at Q; (3) correctly drawn/labeled up-and-right force at the pivot, with no extraneous forces.

Solution 3

(b) Mark scheme

- Balance torques about the pivot ($\Sigma\tau = 0$), with the hanging mass $3m$ at P ($\frac{3L}{8}$ from pivot), the rod's weight mg at C ($\frac{4L}{8} = \frac{L}{2}$ from pivot), and the string tension F_T at Q ($\frac{6L}{8}$ from pivot, acting with perpendicular-component $F_T \cos\theta$ since the string is horizontal and the rod makes angle θ with the vertical pole): $3mg\sin\theta\left(\frac{3L}{8}\right) + mg\sin\theta\left(\frac{4L}{8}\right) - F_T\cos\theta\left(\frac{6L}{8}\right) = 0$, so $\left(\frac{9}{8}\right) mg\sin\theta + \left(\frac{4}{8}\right) mg\sin\theta = \left(\frac{6}{8}\right) F_T\cos\theta$, giving $13mg\sin\theta = 6F_T\cos\theta \Rightarrow F_T = \dfrac{13}{6} mg\tan\theta$. (Scoring note: a maximum of 3 of the 4 points may be earned if sin and cos are reversed throughout for all three torque terms.) Points: (1) net torque on rod equals zero; (2) correct torque

Solution 3

expression for the hanging mass; (3) correct torque expression for the rod's own gravitational force; (4) correct torque expression for the string tension.

Solution 3

(c) Mark scheme

- Because the torque that the string must supply on the rod is always the same (it must still balance the unchanged torques from the rod's weight and the hanging mass), moving the string's attachment point higher on the pole increases the angle between the string and the rod, which increases the string's perpendicular moment arm. To produce the same torque with a larger moment arm, the tension $F_{T,new}$ must be smaller. So as the angle between the string and rod increases, the force exerted on the rod by the string decreases. Points: (1) indicating the torque exerted on the rod by the string is always the same; (2) stating that as the angle between the string and rod increases, the tension force decreases.

Solution 3

(d)(i) Mark scheme

- The nonuniform rod has linear mass density $\lambda(x) = A + Bx$ (with $A = 6.0$ kg/m, $B = 10.0$ kg/m^2) over its 1.2 m length. Total mass: $M = \int dm = \int_0^{1.2} \lambda \, dx = \int_0^{1.2} (6 + 10x) \, dx = \left(6x + \frac{10x^2}{2} \right) \Big|_0^{1.2} = 6(1.2) + 5(1.2)^2 = 7.2 + 7.2 = 14.4$ kg. Points: (1) indicating the total mass is the sum (integral) of differential masses along the rod, $M = \int dm$; (2) correctly writing dm in terms of x (i.e. $dm = \lambda \, dx = (6 + 10x) \, dx$); (3) a correct numeric answer with correct units, $M = 14.4$ kg.

Solution 3

(d)(ii) Mark scheme

- Rotational inertia about the pivot: $I = \int r^2\, dm$ with $dm = \lambda\, dx$ and $r = x$: $I = \int_0^{1.2} (A + Bx)x^2\, dx = \int_0^{1.2} (6 + 10x)x^2\, dx = \left(\frac{Ax^3}{3} + \frac{Bx^4}{4}\right)\Big|_0^{1.2} = \frac{6(1.2)^3}{3} + \frac{10(1.2)^4}{4} = 3.456 + 5.184 = 8.64$ kg · m^2. Points: (1) correct substitution of λ into an integral expression for rotational inertia; (2) correct integration; (3) a correct numeric answer with correct units, $I = 8.64$ kg · m^2.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal

Question 3

string is connected from the pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(a) On the following representation of the clay-disk system, draw and label the external forces (not components) exerted on the system. Each force must be represented by a distinct arrow that starts on, and points away from, the point at which the force is exerted on the system.

[Figure: A shaded disk viewed face-on, with a dot at the center marking the axle, and point A marked on the upper-right edge of the disk (blank, for the student to draw force arrows on).]

Question 3

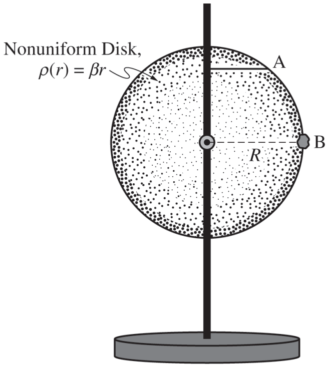

Figure 3

<u>Note</u>: Figure not drawn to scale.

Question 3

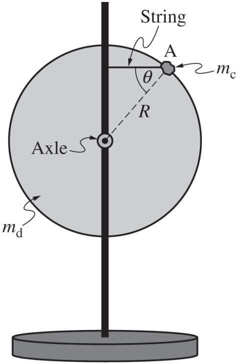

Figure 1

<u>Note:</u> Figure not drawn to scale.

Question 3

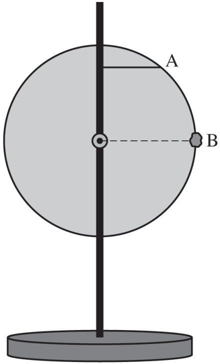

Figure 2

Note: Figure not drawn to scale.

Question 3

2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(b) Derive an expression for the tension F_T in the string when the clay is at Point A, as shown in Figure 1, in terms of R, m_d, m_c, θ, and physical constants, as appropriate.

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2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(c) [Figure 2: The same disk-axle-pole setup, now with point A marked on the upper edge of the disk and point B marked at the right edge of the disk, horizontally in line with the axle; a dashed horizontal line connects the axle to point B. Note: Figure not drawn to scale.]

The string remains connected to the edge of the disk at Point A. The clay is moved to Point B, which is horizontally in line with the axle, as shown in Figure 2. How does the new tension $F_{T,new}$ compare with tension F_T from part (b)? Justify your reasoning.

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2024 Set 2 ▪ Q3

[Figure 1: A uniform disk mounted on a horizontal axle that passes through a vertical pole, with the axle at the disk's center. A string runs horizontally from the pole to point A at the edge of the disk near the top; the string makes angle θ with the line between point A and the axle. A lump of clay of mass m_c is attached to the edge of the disk at point A. Another lump of clay of mass m_d hangs from a string on the lower-left edge of the disk. The disk radius is labeled R, measured from the axle to the edge. The disk sits on a vertical pole mounted on a circular base.]

Note: Figure not drawn to scale.

A uniform disk of radius R and mass m_d is attached to a vertical pole by a horizontal axle that passes through the center of the disk. Friction between the axle and the disk is negligible. A lump of clay of mass m_c is attached to the edge of the disk at Point A. The size of the lump of clay is small compared with the radius of the disk. A horizontal string is connected from the

Question 3

pole to the edge of the disk at Point A. The string makes an angle θ with the line between Point A and the axle, as shown in Figure 1.

(d)(i) [Figure 3: A nonuniform disk (shown with radial shading/stippling denoting varying density) mounted on the same axle-pole setup, labeled "Nonuniform Disk, $\rho(r) = \beta r$". Point A is marked on the upper edge of the disk, and point B is marked at the right edge of the disk, horizontally in line with the axle (dashed line from axle to B). The disk radius is labeled R. Note: Figure not drawn to scale.]

A nonuniform disk is now attached to the axle. The lump of clay is attached to the disk at Point B, as shown in Figure 3. The clay has mass $m_c = 0.60$ kg and the disk has a radius $R = 0.30$ m. The mass density of the disk varies radially and can be modeled by $\rho(r) = \beta r$, where r is the radial distance from the axle and $\beta = 4.0$ kg/m^3. Calculate the rotational inertia of the disk about the axle.

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(d)(ii) The string connecting the disk to the pole is cut. Calculate the magnitude of the initial angular acceleration of the clay-disk system.

Solution 3

(a) Mark scheme

- Force diagram on the clay-disk system with three distinct, correctly placed/labeled external forces (each arrow starting on, and pointing away from, its point of application): (1) a downward gravitational force on the disk, $m_d g$ (or a single downward gravitational force on the clay-disk system as a whole), drawn at the disk's center / appropriate location; (2) a downward gravitational force on the lump of clay, $m_c g$, drawn at Point A; (3) a leftward tension force F_T exerted by the string, drawn at Point A; and (4) a force directed up-and-to-the-right exerted by the axle, F_{axle}, drawn at the axle – with no other (extraneous) forces shown.

Solution 3

(b) Mark scheme

- Net torque about the axle is zero (the disk is in rotational equilibrium): $\Sigma\tau = 0$, i.e. $\tau_{clay} - \tau_{string} = 0$, where only the weight of the clay and the string tension exert torque about the axle. Torque from the clay's weight: $\tau_{clay} = Rm_c g\cos\theta$. Torque from the string tension: $\tau_{string} = RF_T\sin\theta$. Setting them equal: $Rm_c g\cos\theta = RF_T\sin\theta \Rightarrow F_T = \dfrac{m_c g\cos\theta}{\sin\theta} = m_c g\cot\theta$. Points: 1 for a multi-step derivation indicating the net torque is zero, 1 for indicating only the clay's weight and the string tension exert torque on the system about the axle, 1 for a correct torque expression from the clay's weight ($\tau_{clay} = Rm_c g\cos\theta$), 1 for a correct torque expression from the string tension ($\tau_{string} = RF_T\sin\theta$). (A maximum of 3

Solution 3

of these 4 points can be earned if sin and cos are consistently reversed in both torque expressions.)

Solution 3

(c)

Mark scheme

- $F_{T,new} > F_T$ (the tension is greater when the clay is at Point B). Justification: there is a direct relationship between the torque exerted by the clay's weight and the string tension (from part (b), $F_T \propto \tau_{clay}$). At Point B (horizontally in line with the axle) the component of the clay's weight perpendicular to the radial direction – equivalently the lever arm for the weight – is larger than at Point A, so the torque from the clay's weight is greater at B. Since the net torque must still be zero, the tension needed to balance it, $F_{T,new}$, must also be greater than F_T.

Solution 3

(d)(i) Mark scheme

- Rotational inertia of the nonuniform disk ($\rho(r) = \beta r$, radius $R = 0.3$ m) found by integration, treating a thin ring of radius r and thickness dr as $dm = \rho(2\pi r)dr = \beta r(2\pi r)dr = 2\pi\beta r^2\, dr$:

$$I = \int r^2\, dm = \int_0^{0.3\text{ m}} r^2 \rho(2\pi r)\, dr = \int_0^{0.3\text{ m}} 2\pi\beta r^4\, dr = \frac{2\pi\beta R^5}{5}.$$

With $\beta = 4.0\text{ kg/m}^3$ and $R = 0.3$ m: $I = \dfrac{2\pi(4.0\text{ kg/m}^3)(0.3\text{ m})^5}{5} = 0.012\text{ kg}\cdot\text{m}^2$.
Points: 1 for setting up integration to calculate rotational inertia, 1 for either

Solution 3

substituting $\rho(2\pi r)\,dr$ for dm or indicating the correct integration limits, 1 for the correct final answer $I = 0.012$ kg·m^2, including units.

Solution 3

(d)(ii) Mark scheme

- After the string is cut, apply the rotational form of Newton's second law, $\Sigma\tau = I\alpha$, to the clay-disk system: the only remaining torque about the axle is from the clay's weight, $\tau_{net} = Rm_cg$ (clay still at Point A). The system's rotational inertia is the sum of the disk's and the clay's rotational inertia, $I_{system} = I_{disk} + I_{clay}$, where $I_{clay} = m_cR^2$ (point mass at radius R) and $I_{disk} = 0.012\ \text{kg}{\cdot}\text{m}^2$ from part (d)(i). So

$$\alpha = \frac{\tau_{net}}{I_{system}} = \frac{Rm_cg}{I_{disk} + I_{clay}} = \frac{(0.3\ \text{m})(0.60\ \text{kg})(9.8\ \text{m/s}^2)}{(0.012\ \text{kg}\cdot\text{m}^2) + (0.60\ \text{kg})(0.3\ \text{m})^2} = 26.7\ \text{rad/s}^2.$$

Solution 3

Points: 1 for using the rotational form of Newton's second law, 1 for a correct expression for the net torque exerted on the clay-disk system, 1 for indicating that the system's rotational inertia is the sum of the rotational inertia of the disk and the rotational inertia of the lump of clay.

Question 1

2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in

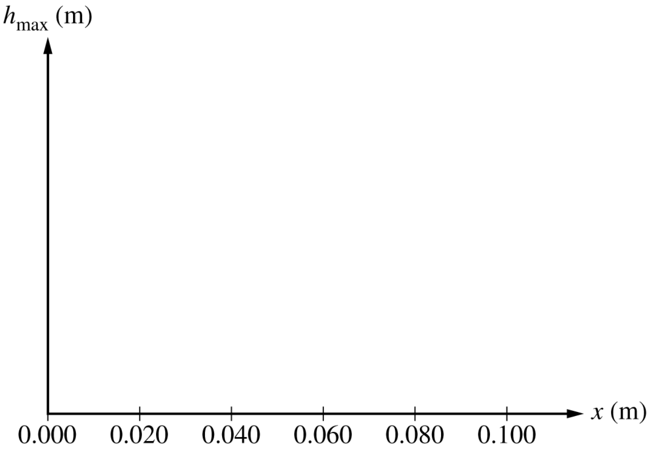
Question 1

Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]
(a) For the experiment, the block is pushed against one of the springs, compressing the spring a distance $x = 0.010$ m. The block is then released from rest. In Trial 1, Spring P is used, and in Trial 2, Spring Q is used. On the following representations of the block in Trials 1 and 2, draw and label the forces (non-components) that are exerted on the block at the instant the block is released. Each force must be

Question 1

represented by a distinct arrow starting on, and pointing away from, the dot. The lengths of the horizontal vectors should represent the relative magnitude of the horizontal forces and the lengths of the vertical vectors should represent the relative magnitude of the vertical forces.
[Two grid diagrams are shown, each with a dot at the center representing the block: "Trial 1 (Spring P)" on the left and "Trial 2 (Spring Q)" on the right. Each grid is for the student to draw labeled force vectors.]

Question 1



Question 1

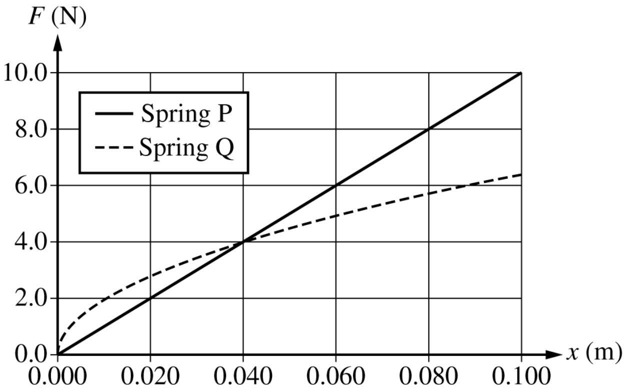
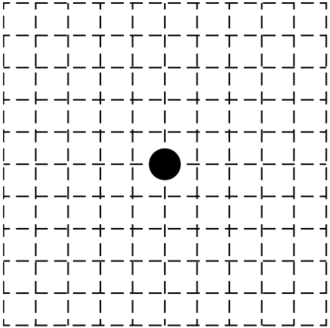


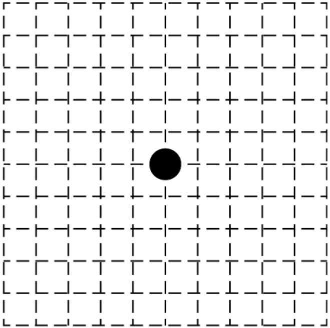
Figure 2

Question 1

Trial 1
(Spring P)



Trial 2
(Spring Q)



Question 1

2023 Set 1 ▪ Q1

[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\mathrm{max,P}}$. Frictional forces between the block and all surfaces are negligible.
A student compares $h_{\mathrm{max,P}}$ for Spring P with the different height $h_{\mathrm{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

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straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(b)(i) What feature(s) of the graph in Figure 2 could be used to estimate the work done on the block by each spring as each spring is compressed?

(b)(ii) There is one compression distance x_0 for which the maximum height h_{max} reached by the block is the same regardless of which spring, Spring P or Spring Q, is used. Predict whether the value of x_0 is greater than, less than, or equal to 0.040 m. Use the graph in Figure 2 to justify your answer.

(b)(iii) On the axes provided, for both Spring P and Spring Q, sketch a graph of the maximum height h_{max} reached by the block as a function of the distance x each spring is compressed for values of x ranging from 0 to 0.100 m. Clearly label the curve for Spring P and Spring Q.

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[Blank axes provided: y-axis labeled h_{max} (m), x-axis labeled x (m) from 0.000 to 0.100 in increments of 0.020, for the student to sketch two curves.]

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[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]

A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.

A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.

[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

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straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(c)(i) [Figure 3: Block A rests at the top of an inclined ramp of height H, connected by a string over the top of the ramp; Block B rests on the horizontal surface at the bottom of the ramp, connected to Spring Q which is attached to a wall, with $x = 0$ marked at the natural length position of the spring.]

Spring Q is attached to the wall and at equilibrium, as shown in Figure 3. Block A has a mass of 0.120 kg and Block B has a mass of 0.070 kg. Block A is held at rest at the top of the ramp on the horizontal surface. The change in vertical height of Block A is $H = 0.75$ m. The student releases Block A, and it moves down the ramp and collides with Block B. After the collision, the blocks stick together and move to the right,

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compressing the spring. Frictional forces between the blocks and all surfaces are negligible.
Calculate the velocity of the two-block system immediately after the collision between Blocks A and B.
(c)(ii) Calculate the maximum compression of the spring.

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[Figure 1: A block on an inclined ramp on the left, connected via the horizontal surface to Spring P mounted against a wall on the right. The spring is compressed a distance x from its natural length, with $x = 0$ marked at the wall-side end of the spring where it is uncompressed.]
A block sitting on a horizontal surface is pushed against a spring, Spring P, that is attached to a wall, compressing the spring a distance x, as shown in Figure 1. The block is then released from rest. The block slides along the horizontal surface and up a ramp, reaching a maximum height $h_{\text{max,P}}$. Frictional forces between the block and all surfaces are negligible.
A student compares $h_{\text{max,P}}$ for Spring P with the different height $h_{\text{max,Q}}$ achieved with a different spring, Spring Q. Each spring exerts a force of magnitude F on the block that varies as a function of the distance x the spring is compressed, as graphed in Figure 2. For Spring P, $F_P(x) = kx$, where $k = 100.0$ N/m, and for Spring Q, $F_Q(x) = Cx^{1/2}$, where $C = 20.0$ N/m$^{1/2}$.
[Figure 2: Graph of F (N) on the y-axis from 0.0 to 10.0, versus x (m) on the x-axis from 0.000 to 0.100, in increments of 0.020. Two curves are plotted: a solid line labeled "Spring P" that is

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straight, passing through the origin and rising linearly (consistent with $F_P(x) = kx$, reaching about 10.0 N at $x = 0.100$ m); a dashed curve labeled "Spring Q" that rises steeply at first and then levels off (consistent with $F_Q(x) = Cx^{1/2}$, reaching about 6.3 N at $x = 0.100$ m).]

(d) Spring Q where $F_Q(x) = Cx^{1/2}$ is replaced by a different nonlinear spring, Spring R, and the procedure described in part (c) is repeated. For Spring R, $F_R(x) = Dx^{1/2}$. The maximum compression of Spring R is greater than the maximum compression of Spring Q. Which of the following correctly compares the constants C and D?

______ $C < D$ ______ $C > D$ ______ $C = D$

Briefly justify your answer.

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[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total

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rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision, the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(a) Derive an expression for the angular speed of the rod just before striking the sphere in terms of the length ℓ and physical constants as appropriate.

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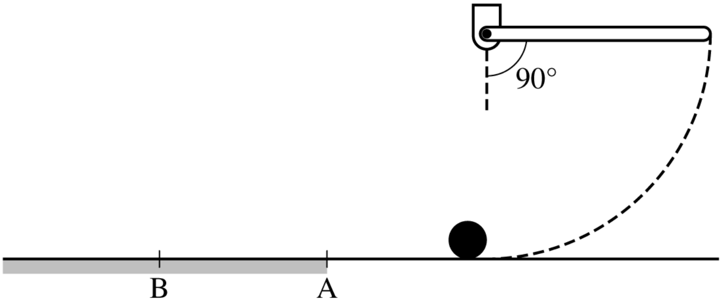

Note: Figure not drawn to scale.

Figure 1

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[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(b) Derive an expression for the linear speed v_0 of the sphere immediately after colliding with the rod in terms of the length ℓ and physical constants as appropriate. After sliding a short distance, at time $t = 0$ the sphere encounters a region of the horizontal surface with a coefficient of kinetic friction μ, beginning at Point A as indicated in Figure 1. The sphere begins rotating while sliding and eventually begins rolling without sliding at Point B, also as indicated.

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[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(c) In the following diagram, which represents the sphere while the sphere is traveling between Points A and B, draw and label the forces (not components) that act on the sphere. Each force must be represented by a distinct arrow starting on, and pointing away from, the point of application on the sphere.
[A circle is shown representing the sphere, for the student to draw and label force vectors starting on the circle.]

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[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a 90° angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(d)(i) Derive an expression for each of the following as the sphere is rotating and sliding between points A and B in terms of v_0, μ, R, t, and physical constants as appropriate.

The linear velocity v of the center of mass of the sphere as a function of time t

(d)(ii) The angular velocity ω of the sphere as a function of time t

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[Figure 1: A horizontal surface with Point B on the left and Point A near the middle marked by a black dot (the sphere's initial position at rest). A rod is shown above Point A, extending upward and connected at a pivot to a small block/weight at the top, with the rod initially vertical making a $90°$ angle at the pivot. A dashed curved arrow shows the rod swinging down from vertical to horizontal, striking the sphere at Point A. Note: Figure not drawn to scale.]

A system consists of a small sphere of mass m and radius R at rest on a horizontal surface and a uniform rod of mass $M = 2m$ and length ℓ attached at one end to a pivot with negligible friction, where $R \ll \ell$. There is negligible friction between the surface and the sphere to the right of Point A and nonnegligible friction to the left of Point A. The rod is held horizontally as shown in Figure 1, then is released from rest. The total rotational inertia of the rod about the pivot is $\frac{1}{3}M\ell^2$ and the rotational inertia of the sphere about its center is $\frac{2}{5}mR^2$. After the rod is released, the rod swings down and strikes the sphere head-on. As a result of this collision,

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the rod is stopped, and the ball initially slides without rotating to the left across the horizontal surface.

(e)(i) Derive an expression for the time it takes the sphere to travel from Point A to Point B in terms of v_0, μ, and physical constants as appropriate.

(e)(ii) Derive an expression for the linear velocity of the sphere upon reaching Point B in terms of v_0.

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A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.
[Diagram: A pulley (marked with a dot in a circle) is mounted at the top-left, at height H above a horizontal table. A dashed horizontal line runs from the pulley to the right, and a "+y" axis arrow points up with "+x" pointing right from a small axes

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marker near the pulley's dashed line. A string runs from the pulley down at an angle θ to a block of mass m resting on the table. The table's left edge is at $x = 0$ and the block starts at $x = L$ (marked on the table surface). Below the table, on the floor, is a motor connected via the string that runs up to the pulley.]

(a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at $x = L$. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

[Diagram: A single dot with a dashed horizontal line through it and a dashed vertical line through it, representing the free body of the block, on which forces should be drawn.]

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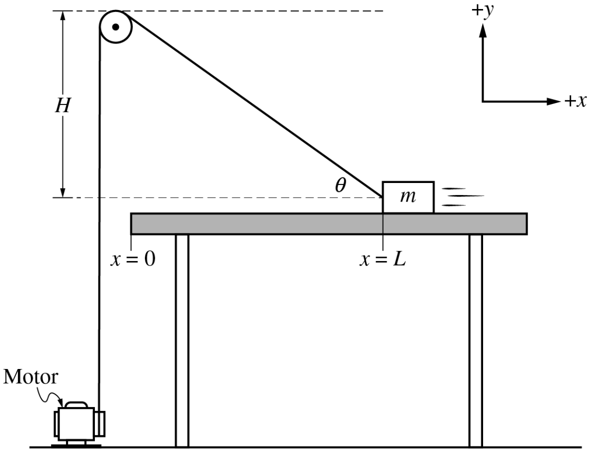

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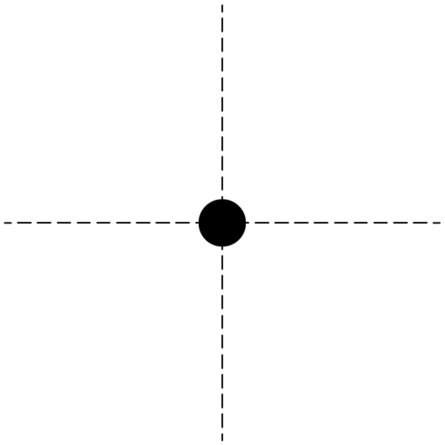

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A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

[Diagram: A pulley (marked with a dot in a circle) is mounted at the top-left, at height H above a horizontal table. A dashed horizontal line runs from the pulley to the right, and a "+y" axis arrow points up with "+x" pointing right from a small axes marker near the pulley's dashed line. A string runs from the pulley down at an angle θ to a block of mass m resting on the table. The table's left edge is at $x = 0$ and the block starts at $x = L$ (marked on the table

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surface). Below the table, on the floor, is a motor connected via the string that runs up to the pulley.]

(b) Derive an expression for the angle θ that the string makes with the horizontal as a function of x.

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A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

[Diagram: A pulley (marked with a dot in a circle) is mounted at the top-left, at height H above a horizontal table. A dashed horizontal line runs from the pulley to the right, and a "+y" axis arrow points up with "+x" pointing right from a small axes marker near the pulley's dashed line. A string runs from the pulley down at an angle θ to a block of mass m resting on the table. The table's left edge is at $x = 0$ and the block starts at $x = L$ (marked on the table

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surface). Below the table, on the floor, is a motor connected via the string that runs up to the pulley.]

(c)(i) Derive an expression for the normal force F_N exerted on the block by the table as a function of the block's position x.

(c)(ii) Derive an expression for the magnitude of the net horizontal force F_{net} exerted on the block as a function of the position x.

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A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension F_T on the string, and the block remains in contact with the table at all times as the block slides across the table from $x = L$ to $x = 0$. The coefficient of kinetic friction between the table and the block is μ_k. Express all algebraic answers in terms of m, H, F_T, x, μ_k, L, and physical constants as appropriate.

[Diagram: A pulley (marked with a dot in a circle) is mounted at the top-left, at height H above a horizontal table. A dashed horizontal line runs from the pulley to the right, and a "+y" axis arrow points up with "+x" pointing right from a small axes marker near the pulley's dashed line. A string runs from the pulley down at an angle θ to a block of mass m resting on the table. The table's left edge is at $x = 0$ and the block starts at $x = L$ (marked on the table

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surface). Below the table, on the floor, is a motor connected via the string that runs up to the pulley.]

(d) Write, but do not solve, an integral expression that could be used to solve for the work W done by the string on the block as the block moves from $x = L$ to $x = 0$.