Past-paper walk-through

Circular Motion ▪ 2.10

eXercise

Questions in this set

- 2021 Set 1 Q2
- 2021 Set 2 Q3
- 2019 Set 1 Q2
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- 2016 Q3
- 2014 Q2

Reading the mark scheme

- **B1** a mark that stands on its own – the point is either made or not.
- **M1** a *method* mark: the right approach, even if the number is wrong.
- **A1** an *answer* mark, and it usually needs its M mark first.
- **C1** a working mark on the way to the answer.
- // separates answers the examiner accepts equally.
- **ecf** = error carried forward: a later part is marked on your own earlier answer.
- **owtte** = "or words to that effect" – the wording need not match.

Question 2

2021 Set 1 ▪ Q2

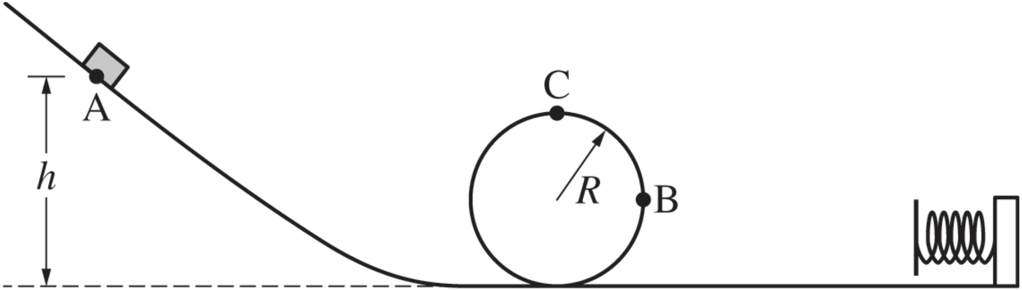
[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

Question 2

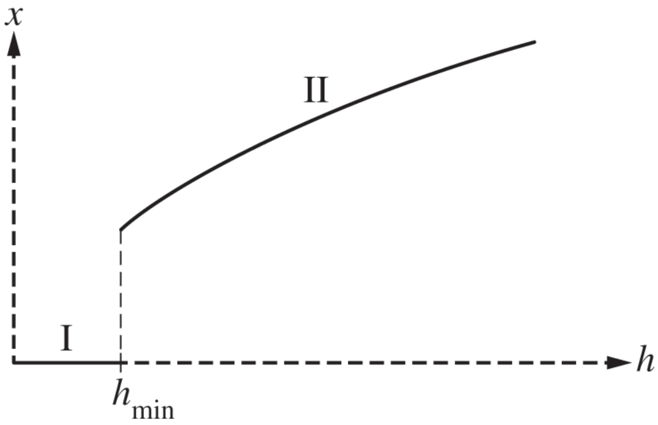
(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.
[Diagram: a single dot with a horizontal dashed line through it and a vertical dashed line through it, representing point B.]
Justify your answer.

Question 2

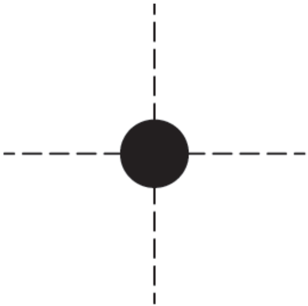


Note: Figure not drawn to scale.

Question 2



Question 2



Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

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(b)(i) Derive an expression for the speed v of the block at point B.

Question 2

(b)(ii) Derive an expression for the magnitude of the net force F on the block at point B.

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

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(c) In terms of R, derive an expression for the minimum height $h_{\min}$ necessary for the block to maintain contact with the track through point C.

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

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Question 2

(d) It is determined that $h = 0.30$ m and $R = 0.10$ m. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{MAX} of the spring?

Question 2

2021 Set 1 ▪ Q2

[Figure: A block sits at point A at the top of a curved incline of height h above a horizontal surface. The incline curves smoothly down and connects to a vertical circular loop of radius R; point C is the highest point on the loop and point B is the rightmost point on the loop. After the loop, the horizontal surface continues to a spring attached to a wall. Note: Figure not drawn to scale.]

A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \dfrac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

(e)(i) A graph of the maximum compression of the spring as a function of height is shown below. The height $h_{\min}$ is the height calculated in part (c).

Question 2

[Graph: axes x (vertical) vs. h (horizontal). For h from 0 to h_{min}, labeled region I, x is a horizontal line segment at a constant low value along the horizontal axis. For $h > h_{\mathrm{min}}$, labeled region II, the curve rises smoothly and continues increasing with an upward-curving-then-leveling shape.]

Explain why section I appears as a horizontal line segment on the horizontal axis.

(e)(ii) Explain the reason for the shape of section II on the graph.

Solution 2

(a)

Mark scheme

- Draw a single acceleration vector from the dot at point B pointing down and to the left (1 point for correct direction with an attempted justification). Justification (1 point): at point B the block moves along a circular arc, so it has a centripetal acceleration component directed toward the center of the circle (horizontally, toward the left in this geometry) plus the usual downward gravitational acceleration; the vector sum of these two components points down and to the left.

Solution 2

(b)(i)

Mark scheme

- Apply conservation of energy between the release point A (height h, at rest) and point B (height $= R$, bottom of the loop):
  $K_A + U_{gA} = K_B + U_{gB} \Rightarrow \frac{1}{2}mv_A^2 + mgh_A = \frac{1}{2}mv_B^2 + mgh_B$ (1 point).
  Substitute $v_A = 0,\ h_A = h,\ h_B = R$ (1 point):
  $mgh = \frac{1}{2}mv_B^2 + mgR \Rightarrow v_B = \sqrt{2g(h-R)}$.

Solution 2

(b)(ii) Mark scheme

- Substitute v_B into the centripetal-force expression (1 point): $F_c = \frac{mv_B^2}{R} = \frac{m\left(\sqrt{2g(h-R)}\right)^2}{R} = \frac{2mg(h-R)}{R}$. Combine F_c (horizontal) with weight mg (vertical) via vector addition to get the net-force magnitude at B (1 point): $F_{net} = \sqrt{F_c^2 + (mg)^2} = \sqrt{\left(\frac{2mg}{R}(h-R)\right)^2 + (mg)^2}$.

Solution 2

(c) Mark scheme

- Find the speed at point C (top of the loop, height $2R$) via conservation of energy (1 point): $0 + mgh = \frac{1}{2}mv_C^2 + mg(2R) \Rightarrow v_C = \sqrt{2g(h-2R)}$. Apply Newton's second law at C with the normal force set to zero (the minimum-speed / minimum-height condition, 1 point): $mg = \dfrac{mv_C^2}{R} \Rightarrow v_C = \sqrt{Rg}$. Combine the two expressions for v_C (1 point):
$\sqrt{Rg} = \sqrt{2g(h-2R)} \Rightarrow R = 2(h-2R) \Rightarrow h_{\min} = 2.5R$.

Solution 2

(d) Mark scheme

- With $h = 0.30$ m, $R = 0.10$ m (above $h_{\min}$), apply conservation of energy from release to maximum spring compression, where all gravitational PE converts to elastic PE (1 point for using energy conservation): $mgh = \frac{1}{2}kx_{MAX}^2$ (1 point for the correctly substituted energy equation). Using the spring-constant relation $k = mg/(2R)$ (established earlier in the problem) and solving for x_{MAX} (1 point): $x_{MAX} = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2mgh}{mg/(2R)}} = \sqrt{4hR} = \sqrt{4(0.30\text{ m})(0.10\text{ m})} = 0.35\text{ m}.$

Solution 2

(e)(i)

Mark scheme

- Justification (1 point) for why section I (for $h \leq h_{\mathrm{min}}$) is flat at $x \approx 0$: because the block cannot make it around the full loop at these heights (it loses contact with the track before reaching the top), it never reaches the spring, so there is no compression.

Solution 2

(e)(ii)

Mark scheme

- For section II ($h > h_{\min}$): as h increases, the compression x of the spring increases (1 point). From part (c)'s relation $x_{MAX} = \sqrt{4hR}$, x_{MAX} is proportional to $\sqrt{h}$, i.e. h is proportional to x^2 – so the curve has the shape of a sideways-opening (x-axis) parabola, rising from $h_{\min}$ and curving upward, rather than a straight line (1 point).

Question 3

2021 Set 2 ▪ Q3

[Figure: A block of mass m sits on top of a vertical spring (spring constant k) which is compressed a distance Δx from its natural length, at the bottom of a vertical track. The track curves upward and to the right in a smooth arc, reaching a highest point labelled A at height $3R$ above the block's release point, where the track has constant radius of curvature R (shown with a radius arrow labelled R from a center point to point A). The track continues curving over the top and down to point B, at the same height $3R$, where the track becomes horizontal (radius R again, shown with a radius arrow labelled R from a center point to B) and the block exits horizontally moving to the right and slightly upward along a dashed projectile trajectory, landing a horizontal distance D from the end of the track (measured from directly below B). A note states "Figure not drawn to scale."]

Question 3

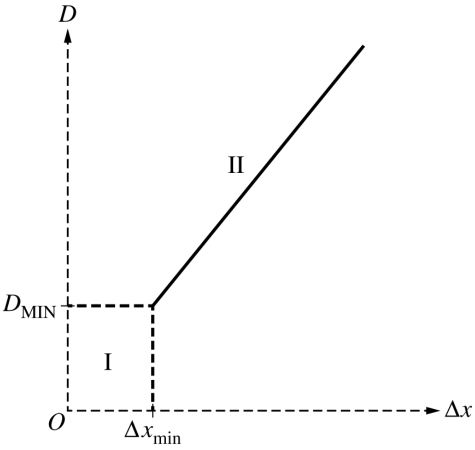
A block of mass m is placed on top of an ideal spring of spring constant k. The block is pushed against the spring, compressing the spring a distance Δx. The block is released from rest, leaves the spring at the position shown in the figure, travels upward, and enters a track with a constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(a) On the dot below, which represents the block, draw and label the forces (not components) that act on the block while still in contact with the track at point B.

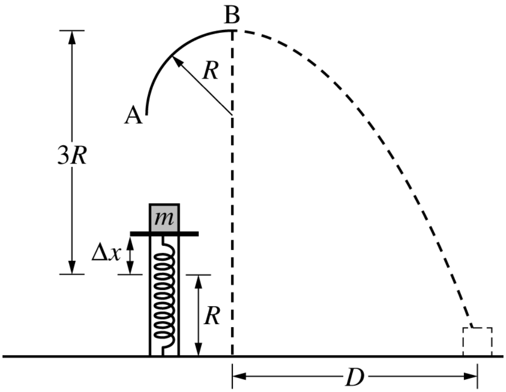
Question 3

Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.
[A single black dot is shown, representing the block at point B, with blank space around it for the student to draw force arrows.]
Justify your choice of vectors.

Question 3



Question 3



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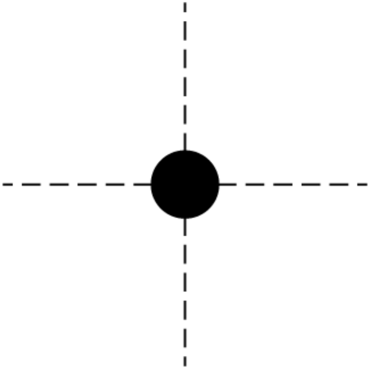
Question 3



Question 3

2021 Set 2 ▪ Q3

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Question 3

constant radius of curvature R that has negligible friction. The block enters the track at point A, maintains contact with the track, and exits horizontally at point B, a distance $3R$ above the point the block was released. The block then falls to the ground and lands a horizontal distance D from the end of the track. Express all algebraic answers in terms of m, k, Δx, R, and physical constants, as appropriate. The size of the block is much smaller than the radius of curvature of the track.

(b)(i) Derive an expression for the speed v of the block at point B.

(b)(ii) Derive an expression for the magnitude of the net force F on the block at point B.

Question 3

2021 Set 2 ▪ Q3

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(c) Derive an expression for the minimum value of Δx_{min} required in order for the block to maintain contact with the track through point B.

The procedure is repeated several times with the distance $\Delta x > \Delta x_{\mathrm{min}}$.

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2021 Set 2 ▪ Q3

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Question 3

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(d) Calculate the distance D that the block travels.

Question 3

2021 Set 2 ▪ Q3

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Question 3

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(e)(i) The graph below shows the best-fit line drawn by the students through their data of D as a function of Δx.

[Graph: vertical axis D, horizontal axis Δx. A dashed horizontal line marks D_{MIN} on the D-axis, and a dashed vertical line marks $\Delta x_{\min}$ on the Δx-axis, meeting at the start of the data trend. Region "I" is labelled in the lower-left area (below/left of the dashed lines, near the origin O, where there is no plotted line). Region "II" is labelled

Question 3

along a straight line that starts at the point $(\Delta x_{\mathrm{min}}, D_{\mathsf{MIN}})$ and rises linearly up and to the right.]
Explain why there are no data for section I of the graph.
(e)(ii) Explain the reason for the shape and minimum value of section II on the graph.

Solution 3

(a) Mark scheme

- At point B, draw two forces on the dot: weight F_g pointing straight down (1 pt), and the normal force from the track F_N pointing toward the center of the circular path / perpendicular to the track surface (1 pt), with a justification consistent with the diagram (1 pt). Example: 'The force F_g represents the weight of the block and always points downward. The force F_N represents the force the track exerts on the block to keep it moving in a circular path and points perpendicular to the surface of the track.' Acceptable gravity labels: F_G, F_g, F_{grav}, W, mg, Mg, etc. (not G or g alone); acceptable normal-force labels: F_n, F_N, N, 'normal force'. Note: if extraneous forces are drawn, a maximum of 2 of the 3 points can be earned.

Solution 3

(b)(i) Mark scheme

- Use conservation of energy for the system, block starting at rest: $U_1 + K_1 = U_2 + K_2$, i.e. $U_1 + 0 = U_2 + K_2$ (1 pt). Relate the elastic PE at maximum spring compression to the gravitational PE at point B: $U_{s1} + 0 = U_{g2} + K_2$, so $K_2 = U_{s1} - U_{g2}$ (1 pt). Substitute: $\frac{1}{2}mv_B^2 = \frac{1}{2}k(\Delta x)^2 - mgh_2$, where $h_2 = 3R$ is the height of point B; so $v_B^2 = \frac{k}{m}(\Delta x)^2 - 2g(3R)$, giving $v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$ (1 pt). Final answer: $v_B = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$.

Solution 3

(b)(ii) Mark scheme

- Relate the centripetal (net) force at point B to the speed found in part (b)(i): $F_C = \frac{mv^2}{r} = \frac{mv_B^2}{R}$ (1 pt). Substitute v_B from (b)(i) for an answer consistent with that result: $F_C = \frac{m}{R}\left(\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}\right)^2 = \frac{k(\Delta x)^2}{R} - 6mg$ (1 pt). Final answer: $F_C = \frac{k(\Delta x)^2}{R} - 6mg$.

Solution 3

(c) Mark scheme

- At the minimum compression, the block barely maintains contact with the track at point B, so the normal force is zero and the net force equals gravity alone: relate the net-force expression from (a)/(b)(ii) to this condition (1 pt): $\dfrac{k(\Delta x)^2}{R} - 6mg = mg$. Substitute and solve (1 pt): $\dfrac{k(\Delta x)^2}{R} = 7mg \implies \Delta x = \sqrt{\dfrac{7mgR}{k}}$. Final answer: $\Delta x_{\mathrm{min}} = \sqrt{\dfrac{7mgR}{k}}$.

Solution 3

(d) Mark scheme

- Relate the height of fall ($4R$, from point B down to the ground) to the time of fall using projectile motion: $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$; with a fall height of $4R$: $t = \sqrt{\frac{2(4R)}{g}} = \sqrt{\frac{8R}{g}}$ (1 pt). Substitute into $D = v_x t$ using the (constant) horizontal velocity from part (b)(i): $D = v_x t = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR} \cdot \sqrt{\frac{8R}{g}}$ (1 pt, must be consistent with the expression for v_B from (b)(i)). Final answer: $D = \sqrt{\frac{k}{m}(\Delta x)^2 - 6gR} \cdot \sqrt{\frac{8R}{g}}$.

Solution 3

(e)(i)

Mark scheme

- Provide a correct justification for the general shape of the D-vs-Δx graph, i.e. why no data/motion occurs for compressions below the minimum (Region I). Example reasoning: 'The block needs a minimum speed to make it through point B on the track; thus, the flat/empty segment (Region I) represents compressions of the spring for which the block does not make it to point B.' (1 pt for a correct justification.)

Solution 3

(e)(ii) Mark scheme

- Explain both the shape and the minimum value of Region II: (1 pt) as the maximum compression Δx increases beyond $\Delta x_{\min}$, the distance D increases — consistent with part (d), D grows with $\sqrt{\frac{k}{m}(\Delta x)^2 - 6gR}$, so beyond the threshold the graph rises monotonically with Δx; (1 pt) the minimum value (onset of Region II, at $\Delta x_{\min}$) occurs because that is exactly the minimum spring compression that gives the block the minimum speed needed to maintain contact with the track through point B (from part (c)).

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

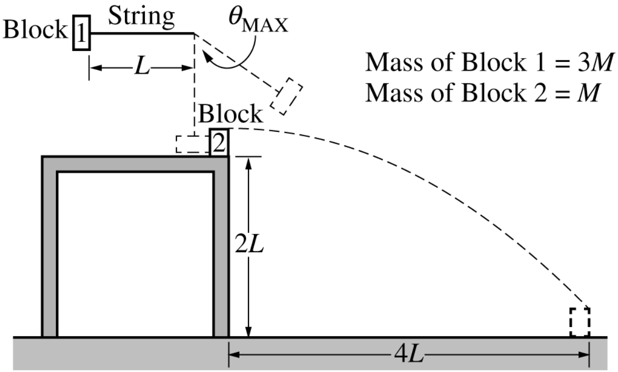
A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a

Question 2

distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

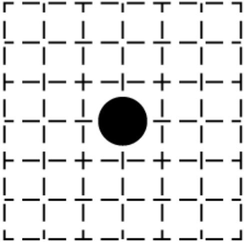
Question 2



Mass of Block 1 = $3M$
Mass of Block 2 = M

Note: Figure not drawn to scale.

Question 2



Question 2

2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.

[A dot on a dashed grid, representing block 1, for the student to draw force vectors on.]

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After

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the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

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Question 2

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(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(e) Calculate the speed of block 2 as it leaves the table.

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2019 Set 1 ▪ Q2

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(f) Calculate the speed of block 1 just after it collides with block 2.

Question 2

2019 Set 1 ▪ Q2

[Figure above the question: A pendulum setup. Block 1 (labeled "Block 1") is attached via a "String" of length L to a pivot, shown horizontal at the top with the string making angle θ_{MAX} with the vertical at its highest point after swinging (dashed arc). Block 2 sits at the edge of a table of height $2L$, at the top of a support structure. The horizontal distance from the base of the table to the landing point on the floor is $4L$ (dashed box shown on the floor at that distance). Labels: "Mass of Block 1 = $3M$", "Mass of Block 2 = M". Note: Figure not drawn to scale.]

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Question 2

the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

Solution 2

(a)

Mark scheme

- Energy conservation for block 1's swing: $U_{g1} = K_2 \Rightarrow mgh = \frac{1}{2}mv^2 \Rightarrow gL = \frac{1}{2}v^2 \Rightarrow v = \sqrt{2gL}$. Full credit even if no work is shown.

Solution 2

(b)

Mark scheme

- Draw two vectors from the dot: the weight F_W pointing straight down, and the string tension F_T pointing straight up, with the tension arrow drawn noticeably LONGER than the weight arrow (net force must be upward/centripetal). 1 pt for correctly drawing and labeling both the weight and the tension; 1 pt for drawing the tension longer than the weight. Only a maximum of 1 point can be earned if any extraneous vectors are included.

Solution 2

(c)

Mark scheme

- Use Newton's second law along the string direction, tension minus weight equals centripetal force (1 pt):
  $F_T = mg + ma_c = mg + \frac{mv^2}{r} = 3Mg + \frac{(3M)(\sqrt{2gL})^2}{L}$. Correct final answer (1 pt): $F_T = 9Mg$.

Solution 2

(d) Mark scheme

- Use a correct kinematic equation for the vertical fall from table height $2L$ (1 pt): $\Delta y = v_i t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2\Delta y}{a}} = \sqrt{\frac{2(2L)}{g}} = \sqrt{\frac{4(0.75\text{ m})}{9.8\text{ m/s}^2}}$. Correct final answer (1 pt): $t = 0.55$ s.

Solution 2

(e)

Mark scheme

- Use a correct kinematic equation for the horizontal (projectile) motion (1 pt): $\Delta x = vt \Rightarrow v = \dfrac{\Delta x}{t} = \dfrac{4L}{t}$. Substitute the time from part (d) (1 pt): $v = \dfrac{(4)(0.75\ \text{m})}{0.55\ \text{s}} = 5.45\ \text{m/s}$.

Solution 2

(f) Mark scheme

- Use conservation of momentum for the collision (1 pt):
$p_i = p_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$. Correctly substitute the masses (1 pt): $(3M)v_{1i} + 0 = (3M)v_{1f} + (M)v_{2f}$. Substitute the speeds found in parts (a) and (e) (1 pt): $v_{1f} = \dfrac{(3M)v_{1i} - (M)v_{2f}}{3M} = \dfrac{(3M)\sqrt{2gL} - (M)v_{2f}}{3M}$, giving
$v_{1f} = \dfrac{3\sqrt{2(9.8)(0.75)} - (1)(5.45)}{3} = 2.02$ m/s (or $v_{1f} = 2.06$ m/s using $g = 10$ m/s^2).

Solution 2

(g) Mark scheme

- Use conservation of energy for block 1's rise after the collision (1 pt): $K_1 = U_{g2} \Rightarrow \frac{1}{2}mv^2 = mgh$. Correctly substitute $h = L(1-\cos\theta)$ (1 pt): $\frac{1}{2}mv^2 = mgL(1-\cos\theta) \Rightarrow \dfrac{v^2}{2gL} = 1-\cos\theta$. Substitute the speed found in part (f) (1 pt): $\theta = \cos^{-1}\left(1-\dfrac{v^2}{2gL}\right) = \cos^{-1}\left(1-\dfrac{(2.02)^2}{2(9.8)(0.75)}\right) = 43.5^\circ$ (or $\theta = 44.1^\circ$ using $g = 10$ m/s^2).

Question 3

2019 Set 2 ▪ Q3

[Figure: A point P at height h above the floor, at the top of a straight incline going down and to the right, with a small circle labeled m, r at P representing the rolling sphere. The incline descends to a circular vertical loop of radius R (labeled with a radius line from the center to the edge of the loop), with point A marked at the top of the loop. After the loop, the track continues to the right and up as a straight incline. A vertical dashed line with arrows shows the height h measured from point P down to the level of the bottom of the loop/floor. Note: Figure not drawn to scale.]

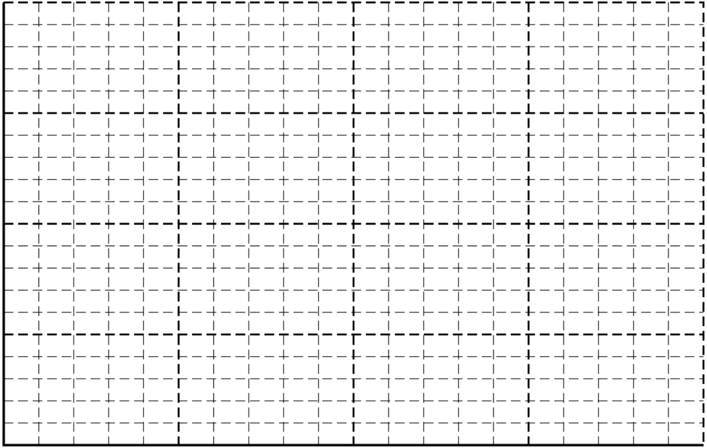
The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius

Question 3

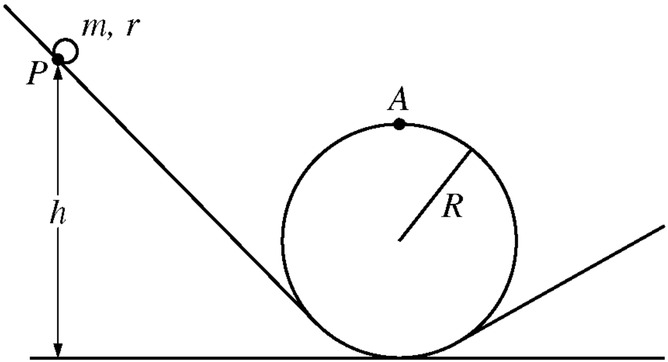
$R \gg r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

Question 3



Question 3



Note: Figure not drawn to scale.

Question 3

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20