Exercise walk-through

Systems and Center of Mass ▪ 2.1

eXercise

You must be able to

- treat a group of objects as a single **system 系统**
- find the **center of mass 质心** of point masses: $x_{cm} = \dfrac{\sum m_i x_i}{\sum m_i}$
- explain that the center of mass moves as if all mass and the net external force acted there
- use symmetry to locate the center of mass of a uniform object

Method — Center of mass of point masses

Weight each position by its mass and divide by the total mass:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}.$$

For 2 kg at $x = 0$ and 3 kg at $x = 5$ m:

$$x_{cm} = \frac{2(0) + 3(5)}{2 + 3} = \frac{15}{5} = 3 \text{ m}.$$

It sits closer to the heavier mass.

Method — The center of mass obeys Newton's second law

The center of mass of a system accelerates as if the total mass were there and the **net external force** acted on it. Internal forces cancel and never move it.

Method — Symmetry

A uniform rod's center of mass is at its midpoint; a uniform disc's is at its centre.

Practice 2.1 ▪ 2 marks

Two masses, 1 kg at $x = 0$ and 1 kg at $x = 4$ m. Find the center of mass.

Solution 2.1

Method: Center of mass of point masses

Worked steps

$$x_{cm} = \frac{1(0) + 1(4)}{2} = 2 \text{ m}$$ M1 A1.

Practice 2.2 ▪ 2 marks

Masses 2 kg at $x = 0$ and 6 kg at $x = 8$ m. Find the center of mass.

Solution 2.2

Method: Center of mass of point masses

Worked steps

$$x_{cm} = \frac{2(0) + 6(8)}{8} = \frac{48}{8} = 6 \text{ m} \quad \text{M1} \; \text{A1}.$$

Practice 2.3 ▪ 1 mark

State where the center of mass of a uniform metre ruler is located.

Solution 2.3

Method: Symmetry

Worked steps

At the 50 cm mark (its midpoint) B1.

Practice 2.4 ▪ 1 mark

Can internal forces alone move a system's center of mass? Answer yes or no.

Solution 2.4

Method: The center of mass obeys Newton's second law

Worked steps

No — only a net external force can move the center of mass B1.

Exam-style 3.1 ▪ 1 mark

The center of mass of a system accelerates in response to the

A internal forces

B net external force

C largest single mass

D total kinetic energy

Solution 3.1

Method: The center of mass obeys Newton's second law

A internal forces

B net external force ✓

C largest single mass

D total kinetic energy

Worked steps

B — the net external force drives the center of mass.

Exam-style 3.2 ▪ 1 mark

Two equal masses sit at $x = 2$ m and $x = 6$ m. Their center of mass is at

A 2 m

B 4 m

C 6 m

D 8 m

Solution 3.2

Method: Center of mass of point masses

A 2 m

B 4 m ✓

C 6 m

D 8 m

Worked steps

B — equal masses give the midpoint, 4 m.

Exam-style 3.3 ▪ 2 marks

Point masses: 4 kg at $x = 1$ m and 2 kg at $x = 7$ m.
(a) Find the center of mass.
(b) State whether it is closer to the 4 kg or the 2 kg mass, and why.

Solution 3.3

Method: Center of mass of point masses

Worked steps

(a) $x_{cm} = \dfrac{4(1) + 2(7)}{6} = \dfrac{18}{6} = 3$ m M1 A1. (b) Closer to the 4 kg mass, because it is weighted more heavily B1.

Exam-style 3.4 ▪ 1 mark

An astronaut floating in space throws a tool.
(a) State what happens to the center of mass of (astronaut + tool).
(b) Explain your answer using external forces.

Solution 3.4

Method: The center of mass obeys Newton's second law

Worked steps

(a) It keeps moving as before (stays at rest if it was at rest) B1. (b) Throwing is an internal force, and internal forces cannot move the center of mass B1.