Exercise walk-through

Motion in Two or Three Dimensions ▪ 1.5

eXercise

You must be able to

- treat the horizontal and vertical motions as **independent** 独立
- use constant velocity horizontally and free-fall acceleration g vertically
- find the **time of flight** 飞行时间 from the vertical motion, then the range from the horizontal
- combine perpendicular velocity components into a resultant

Method — Independent axes

In projectile motion the two axes do not affect each other:

- horizontal: constant velocity, $x = v_x t$;
- vertical: free fall, $a_y = -g$.

They share only the **time** t.

Method — Horizontal launch

A ball rolls off a table at $v_x = 5$ m s^{-1} from height $h = 1.25$ m. Vertical: $h = \frac{1}{2}gt^2$, so

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.25)}{10}} = 0.5 \text{ s}.$$

Horizontal range: $x = v_x t = 5 \times 0.5 = 2.5$ m.

Method — Combining velocity components

At any instant the speed is $\sqrt{v_x^2 + v_y^2}$. If $v_x = 5$ and $v_y = 5$, speed $= \sqrt{50} \approx 7.1$ m s^{-1}.

Practice 2.1 ▪ 1 mark

State what the horizontal and vertical motions of a projectile share in common.

Solution 2.1

Method: Independent axes

Worked steps

The time of flight (both motions happen over the same time) B1.

Practice 2.2 ▪ 1 mark

A ball is thrown horizontally at 8 m s^{-1}. State its horizontal acceleration (ignore air resistance).

Solution 2.2

Method: Horizontal launch

Worked steps

Zero — there is no horizontal force B1.

Practice 2.3 ▪ 2 marks

A stone falls from rest for 2 s. Find its vertical velocity (use $g = 10$ m s^{-2}).

Solution 2.3

Worked steps

$v = gt = 10 \times 2 = 20$ m s^{-1} M1 A1.

Practice 2.4 ▪ 2 marks

A projectile has $v_x = 6$ m s^{-1} and $v_y = 8$ m s^{-1} at some instant. Find its speed.

Solution 2.4

Method: Combining velocity components

Worked steps

$\sqrt{6^2 + 8^2} = 10$ m s^{-1} M1 A1.

Exam-style 3.1 ▪ 1 mark

For a projectile (no air resistance), the horizontal velocity

A increases

B decreases

C stays constant

D becomes zero at the top

Solution 3.1

Method: Independent axes

A increases

B decreases

C stays constant ✓

D becomes zero at the top

Worked steps

C — with no horizontal force, the horizontal velocity is constant.

Exam-style 3.2 ▪ 1 mark

Two balls leave a table at the same instant; one is dropped and one is thrown horizontally. They

A land at the same time

B the dropped one lands first

C the thrown one lands first

D never land

Solution 3.2

A land at the same time ✓

B the dropped one lands first

C the thrown one lands first

D never land

Worked steps

A — the vertical motions are identical, so they land together.

Exam-style 3.3 ▪ 2 marks

A ball is launched horizontally at 10 m s^{-1} from a height of 5 m ($g = 10$ m s^{-2}).
(a) Find the time to reach the ground.
(b) Find the horizontal range.

Solution 3.3

Method: Horizontal launch

Worked steps

(a) $t = \sqrt{2h/g} = \sqrt{2(5)/10} = 1$ s M1 A1. (b) $x = v_x t = 10 \times 1 = 10$ m M1 A1.

Exam-style 3.4 ▪ 2 marks

A projectile's velocity components are $v_x = 12$ m s^{-1} and $v_y = -5$ m s^{-1}.
(a) Find the speed.
(b) Find the angle below the horizontal.

Solution 3.4

Method: Combining velocity components

Worked steps

(a) $\sqrt{12^2 + 5^2} = 13$ m s^{-1} M1 A1. (b) $\tan^{-1}(5/12) = 23^\circ$ below the horizontal B1.