Exercise walk-through

Displacement, Velocity, and Acceleration ▪ 1.2

eXercise

You must be able to

- use **velocity 速度** as the derivative of position, $v = \dfrac{dx}{dt}$
- use **acceleration 加速度** as the derivative of velocity, $a = \dfrac{dv}{dt}$
- go the other way by **integration 积分**: $x = \int v\,dt$ and $v = \int a\,dt$
- read velocity as the slope of an x-t graph and displacement as the area under a v-t graph

Method — Derivatives: position to velocity to acceleration

Differentiate to move down the chain. If $x(t) = 3t^2$, then

$$v = \frac{dx}{dt} = 6t, \qquad a = \frac{dv}{dt} = 6.$$

At $t = 2$ s: $v = 12$ m s^{-1}, $a = 6$ m s^{-2}.

Method — Integration: acceleration back to velocity

Integrate to move up the chain, adding the starting value. If $a = 4$ (constant) and $v_0 = 2$, then

$$v = \int a \, dt = 4t + 2.$$

Method — Slopes and areas on graphs

- The **slope** of an *x*-*t* graph is the velocity.
- The **area** under a *v*-*t* graph is the displacement.

A *v*-*t* graph holds a steady 5 m s^{-1} for 4 s: displacement $= 5 \times 4 = 20$ m (the area).

Practice 2.1 ▪ 2 marks

A particle has $x(t) = 2t^3$. Find $v(t)$.

Solution 2.1

Worked steps

$v = \dfrac{dx}{dt} = 6t^2$ **M1** **A1**.

Practice 2.2 ▪ 1 mark

A particle has $v(t) = 6t$. Find $a(t)$.

Solution 2.2

Worked steps

$$a = \frac{dv}{dt} = 6 \text{ m s}^{-2}$$ **B1**.

Practice 2.3 ▪ 2 marks

A constant acceleration $a = 3$ m s^{-2} acts, with $v_0 = 5$ m s^{-1}. Write $v(t)$.

Solution 2.3

Worked steps

$v = \int a\,dt = 3t + 5$ **M1** **A1**.

Practice 2.4 ▪ 2 marks

A *v*-*t* graph is a constant 8 m s^{-1} for 3 s. Find the displacement.

Solution 2.4

Method: Slopes and areas on graphs

Worked steps

Displacement $= \text{area} = 8 \times 3 = 24$ m M1 A1.

Exam-style 3.1 ▪ 1 mark

The velocity of an object is the ______ of its position with respect to time.

A integral

B derivative

C reciprocal

D square

Solution 3.1

Method: Derivatives: position to velocity to acceleration

A integral

B derivative ✓

C reciprocal

D square

Worked steps

B — velocity is the derivative of position.

Exam-style 3.2 ▪ 1 mark

On a position-time graph, the velocity at a point is given by the

A area under the curve

B slope of the tangent

C height of the curve

D curvature

Solution 3.2

- **A** area under the curve
- **B** slope of the tangent ✓
- **C** height of the curve
- **D** curvature

Worked steps

B — the slope of the tangent gives the velocity.

Exam-style 3.3 ▪ 2 marks

A particle moves with $x(t) = t^3 - 6t$.

(a) Find $v(t)$.

(b) Find the acceleration $a(t)$.

Solution 3.3

Worked steps

(a) $v = 3t^2 - 6$ **M1** **A1**. (b) $a = 6t$ **B1**.

Exam-style 3.4 ▪ 2 marks

A particle starts from rest with acceleration $a(t) = 6t$.
(a) Find $v(t)$.
(b) Find the velocity at $t = 2$ s.

Solution 3.4

Method: Derivatives: position to velocity to acceleration

Worked steps

(a) $v = \int 6t\,dt = 3t^2$ M1 A1. (b) $v(2) = 3(2)^2 = 12$ m s^{-1} B1.