Exercise walk-through

Scalars and Vectors ▪ 1.1

eXercise

You must be able to

- distinguish a **scalar 标量** (magnitude only) from a **vector 矢量** (magnitude **and** direction 方向)
- resolve a vector into **components 分量**: $A_x = A\cos\theta$, $A_y = A\sin\theta$
- find a vector's **magnitude 大小** $A = \sqrt{A_x^2 + A_y^2}$ and its direction $\theta = \tan^{-1}(A_y/A_x)$
- add vectors by adding their components separately

Method — Scalar or vector?

A **scalar** has size only; a **vector** has size **and** direction.

scalar	**vector**
distance, speed, mass time, energy, temperature	displacement, velocity, force acceleration, momentum

Method — Resolving into components

A vector of magnitude A at angle θ above the x-axis splits into

$$A_x = A\cos\theta, \qquad A_y = A\sin\theta.$$

For $A = 10$ at $\theta = 30^\circ$: $A_x = 10\cos 30^\circ = 8.7$, $A_y = 10\sin 30^\circ = 5.0$.

Method — Magnitude and direction from components

Reverse the process with Pythagoras and the tangent:

$$A = \sqrt{A_x^2 + A_y^2}, \qquad \theta = \tan^{-1}\frac{A_y}{A_x}.$$

For $A_x = 3$, $A_y = 4$: $A = \sqrt{3^2 + 4^2} = 5$, and $\theta = \tan^{-1}(4/3) = 53^\circ$.

Method — Adding vectors by components

Add the x-components and the y-components **separately**, then recombine. Two forces $\vec{F}_1 = (3, 0)$ N and $\vec{F}_2 = (0, 4)$ N:

$$\vec{F}_{\text{net}} = (3 + 0,\ 0 + 4) = (3, 4)\ \text{N}, \qquad |\vec{F}_{\text{net}}| = 5\ \text{N}.$$

Practice 2.1 ▪ 2 marks

State whether each is a scalar or a vector: (a) speed, (b) velocity, (c) mass, (d) acceleration.

Solution 2.1

Method: Scalar or vector?

Worked steps

(a) scalar; (b) vector; (c) scalar; (d) vector **B1** **B1** *any two correct; all four.*

Practice 2.2 ▪ 2 marks

A displacement has magnitude 20 m at 60° above the x-axis. Find its x- and y-components.

Solution 2.2

Method: Resolving into components

Worked steps

$A_x = 20\cos 60^\circ = 10$ m **M1**; $A_y = 20\sin 60^\circ = 17$ m **A1**.

Practice 2.3 ▪ 2 marks

A vector has components $A_x = 6$ and $A_y = 8$. Find its magnitude.

Solution 2.3

Method: Resolving into components

Worked steps

$A = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ **M1** **A1**.

Practice 2.4 ▪ 2 marks

Add $\vec{A} = (5, 2)$ and $\vec{B} = (-1, 3)$ by components, and give the magnitude of the resultant.

Solution 2.4

Worked steps

$(5-1,\ 2+3) = (4,5)$ **M1**; magnitude $= \sqrt{4^2+5^2} = \sqrt{41} \approx 6.4$ **A1**.

Exam-style 3.1 ▪ 1 mark

Which quantity is a vector?

A speed

B distance

C acceleration

D mass

Solution 3.1

A speed

B distance

C acceleration ✓

D mass

Worked steps

C — acceleration is a vector; the others are scalars.

Exam-style 3.2 ▪ 1 mark

A vector of magnitude 12 makes an angle of 30° with the x-axis. Its x-component is closest to

A 6.0

B 10.4

C 12

D 0

Solution 3.2

Method: Resolving into components

A 6.0

B 10.4 ✓

C 12

D 0

Worked steps

B — $12\cos 30^\circ = 10.4$.

Exam-style 3.3 ▪ 2 marks

A vector has components $A_x = 9$ and $A_y = 12$.
(a) Find the magnitude of the vector.
(b) Find the angle it makes with the x-axis.

Solution 3.3

Method: Resolving into components

Worked steps

(a) $A = \sqrt{9^2 + 12^2} = \sqrt{225} = 15$ **M1** **A1**. (b) $\theta = \tan^{-1}(12/9) = 53^\circ$ **B1**.

Exam-style 3.4 ▪ 1 mark

Two velocity vectors are $\vec{v}_1 = (4, 0)$ m s^{-1} and $\vec{v}_2 = (0, -3)$ m s^{-1}.

(a) Find the components of the resultant.

(b) Find the magnitude and direction of the resultant.

Solution 3.4

Worked steps

(a) $(4, -3)$ m s^{-1} **B1**. (b) magnitude $\sqrt{4^2 + 3^2} = 5$ m s^{-1} **M1**; direction $\tan^{-1}(3/4) = 37°$ below the x-axis **A1**.