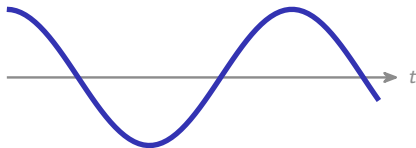


Oscillations

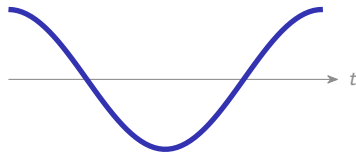
AP Physics C: Mechanics ■ Unit 7

AP Physics C: Mechanics



What we will cover

- 1 Defining SHM as a differential equation
- 2 Frequency and period
- 3 Representing and analyzing SHM
- 4 Energy of an oscillator
- 5 Simple and physical pendulums





SHM is a differential equation

A **restoring force** $F = -kx$ put into $\vec{F} = m\vec{a}$ gives the defining law:

The SHM equation

$$\frac{d^2x}{dt^2} = -\omega^2 x, \quad \omega = \sqrt{\frac{k}{m}}$$

Any system whose acceleration is $-\omega^2$ times its displacement oscillates in SHM.



Foucault pendulum

What makes a motion simple harmonic

periodic motion

anything that repeats; SHM is one special case of it

condition 1

there is an **equilibrium** position the system returns to

condition 2

the restoring force is **proportional to the displacement** from it:
 $F = -kx$

the consequence

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$ – and every solution of that is a sinusoid

the payoff

the period is **independent of amplitude**: $T = 2\pi\sqrt{m/k}$

Whenever you can show a restoring force is proportional to displacement, you have proved the motion is SHM and may quote T immediately – that is the marked step.



Frequency and period

Angular frequency

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}$$

The period does **not** depend on amplitude – a bigger swing just moves faster.

0.5 kg on $k = 50 \text{ N/m}$:

$$T = 2\pi\sqrt{\frac{0.5}{50}} \approx 0.63 \text{ s}$$

Pull it 2 or 10 cm – same period. Heavier mass or softer spring lengthens T . Each system also **ignores** one obvious variable: a pendulum's period does not depend on its mass, and a spring's does not depend on g .



Representing and analyzing SHM

The solution is a cosine; differentiate for v and a :

Position, velocity, acceleration

$$x = A \cos(\omega t + \phi)$$

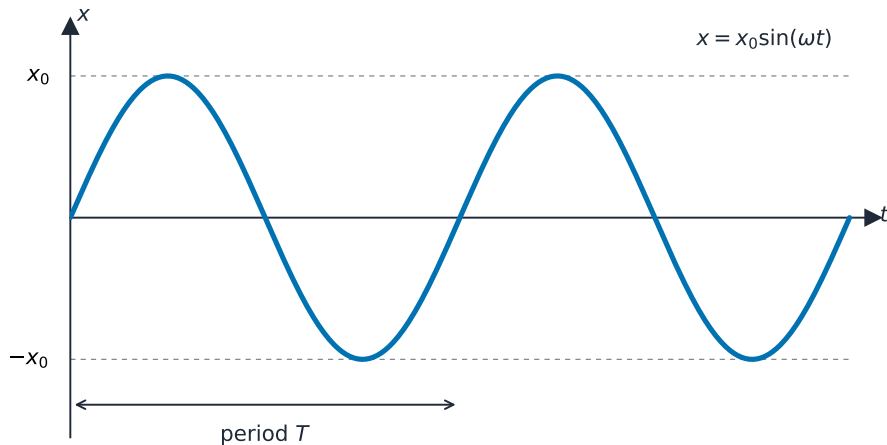
$$v = -A\omega \sin(\omega t + \phi)$$

$$a = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 x$$

$$v_{\max} = A\omega \text{ (at the middle);}$$
$$a_{\max} = A\omega^2 \text{ (at the extremes).}$$

$A = 0.1$, $\omega = 5$: $v_{\max} = 0.5 \text{ m/s}$,
 $a_{\max} = 2.5 \text{ m/s}^2$. Velocity peaks a
quarter-cycle before position.

Displacement varies sinusoidally with time in simple



Displacement varies sinusoidally with time in simple harmonic motion

Where the speed is greatest, and where the acceleration is

at the ends $x = \pm A$ speed is **zero**, acceleration is **maximum** ($a_{\max} = \omega^2 A$) and points back to centre

at $x = 0$ acceleration is **zero**, speed is **maximum** ($v_{\max} = \omega A$)

the phase x , v and a are each a quarter cycle apart – v leads x by 90° , a is 180° out of phase with x

the energy $E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$ – constant, traded back and forth twice per cycle

The two extremes are never at the same place. If your answer has maximum speed *and* maximum acceleration at one point, a sign has gone astray.



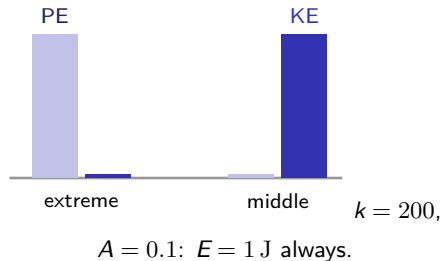
Energy of an oscillator

Energy sloshes between two forms, but the total is constant:

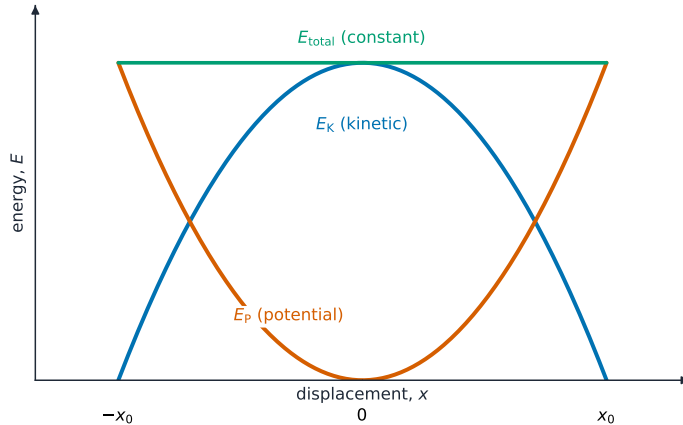
Set by the amplitude

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

All **potential** at the extremes; all **kinetic** in the middle – solve for the speed at any x .



Energy of Simple Harmonic Oscillators



Kinetic and potential energy swap over a cycle while the total energy stays constant

Worked example – where is the energy split evenly?

At what displacement is the potential energy exactly half the total?

- $\frac{1}{2}kx^2 = \frac{1}{2}E = \frac{1}{4}kA^2$
- $x^2 = \frac{1}{2}A^2$
- $x = A/\sqrt{2} \approx 0.71A$

Much closer to the **end** than to the middle – because U grows as x^2 , so the outer part of the swing holds most of the potential energy.



Simple and physical pendulums

Simple pendulum (small angle)

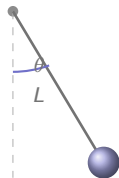
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Depends on L and g only – not mass or amplitude.

Physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

d = pivot to centre of mass.



1 m pendulum: $T \approx 2.0$ s.

The pendulum, and why small angles matter

small-angle approximation

$\sin \theta \approx \theta$ in radians – accurate to 1% below about 14°

the equation

$\tau = I\alpha$ gives $-mgL \sin \theta = I\ddot{\theta}$, which is SHM **only** after that approximation

simple pendulum

$T = 2\pi\sqrt{L/g}$ – mass cancels, amplitude does not appear

physical pendulum

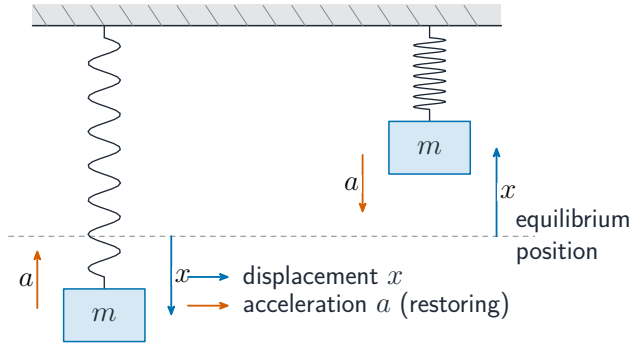
$T = 2\pi\sqrt{\frac{I}{mgd}}$, with d from pivot to centre of mass

natural frequency

$f_0 = \frac{1}{2\pi}\sqrt{k/m}$; driving at f_0 gives **resonance** and a growing amplitude

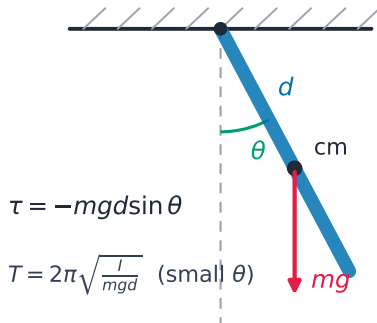
At large angles a pendulum is **not** simple harmonic and its period grows with amplitude. Every quoted pendulum formula silently assumes the approximation.

Defining Simple Harmonic Motion



Simple harmonic motion needs exactly two things: an **equilibrium** point, and a restoring force **proportional to the displacement** and directed back towards it. Recognise the equation $\ddot{x} = -\omega^2 x$ and you know the motion is sinusoidal – you are not asked to solve it.

Simple and Physical Pendulums, continued



$$\text{physical pendulum: } T = 2\pi \sqrt{I/mgd}$$

Gravity acting at the center of mass provides a physical pendulum's restoring torque

The phase constant, and one more oscillator

Phase constant

In $x = A \cos(\omega t + \phi)$ the **phase constant** ϕ sets where the motion starts: $\phi = 0$ from the amplitude, $\phi = -\pi/2$ from the equilibrium moving positive.

Reading it off

Two initial conditions – $x(0)$ and $v(0)$ – fix A and ϕ together.

Torsion pendulum

A **torsion pendulum** twists a wire instead of stretching a spring: $\tau = -\kappa\theta$, so $\omega = \sqrt{\kappa/I}$.

The pattern

Every simple harmonic oscillator has the same form: a restoring quantity over an inertial one, square-rooted.

Identify the restoring constant and the inertia, and ω follows – k/m , κ/I , mgd/I for a physical pendulum.

Exam tips

- Identify SHM from a **linear restoring force** $F = -kx$, which gives $\frac{d^2x}{dt^2} = -\omega^2x$ with $\omega = \sqrt{k/m}$.
- Write the solution $x = A \cos(\omega t + \phi)$ and get v, a by differentiating; period $T = \frac{2\pi}{\omega}$ is amplitude-independent.
- Energy trades between $\frac{1}{2}kx^2$ and $\frac{1}{2}mv^2$, with total $\frac{1}{2}kA^2$.
- For a pendulum, use the small-angle approximation $\sin \theta \approx \theta$ to reach SHM.
- Match phase to the start: released from rest at maximum displacement uses cosine.



Unit 7 – key takeaways

1

SHM

$$d^2x/dt^2 = -\omega^2x \text{ with } \omega = \sqrt{k/m}$$

2

Motion

$$x = A \cos(\omega t + \phi); v_{\max} = A\omega, \\ a_{\max} = A\omega^2$$

3

Energy

$$E = \frac{1}{2}kA^2; \text{ trades between PE} \\ \text{and KE}$$

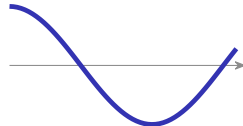
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Pendulums

$$2\pi\sqrt{L/g} \text{ (simple), } 2\pi\sqrt{I/mgd} \\ \text{(physical)}$$

Check yourself

- 1 What differential equation defines simple harmonic motion?
- 2 Does a mass-spring period depend on the amplitude?
- 3 Where in the swing is the speed greatest?
- 4 Does a simple pendulum's period depend on the bob's mass?



Answers 1. $d^2x/dt^2 = -\omega^2x$ 2. no 3. in the middle 4. no