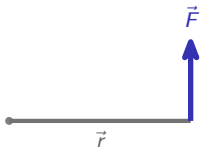


Torque & Rotational Dynamics

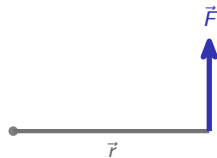
AP Physics C: Mechanics ■ Unit 5

AP Physics C: Mechanics



What we will cover

- 1 Rotational kinematics
- 2 Linking linear and rotational motion
- 3 Torque as a cross product
- 4 Rotational inertia
- 5 Rotational equilibrium
- 6 Newton's second law for rotation





Rotational kinematics

Angular motion mirrors the linear case – through calculus:

Derivatives

$$\omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$

Constant- α equations have the same form as SUVAT; for a **variable** α , integrate: $\omega = \int \alpha dt$.

Worked example ($\alpha = 4t$)

$$\omega = \int_0^t 4t' dt' = 2t^2$$

$$\theta = \int_0^t 2t'^2 dt' = \frac{2}{3}t^3$$

Only calculus handles a non-constant α .

Worked example – a fan blade slowing to rest

A fan blade slows from $\omega_0 = 30 \text{ rad/s}$ to rest with $\alpha = -6.0 \text{ rad/s}^2$. How long, and through what angle?

- $\omega = \omega_0 + \alpha t \Rightarrow 0 = 30 - 6.0t$, so $t = 5.0 \text{ s}$
- $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta \Rightarrow \Delta\theta = \frac{30^2}{2(6.0)}$
- = **75 rad**, about 12 revolutions

Identical in form to a car braking from 30 m/s at 6.0 m/s^2 . Every rotational kinematics problem is a linear one you have already solved, with the symbols swapped.

The rotational quantities, and their radian rule

angular displacement θ in **radians** – AP works in radians throughout, so degrees must be converted first

angular velocity ω $\omega = d\theta/dt$, in rad/s

angular acceleration α $\alpha = d\omega/dt$, in rad/s²

linking to linear $v = \omega r$, $a_t = \alpha r$ (**tangential**), $a_c = \omega^2 r$ (**centripetal**)

the equations the SUVAT set carries over symbol for symbol, for constant α only

$s = r\theta$ is true **only** in radians. Every one of those linking formulas fails silently if a degree slips in.

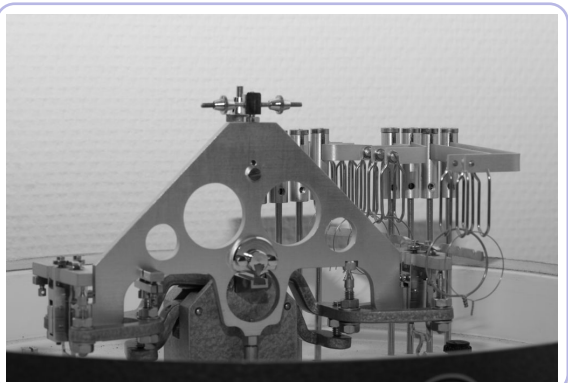
Linking linear and rotational motion

Every point on a **rigid body** moves along a curve, tied to the radius r :

Same ω , different v

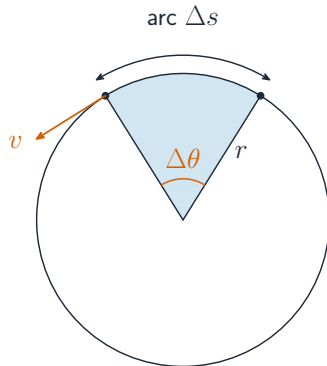
$$v = \omega r, \quad a_t = \alpha r$$

All points share one ω ; points farther out move **faster**.



Balance scale

As the radius turns through an angle



As the radius turns through an angle, a point moves along an arc at speed v

Two accelerations of a point on a wheel

Centripetal acceleration

Centripetal acceleration $a_c = \omega^2 r$ points to the axis. It is present whenever the point is going round, spinning up or not.

Tangential acceleration

Tangential acceleration $a_t = \alpha r$ points along the motion. It is zero at constant ω .

Together

They are perpendicular, so the total is $\sqrt{a_c^2 + a_t^2}$ – and the direction turns towards the axis as ω grows.

“Uniform” rotation means $a_t = 0$, never $a = 0$. The point is still accelerating, towards the centre.



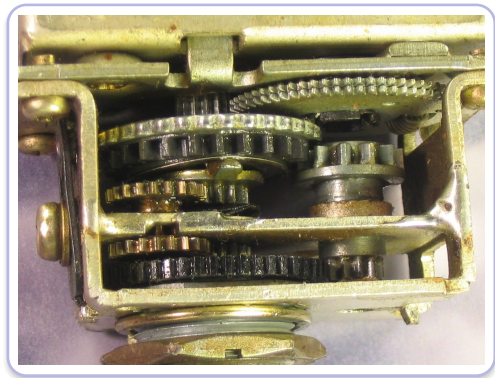
Torque

Torque is the rotational effect of a force:

A cross product

$$\tau = rF \sin \theta, \quad \vec{\tau} = \vec{r} \times \vec{F}$$

The **lever arm** is the perpendicular distance to the force – a force farther out and $\perp$ to $\vec{r}$ twists most.



Gear train

Torque is a vector product

the definition

$$\vec{\tau} = \vec{r} \times \vec{F}, \text{ magnitude } rF \sin \theta$$

the lever arm

only the component of $\vec{F}$ **perpendicular** to $\vec{r}$ turns anything – a force pointing at the pivot has zero torque

direction

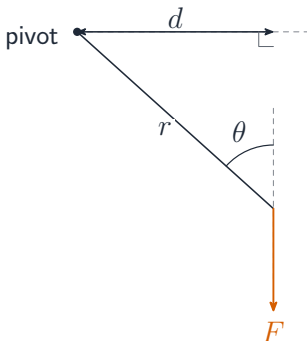
perpendicular to the plane of $\vec{r}$ and $\vec{F}$, fixed by the **right-hand rule**

why a vector product

it captures both the size *and* the sense of the turn in one object

Push a door at the hinge and nothing happens; push at the handle, perpendicular, and it swings. $\sin \theta$ and r are that sentence in symbols.

The torque of a force depends on



The torque of a force depends on the perpendicular distance from the axis



Rotational inertia

Rotational inertia resists angular acceleration, and depends on **how mass is spread**:

Sum, or integrate

$$I = \sum m_i r_i^2 = \int r^2 dm$$

Parallel-axis theorem : $I = I_{cm} + Md^2$.

axis



$$I = \frac{1}{3} ML^2$$

axis



$$I = \frac{1}{12} ML^2$$

A rod is harder to spin about its end.

Building a rotational inertia by integration

the definition

$I = \int r^2 dm$ – r is the distance from the **axis**, not from the centre

the mass element

use the right density: $dm = \lambda dl$ for a rod (**linear density**), σdA for a plate, ρdV for a solid

coaxial shells

build a solid cylinder from thin shells of radius r , thickness dr : $dm = \rho(2\pi rL) dr$

the same method

handles cylindrical shells and annular rings – change only the limits

parallel axis

$I = I_{\text{cm}} + Md^2$ for an axis a distance d off the centre of mass

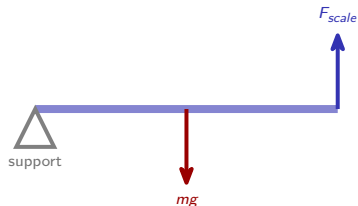
I depends on **where the axis is**, not just on the object. The same rod has $\frac{1}{12}ML^2$ about its centre and $\frac{1}{3}ML^2$ about its end.



Rotational equilibrium

In **rotational equilibrium** the **net torque** is zero. Full **static equilibrium** : both net force and net torque zero.

- draw an **extended free-body diagram** (where each force acts)
- put the axis at an unknown force to drop it out



$$mg \times 2 = F_{scale} \times 4 \Rightarrow F_{scale} = 49 \text{ N}.$$

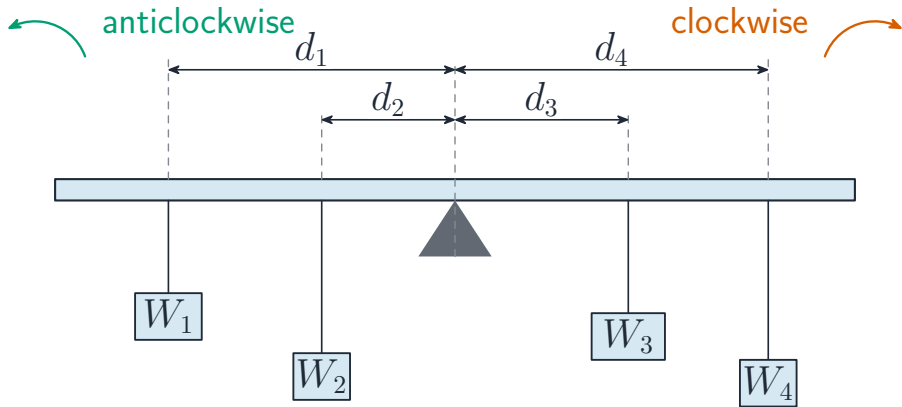
Worked example – a beam on a pivot

A uniform 20 kg beam, 4.0 m long, is pivoted at its left end and held horizontal by a vertical rope at its right end. Find the rope tension.

- take torques about the **pivot**, so its unknown force drops out
- weight acts at the centre:
 $\tau_w = (20)(9.8)(2.0) = 392 \text{ N m}$
- rope: $T(4.0) = 392$, so $T = 98 \text{ N}$

Choosing the pivot as the axis is the whole technique: any unknown force *through* your axis contributes no torque, so it never enters the equation.

At balance the clockwise and anticlockwise torques



At balance the clockwise and anticlockwise torques about the axis are equal



Newton's second law for rotation

A nonzero net torque gives an angular acceleration – exactly like $F = ma$:

Rotational form

$$\tau_{net} = I\alpha$$

torque $\leftrightarrow$ force, $I \leftrightarrow$ mass, $\alpha \leftrightarrow$ acceleration.

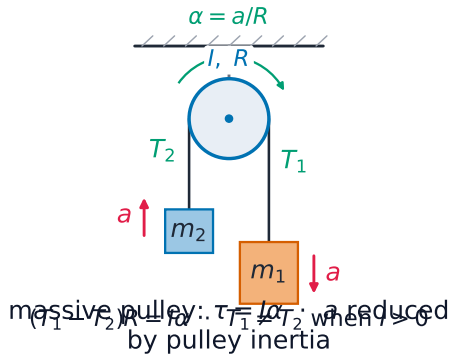
Worked example

$\tau = 6 \text{ N m}$ on $I = 2 \text{ kg m}^2$:

$$\alpha = \frac{6}{2} = 3 \text{ rad/s}^2$$

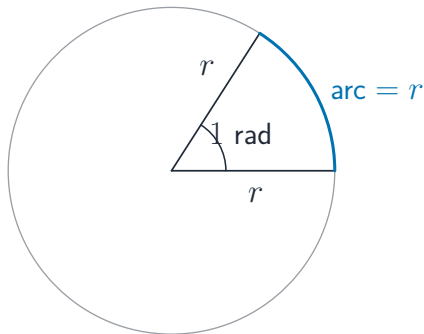
Double $I \Rightarrow$ half the α . For a pulley, pair it with $F = ma$ (linked by $a = \alpha r$).

A massive pulley



A massive pulley: the two tensions differ because the pulley needs net torque

Rotational Kinematics, continued



one **radian**: the angle whose arc is exactly one radius long

$\pi \text{ rad} = 180^\circ$, so $1 \text{ rad} \approx 57.3^\circ$

degrees $\rightarrow$ radians: $\times \frac{\pi}{180}$

radians $\rightarrow$ degrees: $\times \frac{180}{\pi}$

One radian is the angle whose arc length equals the radius

Rotational FRQs score in the setup

1. separate diagrams

one free-body diagram for **each** mass *and* one for the pulley – the pulley's is the one candidates omit

2. state the constraint

$a = R\alpha$, linking the string's linear acceleration to the pulley's rotation

3. state the sign convention

which way is positive for rotation, and keep it for every equation

4. then solve

e.g. with $I/R^2 = 12.5 \text{ kg}$: $a = \frac{(m_1 - m_2)g}{m_1 + m_2 + I/R^2} = \frac{19.6}{18.5} = 1.1 \text{ m/s}^2$,
giving $T_1 = m_1(g - a) = 35 \text{ N}$

A massive pulley makes the two tensions **different** – that difference is exactly what supplies the net torque that spins it up.

Exam tips

- Use the rotational analogues: $\theta, \omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt}$, with the constant- α equations mirroring linear kinematics.
- Convert between linear and angular with $v = r\omega$ and $a_t = r\alpha$ (plus centripetal $a_c = \frac{v^2}{r} = \omega^2 r$).
- Keep radians throughout and fix a positive sense of rotation.
- Relate torque to angular acceleration through $\tau = I\alpha$.
- Draw the rotation axis – the moment arm is the perpendicular distance to it.



Unit 5 – key takeaways

1

Kinematics

$\omega = d\theta/dt$, $\alpha = d\omega/dt$; integrate for variable α

2

Torque

$\vec{\tau} = \vec{r} \times \vec{F}$; the lever arm sets the twist

3

Inertia

$I = \int r^2 dm$; mass far from the axis counts most

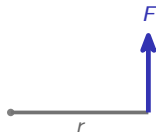
4

Dynamics

equilibrium: net $\tau = 0$; otherwise $\tau = I\alpha$

Check yourself

- 1 What integral gives the rotational inertia of a continuous body?
- 2 Is a rod harder to spin about its end or its center?
- 3 Torque is the cross product of which two vectors?
- 4 $\tau = 10 \text{ N m}$ on $I = 5 \text{ kg m}^2$ – find α .



Answers 1. $\int r^2 dm$ 2. its end 3. $\vec{r}$ and $\vec{F}$ 4. 2 rad/s^2