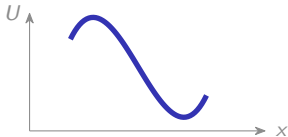


Work, Energy, and Power

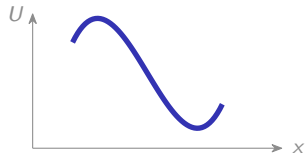
AP Physics C: Mechanics ■ Unit 3

AP Physics C: Mechanics



What we will cover

- 1 Kinetic energy and the work-energy theorem
- 2 Work as an integral
- 3 Potential energy and force
- 4 Conservation of energy
- 5 Power





Kinetic energy and the work-energy theorem

Energy of motion

$$K = \frac{1}{2}mv^2$$

A **scalar** ; $\propto v^2$, so double the speed quadruples K .

Work-energy theorem

$$W_{\text{net}} = \Delta K$$



Coaster energy

Kinetic energy, and the theorem that uses it

the definition

$$K = \frac{1}{2}mv^2, \text{ measured in joules}$$

the square

doubling the speed **quadruples** K – which is why stopping distance grows so fast with speed

work–energy theorem

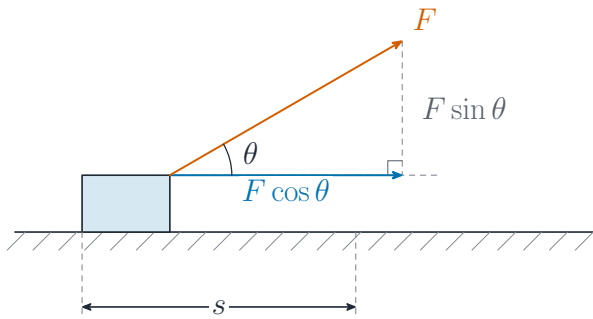
$W_{\text{net}} = \Delta K$: the net work on an object equals its change in kinetic energy

reference frame

K depends on the observer – a passenger has zero K in the train's frame and a great deal in the platform's

Kinetic energy is a **scalar**: no direction, and never negative. Work can be negative, which is how a force removes energy.

Only the force component along the displacement



Only the component **along** the displacement does work. So a force at right angles does **zero** – the **normal force** on a sliding block, gravity on a horizontal move, the tension in a circular swing – and a force opposing the motion does **negative** work.



Work as an integral

Constant vs variable force

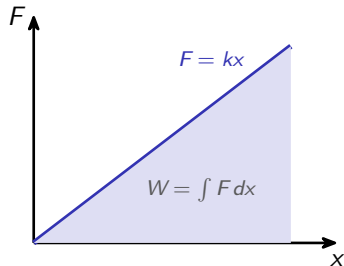
$$W = \vec{F} \cdot \vec{d} = Fd \cos \theta$$

$$W = \int \vec{F} \cdot d\vec{r}$$

The integral is the **area** under an F - x graph.

Stretching a spring: $W =$

$$\int_0^x kx' dx' = \frac{1}{2} kx^2.$$





Potential energy and force

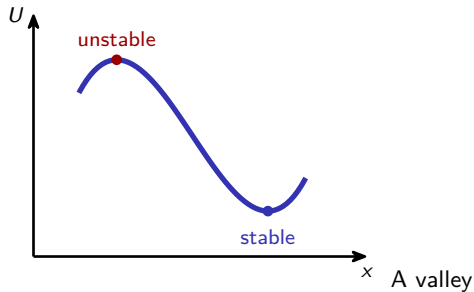
A **conservative force** stores **potential energy** :

Two stored forms, and the link

$$U_g = mgy, \quad U_s = \frac{1}{2}kx^2$$

$$F_x = -\frac{dU}{dx}$$

For a spring $-\frac{d}{dx}(\frac{1}{2}kx^2) = -kx$ – Hooke's law drops out.



is stable; a hilltop unstable.

The three potential energies, and where they come from

near a surface

$\Delta U_g = mg \Delta y$ – valid only where g is effectively constant

far from a planet

$U_g = -\frac{GMm}{r}$, taking zero at infinity

a spring

$U_s = \frac{1}{2}k(\Delta x)^2$, with Δx from **natural length**, not from the floor

in general

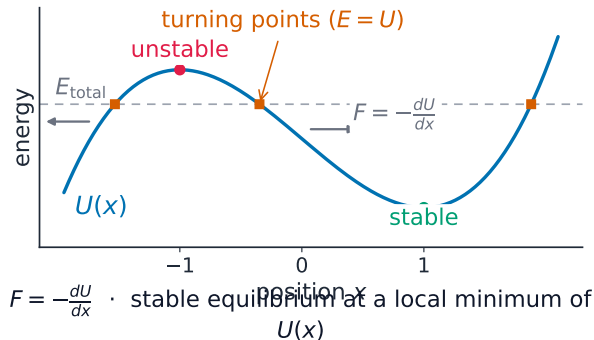
a conservative force is minus the **derivative** of its potential: $F_x = -\frac{dU}{dx}$

several objects

add the potential energy of each **pair**, once each

$F = -dU/dx$ means the force points **downhill** on a U - x graph, and a minimum of U is a point of stable equilibrium.

Potential Energy



The force is $-dU/dx$: it points **downhill** on the curve, and $E = U$ marks the **turning points**. You may put $U = 0$ wherever you like – only *changes* in U matter – so choose the zero that makes the algebra shortest.



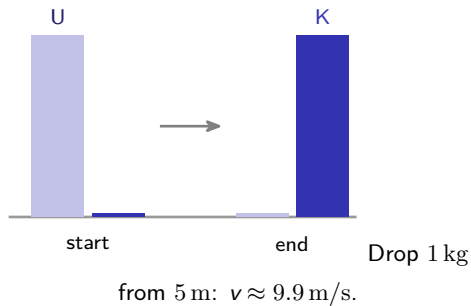
Conservation of energy

With only conservative forces, **mechanical energy is conserved** :

Before = after

$$K_i + U_i = K_f + U_f$$

A **non-conservative force** like friction removes mechanical energy as heat.



Energy is always conserved – the question is where it went

mechanical energy

$E = K + U$; conserved when only conservative forces do work

external work

$W_{\text{ext}} = \Delta E_{\text{mech}}$ – work done *on* the system from outside changes it

nonconservative forces

friction and drag convert mechanical energy into **thermal energy**: $\Delta E_{\text{mech}} = -F_f d$

the balance

$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{thermal}}$ – nothing is ever lost, only relabelled

“Energy is lost to friction” scores nothing on its own. Say **how much**, as $F_f d$, and where it went.



Power

Power is the **rate** of doing work:

Two forms

$$P = \frac{dW}{dt}, \quad P = \vec{F} \cdot \vec{v}$$

Instantaneous power varies as force or speed changes.

Worked example

Lift 50 kg at a steady 2 m/s:

$$P = Fv = (490)(2) = 980 \text{ W}$$

Twice as fast $\Rightarrow$ 1960 W – same work per metre, half the time.

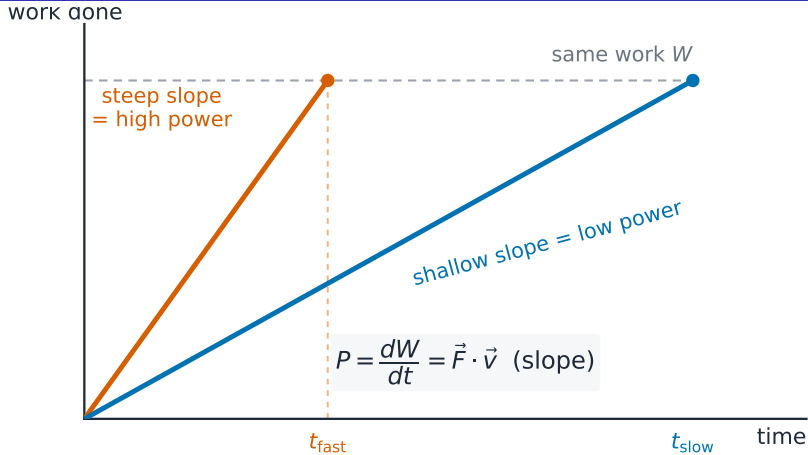
Worked example – the car climbing a hill

A 1200 kg car climbs a road that rises 1.0 m per 20 m travelled, at a steady 15 m/s. Find the power against gravity.

- the climb rate: $15 \times \frac{1.0}{20} = 0.75 \text{ m/s}$ vertically
- $P = mgv_{\text{vertical}} = (1200)(9.8)(0.75)$
- = **8.8 kW**, about 12 horsepower

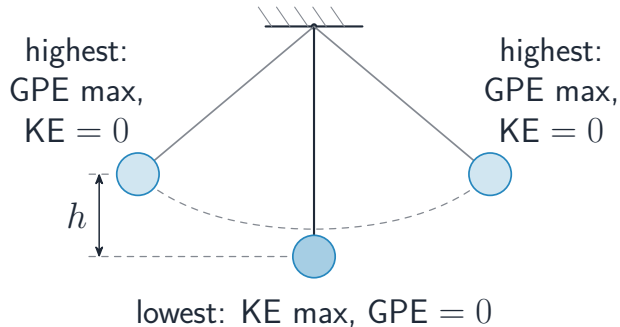
Power is measured in **watts**, $P = dW/dt = \vec{F} \cdot \vec{v}$. Also worth knowing: a block from rest down a frictionless 3.0 m ramp arrives at $v = \sqrt{2gh} = 7.7 \text{ m/s}$, **whatever the ramp's shape**.

Power is the slope of the work-time graph



Power is the slope of the work-time graph: the same work in less time means more power

Conservation of Energy, continued



A pendulum trades gravitational potential energy for kinetic energy and back

The accounting sentence that earns the marks

1. name the system

say what is inside it – block alone, or block plus Earth, or block plus spring

2. name the forces

which ones do work *on* that system, and which are internal

3. write the balance

$$W_{\text{ext}} = \Delta K + \Delta U + \Delta E_{\text{thermal}}$$

4. then substitute

numbers last, once the accounting is on the page

Gravity is an **internal** force if the Earth is in your system (so it has a U) and an **external** one if it is not (so it does work). Both are correct; mixing them in one line is not.

Exam tips

- Use the **work–energy theorem** $W_{net} = \Delta KE$ and compute work as $W = \int \vec{F} \cdot d\vec{r}$ for a variable force.
- Read work off a **force–position graph** as the area under the curve.
- Power is $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$; watch instantaneous vs average.
- Split forces into conservative (define a potential energy) and non-conservative (dissipate energy).
- Choose energy methods over kinematics when the force varies or the path is complex.



Unit 3 – key takeaways

1

Work

$W = \int \vec{F} \cdot d\vec{r}$; the area under an F - x graph

2

Energy

$K = \frac{1}{2}mv^2$; net work equals ΔK

3

Potential

$F_x = -dU/dx$; valley stable, hill-top unstable

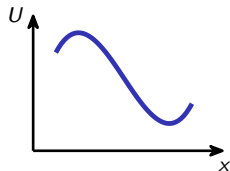
4

Conservation

$K + U$ constant; $P = dW/dt = \vec{F} \cdot \vec{v}$

Check yourself

- 1 What integral gives the work done by a variable force?
- 2 How do you get the force from a potential-energy function?
- 3 Is a potential-energy valley stable or unstable equilibrium?
- 4 Write instantaneous power as a dot product.



Answers 1. $\int \vec{F} \cdot d\vec{r}$ 2. $F_x = -dU/dx$ 3. stable 4. $P = \vec{F} \cdot \vec{v}$