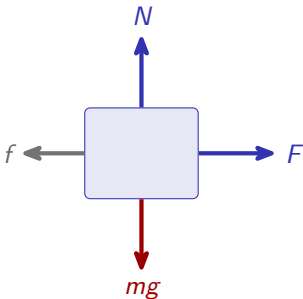


Force & Translational Dynamics

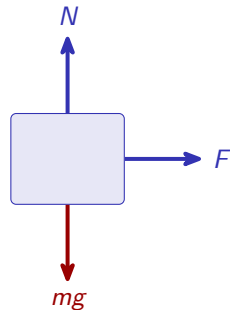
AP Physics C: Mechanics ■ Unit 2

AP Physics C: Mechanics



What we will cover

- 1 Systems and center of mass
- 2 Free-body diagrams and Newton's laws
- 3 Gravitation
- 4 Friction
- 5 Spring forces
- 6 Resistive forces and terminal velocity
- 7 Circular motion





Systems and center of mass

A **system** moves as if all its mass sat at its **center of mass** :

Discrete and continuous

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1}{M} \int x dm$$

Only the **net external force** accelerates the center of mass – internal forces cannot.

Worked example – a uniform rod

With $\lambda = M/L$, $dm = \lambda dx$:

$$x_{cm} = \frac{1}{M} \int_0^L x \lambda dx = \frac{L}{2}$$

The integral confirms it balances at the middle.



Free-body diagrams and Newton's laws

Draw every force on a **free-body diagram** ,
resolve into components, add to the **net force** .

The three laws

1st: inertia $\vec{F}_{net} = m\vec{a}$

3rd: equal-opposite pairs, different bodies



Rocket force

Worked example – two bodies, one string

A 6.0 kg cart on a frictionless table is pulled by a light string over a pulley to a hanging 2.0 kg mass. Find a and the tension.

- treat them as one system:

$$a = \frac{m_2 g}{m_1 + m_2} = \frac{(2.0)(9.8)}{8.0} = 2.45 \text{ m/s}^2$$

- then one body alone for T : cart,

$$T = m_1 a = 14.7 \text{ N}$$

- check with the hanging mass:

$$m_2 g - T = 19.6 - 14.7 = 4.9 = m_2 a \checkmark$$

The string constraint – same $|a|$, same $|T|$ – is what lets you do this. Solve for a with the whole system, then use one body to get an internal force.

Newton's laws, one axis at a time

third law

A pushes B, B pushes back equally and oppositely. The pair acts on **different objects**, so it never cancels inside one free-body diagram

second law

$\sum \vec{F} = \frac{d\vec{p}}{dt} = m\vec{a}$ for constant mass – apply it **one axis at a time**

when F depends on v

$m dv/dt = F(v)$ is a differential equation; separate and integrate

translational equilibrium

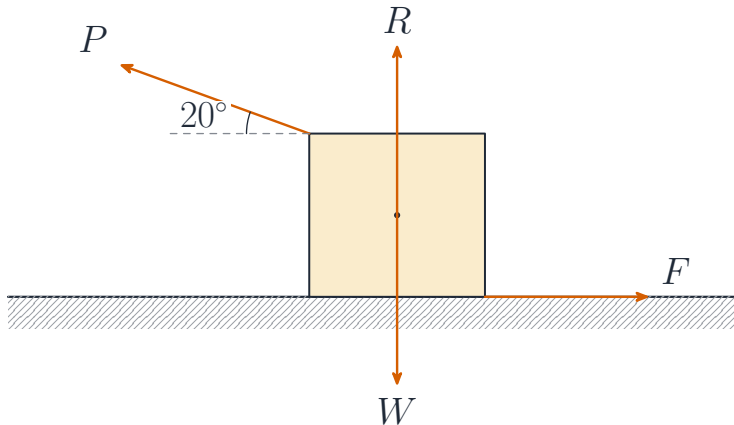
$\sum \vec{F} = 0$, so $\vec{a} = 0$ – at rest *or* at constant velocity

normal force

perpendicular to the surface; it is whatever it needs to be, and is not always mg

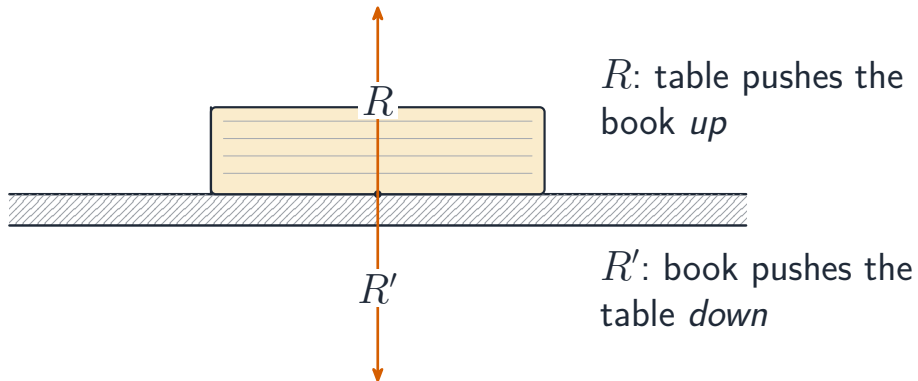
An action–reaction pair on one diagram is the commonest error in the unit. If both arrows are on the same body, one of them does not belong there.

Forces and Free-Body Diagrams



A free-body diagram shows every force acting on one object

A Newton's third-law pair



A Newton's third-law pair: equal and opposite forces on two different objects



Gravitation

Universal gravitation

$$F_g = \frac{Gm_1m_2}{r^2}, \quad g = \frac{GM}{r^2}$$

- **mass** is intrinsic; **weight** = mg
- a sphere pulls as if all mass sat at its centre
- g falls off as $1/r^2$ with **altitude**

Worked example

At one Earth radius up, r doubles:

$$g = \frac{9.8}{2^2} = 2.45 \text{ m/s}^2$$

The gentler pull is what lets satellites orbit high up.



Friction and spring forces

Friction

$$f_s \leq \mu_s N, \quad f_k = \mu_k N$$

Depends on N and μ – not on the contact area.

Hooke's law

$$F_s = -kx$$

Combining springs: **parallel** adds ($k_{eq} = k_1 + k_2$, stiffer); **series** is softer ($\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$).

Two 100 N/m springs: parallel 200, series 50 N/m.

Worked example – a block sliding down a rough incline

A block slides down a 30° incline with $\mu_k = 0.20$. Find its acceleration.

- perpendicular: $N = mg \cos 30^\circ$
- along: $ma = mg \sin 30^\circ - \mu_k mg \cos 30^\circ$
- $a = g(\sin 30^\circ - 0.20 \cos 30^\circ) = 3.2 \text{ m/s}^2$

The mass cancels – the answer is the same for a marble and a boulder. Note $N \neq mg$ on an incline; using mg there is the standard lost mark.

Static and kinetic friction are not the same law

kinetic

$f_k = \mu_k N$ – a **fixed** value while sliding, opposing the slide

static

$f_s \leq \mu_s N$ – an **inequality**: it takes whatever value stops the slide, up to that maximum

usually

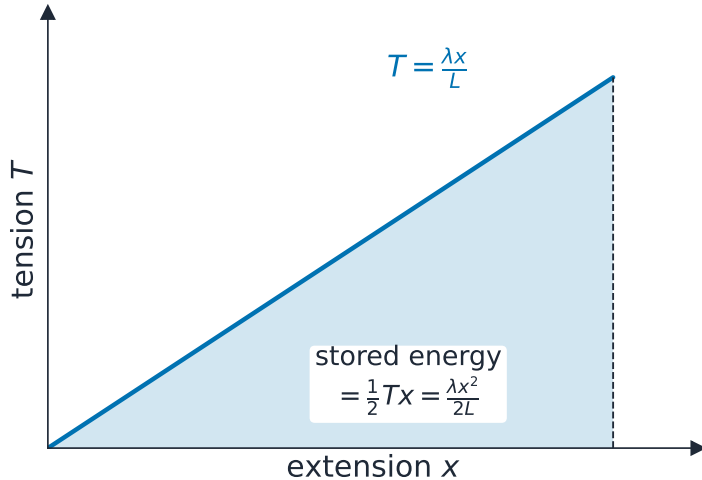
$\mu_s > \mu_k$, which is why a block is harder to start than to keep going

on an incline

resolve along and across the ramp: $N = mg \cos \theta$, and the driving component is $mg \sin \theta$

Writing $f_s = \mu_s N$ when nothing is on the point of slipping is wrong: static friction is only at its maximum at the instant of breaking away.

Hooke's law



Hooke's law: extension is proportional to load up to the limit of proportionality

Combining springs, and the sign that trips everyone

in parallel

side by side, sharing the load: $k_{\text{eq}} = k_1 + k_2$ – **stiffer** than either

in series

end to end, sharing the stretch: $\frac{1}{k_{\text{eq}}} = \frac{1}{k_1} + \frac{1}{k_2}$ – **softer** than either

the spring constant

$F = -kx$; the minus sign says the force is a **restoring** one, always back toward natural length

the memory hook

exactly the *opposite* of resistors – springs in parallel add, resistors in series add

Two identical springs side by side hold twice the load at the same stretch; end to end they stretch twice as far under the same load.



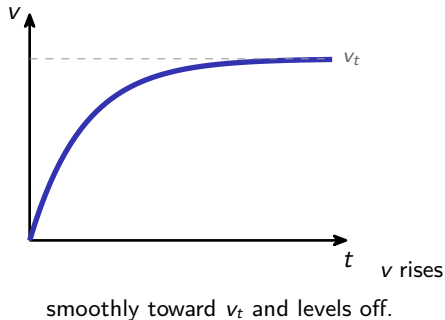
Resistive forces and terminal velocity

Drag grows with speed ($F = -bv$), so Newton's law becomes a **differential equation** :

Falling with drag

$$m \frac{dv}{dt} = mg - bv$$

At **terminal velocity** , $\frac{dv}{dt} = 0$, so
$$v_t = \frac{mg}{b}.$$



Separating the variables, and why you must

the problem

with drag, $m \frac{dv}{dt} = mg - bv$ – the acceleration **depends on v** ,
so no SUVAT equation applies

separate the variables

$$\int_0^v \frac{dv'}{mg - bv'} = \int_0^t \frac{dt'}{m}$$

the result

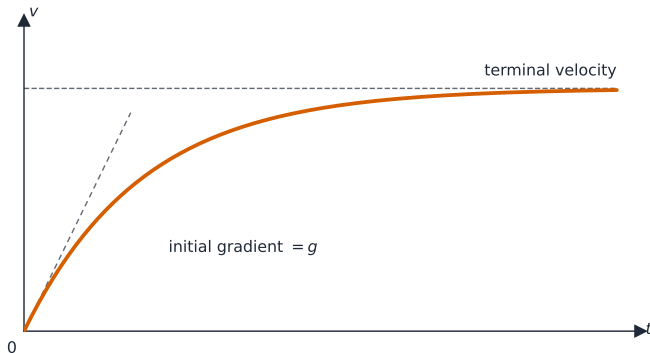
$$v(t) = \frac{mg}{b} (1 - e^{-bt/m})$$

terminal velocity

set $dv/dt = 0$: $v_T = \frac{mg}{b}$ – reachable without solving the
equation at all

If a question asks only for the **terminal** value, set the acceleration to zero and stop. Integrate only when it asks for $v(t)$.

An object falling through a fluid speeds



An object falling through a fluid speeds up to a terminal velocity



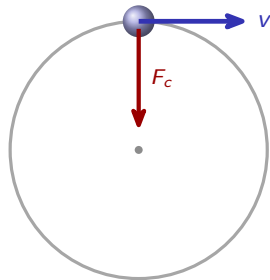
Circular motion

Uniform circular motion : steady speed, changing direction – so it accelerates toward the centre:

Centripetal quantities

$$a_c = \frac{v^2}{r}, \quad F_c = \frac{mv^2}{r}$$

Non-uniform motion also has a **tangential** force that changes the speed.



A car needs F_c from tyre friction – else it skids.

Circular motion needs a net inward force

centripetal acceleration

$$a_c = \frac{v^2}{r} = \omega^2 r, \text{ always pointing to the centre}$$

the force

$$F_c = \frac{mv^2}{r} - \text{not a new force. It is the name for whatever the net inward force already is}$$

identify it

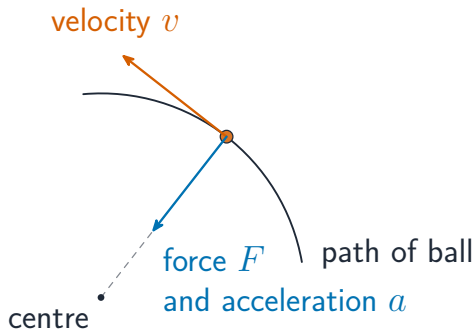
tension, gravity, friction, the normal force – say *which*, every time

Newton's law of gravitation

$$F = \frac{Gm_1 m_2}{r^2} - \text{the inward force for an orbit}$$

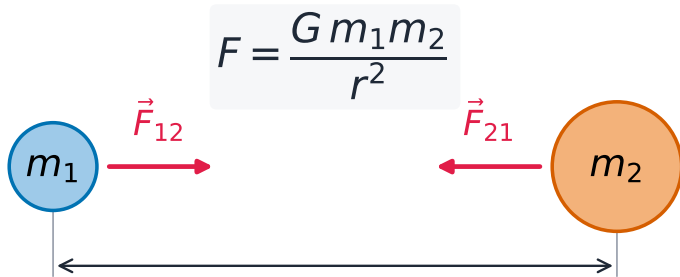
There is no outward force. Writing 'centrifugal force' on a free-body diagram loses the mark; the sensation of being flung out is the body's **inertia**, not a force.

The velocity points along the tangent; the acceleration inward



The velocity points along the tangent; the centripetal force points to the centre

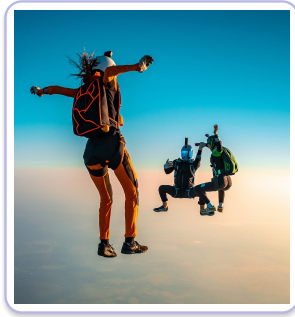
Gravitational Force



$$|\vec{F}_{12}| = |\vec{F}_{21}| \quad (\text{action-reaction; inverse-square})$$

Two masses attract each other with equal, opposite, inverse-square forces along the line joining them

Resistive Forces



Skydivers in freefall: drag grows with speed until it balances weight at terminal velocity

Exam tips

- Locate the **center of mass** with $x_{cm} = \frac{1}{M} \int x dm$ (or $\frac{\sum m_i x_i}{\sum m_i}$ for point masses) and use λ, σ, ρ for the mass element.
- The net external force moves the center of mass as if all mass sat there:
$$\vec{F}_{net} = M\vec{a}_{cm}.$$
- Define your **system** clearly – internal forces cancel, so only external forces change its momentum.
- Exploit symmetry to shortcut a center-of-mass integral.
- Distinguish center of mass from center of gravity (identical in a uniform field).



Unit 2 – key takeaways

1

Center of mass

$x_{cm} = \frac{1}{M} \int x dm$; moved only by external force

2

Newton

$\vec{F}_{net} = m\vec{a}$; third-law pairs act on different bodies

3

Drag

$m dv/dt = mg - bv$; terminal velocity $v_t = mg/b$

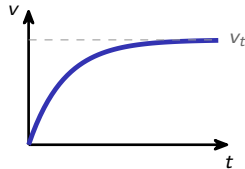
4

Circular

$a_c = v^2/r$ toward the centre; a role, not a new force

Check yourself

- 1 What integral gives the center of mass of a continuous object?
- 2 At terminal velocity, what is the acceleration?
- 3 Two springs in parallel – stiffer or softer than one?
- 4 Rise altitude by one Earth radius – what happens to g ?



Answers 1. $\frac{1}{M} \int x dm$ 2. zero 3. stiffer 4. it drops to one quarter