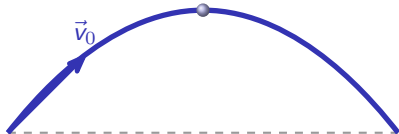


Kinematics

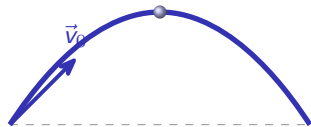
AP Physics C: Mechanics ■ Unit 1

AP Physics C: Mechanics



What we will cover

- 1 Vectors, components, products
- 2 Velocity and acceleration as derivatives
- 3 Motion graphs
- 4 Reference frames and relative motion
- 5 Motion in two dimensions





Vectors and components

Resolve every **vector** onto the axes with **unit vectors** $\hat{i}, \hat{j}, \hat{k}$:

Components

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Add vectors axis by axis to get the **resultant** .



Speedometer

Vectors are added component by component

components

resolve every vector onto the axes first: $v_x = v \cos \theta$, $v_y = v \sin \theta$

adding

add the x's, add the y's – never add magnitudes

magnitude

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

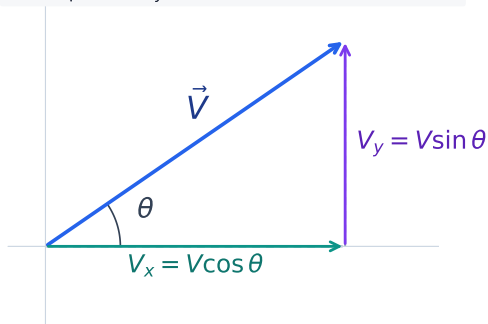
direction

$\theta = \tan^{-1}(v_y/v_x)$ – and check the quadrant against the signs of the components

A **scalar** carries only size (mass, time, energy); a **vector** carries size and direction. Half the sign errors in this course come from treating one as the other.

Resolving a vector into its x and y components

$$|\vec{V}| = \sqrt{V_x^2 + V_y^2} \cdot V_x = V \cos \theta, V_y = V \sin \theta$$



Resolving a vector into its x and y components

Two ways to multiply vectors

Dot product $\rightarrow$ a *scalar*

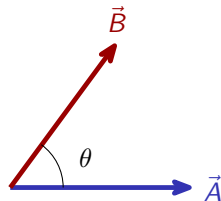
$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Work is defined this way.

Cross product $\rightarrow$ a *vector*

$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta$$

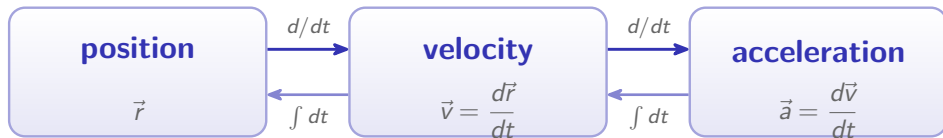
$\perp$ to both (right-hand rule). Torque and angular momentum use this.





Velocity and acceleration are derivatives

Differentiate to move **down** the chain; integrate to move **back up**:



So $\vec{a} = \frac{d^2\vec{r}}{dt^2}$, and running it backwards: $\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a} dt'$ – valid even when a is **not** constant.

In this course, motion is calculus

the derivatives

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

reversed

$$v = \int a \, dt \text{ and } x = \int v \, dt, \text{ each with a constant fixed by the initial condition}$$

average velocity

$\Delta x / \Delta t$ over an interval – a chord on the graph

instantaneous velocity the derivative at one instant – the tangent

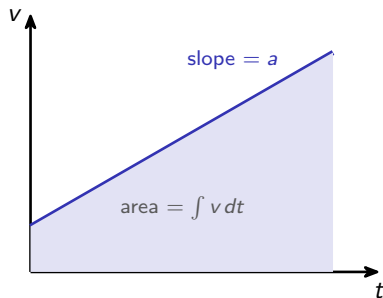
the SUVAT equations

$$v = v_0 + at, \quad x = x_0 + v_0 t + \frac{1}{2}at^2, \quad v^2 = v_0^2 + 2a\Delta x - \text{valid only for constant } a$$

The most important habit in this unit: check whether a is constant before reaching for a SUVAT equation. If it is not, integrate.



Motion graphs: slope and area



$$x-t: \text{slope} = v = \frac{dx}{dt}.$$

$$v-t: \text{slope} = a; \text{ area} = \int v dt = \Delta x.$$

Constant a (a special case)

$$v = v_0 + at, \quad x = x_0 + v_0 t + \frac{1}{2}at^2$$

Worked example – differentiate, then integrate

(a) $x(t) = 2t^3 - 3t^2$ m: find v and a . **(b)**
From rest at the origin with $a(t) = 6t$: find v and x at $t = 2$ s.

■ (a) $v = \frac{dx}{dt} = 6t^2 - 6t, \quad a = \frac{dv}{dt} = 12t - 6$

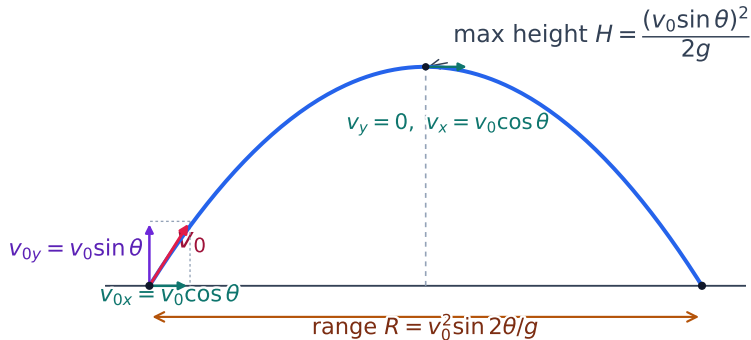
■ (b) $v = \int 6t \, dt = 3t^2,$

$$x = \int 3t^2 \, dt = t^3$$

■ at $t = 2$: $v = 12$ m/s, $x = 8$ m

Slopes are derivatives; areas are integrals – the two are inverse operations. Both constants of integration are zero here only because the particle starts from rest at the origin.

Motion in Two or Three Dimensions – one



projectile: v_x constant $\cdot a_y = -g$ $\cdot x$ and y linked only by t

A projectile launched at an angle: the velocity splits into a constant horizontal part and a vertical part that gravity changes – the two motions share only the time.

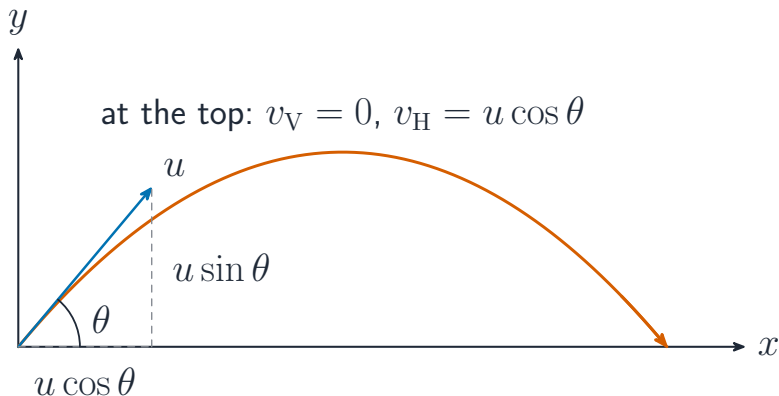
Worked example – motion in two dimensions

$\vec{r}(t) = (2t^2)\hat{i} + (4t - t^3)\hat{j}$ metres. Find $\vec{v}$ and $\vec{a}$.

- differentiate each component separately:
- $\vec{v} = 4t\hat{i} + (4 - 3t^2)\hat{j}$
- $\vec{a} = 4\hat{i} - 6t\hat{j}$

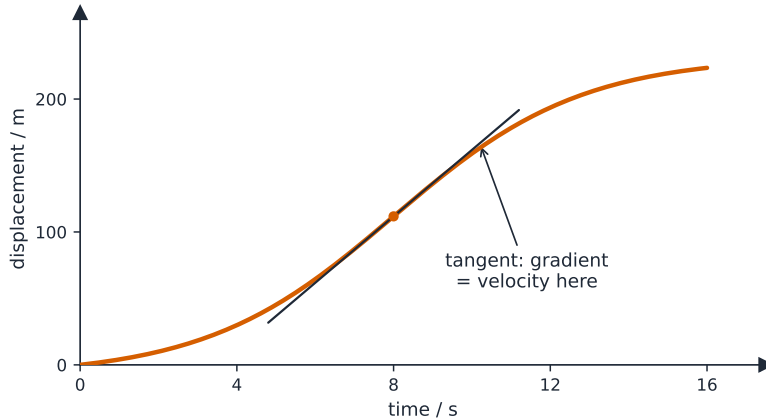
Components are **independent**: the $\hat{i}$ motion never enters the $\hat{j}$ derivative. That independence is the whole reason projectile motion splits into two one-dimensional problems.

Motion in Two or Three Dimensions – two



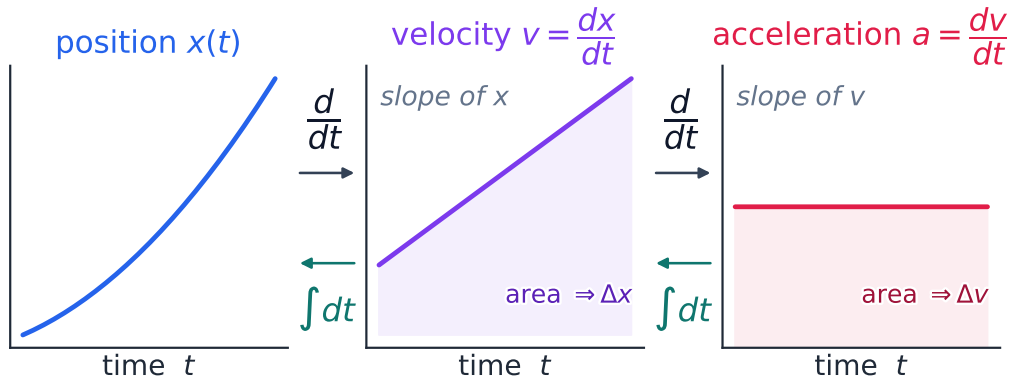
A projectile launched at an angle: the horizontal and vertical motions are independent

Representing Motion – one



A displacement-time graph: the slope at any instant is the velocity

Representing Motion – two



AP Physics C Mech · U1 · differentiate to the right, integrate to the left

Position, velocity, and acceleration are linked by differentiation



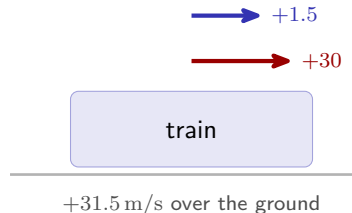
Reference frames and relative motion

Velocities add as vectors between frames:

Frame conversion

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$$

Frames moving at **constant** relative velocity share the same acceleration – they are **inertial**, and Newton's laws hold identically.



Relative motion is vector addition

the rule

$$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C} - \text{the inner subscripts cancel}$$

reversing

$$\vec{v}_{B/A} = -\vec{v}_{A/B}$$

a boat crossing

add the boat's velocity through the water to the water's velocity over the ground

reference frame

all motion is measured relative to one; there is no absolute rest

Write the subscripts before the numbers. If the inner pair does not cancel, you have chained the frames the wrong way round.

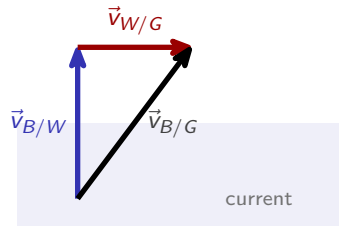
A boat crossing a river

The boat's ground velocity is the **vector sum** of its velocity through the water and the current:

$$\vec{v}_{B/G} = \vec{v}_{B/W} + \vec{v}_{W/G}$$

Across 3.0, downstream 2.0

$$|\vec{v}_{B/G}| = \sqrt{3.0^2 + 2.0^2} \approx 3.6 \text{ m/s}$$





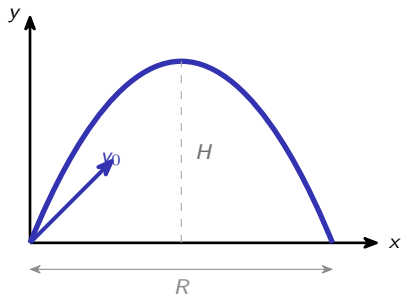
Projectile motion

The axes are **independent**, joined only by time t :

- horizontal: $a_x = 0$, so v_x constant
- vertical: $a_y = -g$ (free fall)

Eliminating t makes y a quadratic in x – the **trajectory** is a **parabola**.

A dropped and a launched ball land **together**.



Projectile formulas

Fired from the ground at speed v_0 , angle θ :

Three key results

$$t_{\text{flight}} = \frac{2v_0 \sin \theta}{g}$$

$$H = \frac{v_0^2 \sin^2 \theta}{2g}, \quad R = \frac{v_0^2 \sin 2\theta}{g}$$

Range is greatest at $\theta = 45^\circ$; angles like 30° and 60° land at the **same** distance.

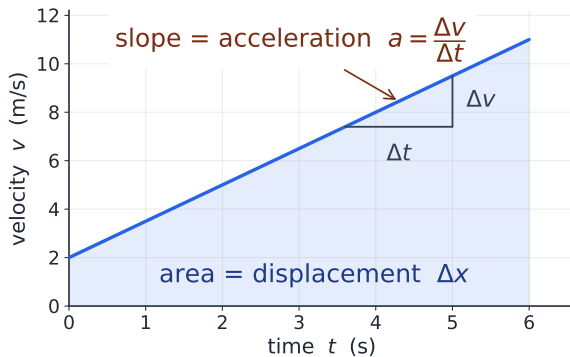
Uniform circular motion : $a_c = \frac{v^2}{r}$ toward the centre – a preview of dynamics.

Displacement, Velocity, and Acceleration



A high-speed train: kinematics describes displacement, velocity and acceleration over time

On a velocity-time graph the gradient is



On a velocity-time graph the gradient is the acceleration and the area is the displacement

Exam tips

- Resolve every vector into components before adding – never add magnitudes at an angle directly.
- On Physics C you are expected to use **calculus**: velocity is $\vec{v} = \frac{d\vec{r}}{dt}$ and acceleration $\vec{a} = \frac{d\vec{v}}{dt}$; reverse with integration.
- Carry and check **units** and treat direction with signs (choose a positive axis and stick to it).
- Use the dot product for work-type quantities and the cross product for torque and angular momentum.
- Sketch the vectors – a diagram catches sign and direction errors the algebra hides.



Unit 1 – key takeaways

1

Vectors

components $\hat{i}, \hat{j}, \hat{k}$; dot $\rightarrow$ scalar,
cross $\rightarrow$ vector

2

Calculus

$\vec{v} = d\vec{r}/dt$, $\vec{a} = d\vec{v}/dt$; integrate to
reverse

3

Relative

$\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$; inertial frames

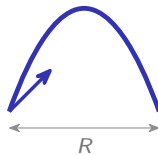
4

Projectiles

independent axes; range peaks at
 45°

Check yourself

- 1 What operation takes velocity back to position?
- 2 Dot product or cross product – which gives a scalar?
- 3 Two frames move at constant relative velocity – do they measure the same acceleration?
- 4 Which launch angle gives the greatest range?



Answers 1. integration 2. the dot product 3. yes 4. 45°