

5 Rotational inertia, balance, and torque

= I alpha – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary

Write each word three times.

English	Write it 3 times (English)
Rotational inertia	_____
rotational equilibrium	_____

Recall

In Part A you met torque – the turning effect of a force. Now use it two ways:

- to **balance** things (a seesaw, a shelf), which you have done before;
- to find how fast something starts to spin, the rotational copy of $F = ma$.

Each idea has a worked example before the Practice.

New ideas

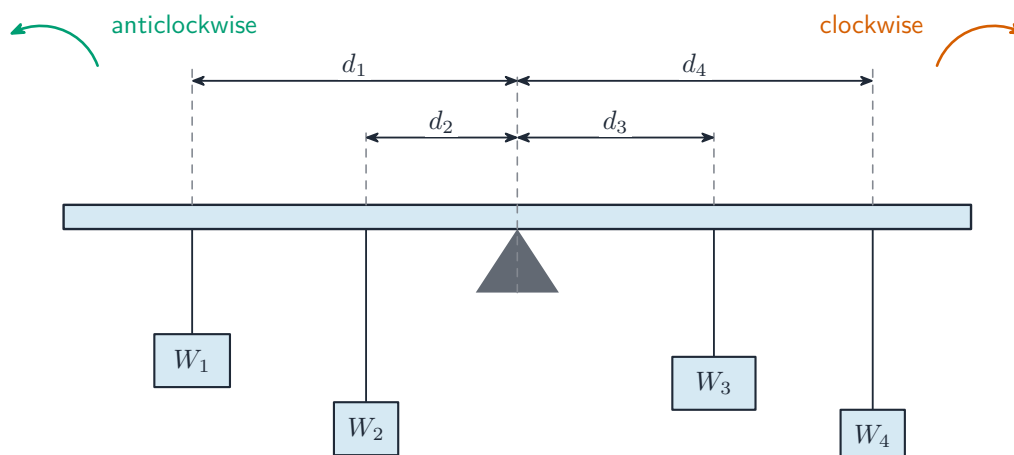
■ 1 Rotational inertia

Rotational inertia (symbol I) measures how hard it is to change something's spin – the rotational version of mass. It depends on the mass **and how far** that mass sits from the axis: mass spread farther out makes I larger.

This is why a spinning skater speeds up when she pulls her arms **in** – she brings her mass closer to the axis, making I smaller.

■ 2 Rotational equilibrium

An object is in **rotational equilibrium** when the total turning effect is zero – the clockwise torques balance the anticlockwise torques. Then it does not start to spin.



Worked example. A 30 kg child sits 2.0 m from the pivot of a seesaw. Where must a 40 kg child sit to balance it? Set the two torques equal (the g cancels):

$$30 \times 2.0 = 40 \times d \Rightarrow d = \frac{60}{40} = 1.5 \text{ m.}$$

The heavier child sits closer to the pivot.

■ 3 The rotational Newton's second law

Just as a net force gives linear acceleration, a **net torque** gives angular acceleration:

$$\sum \tau = I\alpha,$$

the exact twin of $\sum F = ma$ (with $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$).

Worked example. A net torque of 12 N m acts on a wheel with $I = 3.0 \text{ kg m}^2$. Its angular acceleration is $\alpha = \frac{\tau}{I} = \frac{12}{3.0} = 4.0 \text{ rad/s}^2$.

Watch out: the same mass can have different rotational inertia depending on where it sits – a ring is harder to spin than a solid disc of the same mass, because its mass is all at the edge.

Practice

Try these in order. The methods are all above.

2.1 State what two things the rotational inertia of an object depends on. [2]

2.2 A 20 kg child sits 1.5 m from a seesaw pivot. A 25 kg child sits on the other side. How far from the pivot must the second child sit to balance? [3]

2.3 A net torque of 8.0 N m acts on a wheel of rotational inertia 2.0 kg m^2 . Find the angular acceleration. [2]

2.4 Explain why a spinning ice skater speeds up when she pulls her arms in. [2]

2.5 A shelf is held by a bracket. State the condition on the torques for the shelf to stay balanced (not rotate). [1]

2.6 A wheel with $I = 5.0 \text{ kg m}^2$ must reach $\alpha = 3.0 \text{ rad/s}^2$. Find the net torque needed. [2]