

Exercise walk-through

7.5

Simple and Physical Pendulums

AP Physics C: Mechanics

Simple and physical pendulums

Simple pendulum (small angle)

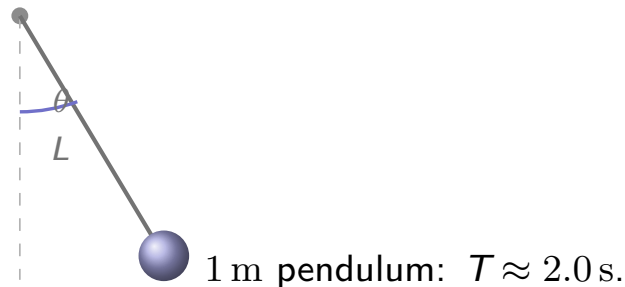
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Depends on L and g only – not mass or amplitude.

Physical pendulum

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

d = pivot to centre of mass.



The pendulum, and why small angles matter

small-angle approximation

$\sin \theta \approx \theta$ in radians – accurate to 1% below about 14°

the equation

$\tau = I\alpha$ gives $-mgL \sin \theta = I\ddot{\theta}$, which is SHM **only** after that approximation

simple pendulum

$T = 2\pi\sqrt{L/g}$ – mass cancels, amplitude does not appear

physical pendulum

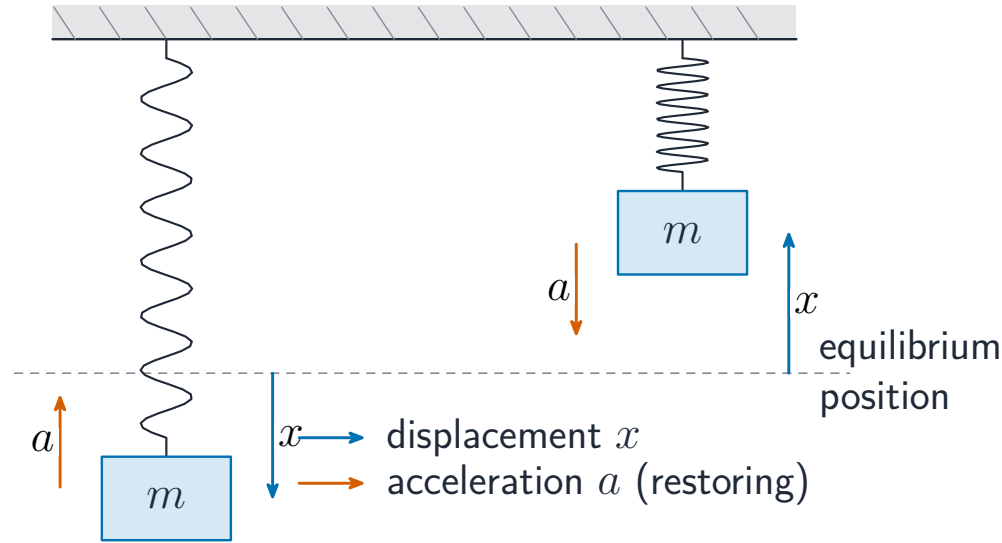
$T = 2\pi\sqrt{\frac{I}{mgd}}$, with d from pivot to centre of mass

natural frequency

$f_0 = \frac{1}{2\pi}\sqrt{k/m}$; driving at f_0 gives **resonance** and a growing amplitude

At large angles a pendulum is **not** simple harmonic and its period grows with amplitude. Every quoted pendulum formula silently assumes the approximation.

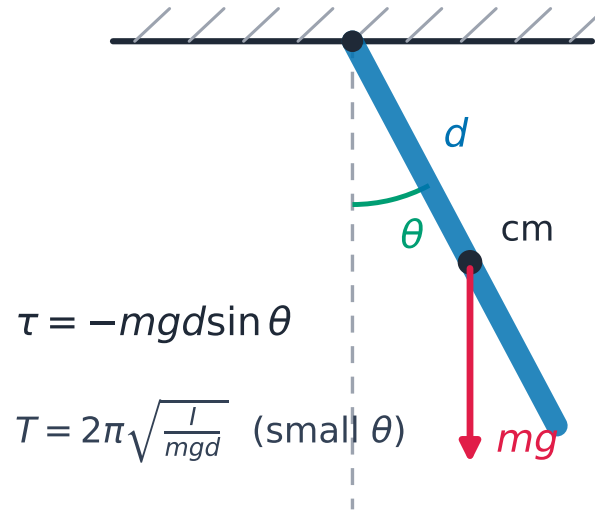
Defining Simple Harmonic Motion



Simple harmonic motion
equilibrium

needs exactly two things: an **equilibrium** point, and a restoring force **proportional to the displacement** and directed back towards it. Recognise the equation $\ddot{x} = -\omega^2 x$ and you know the motion is sinusoidal – you are not asked to solve it.

Simple and Physical Pendulums, continued



physical pendulum: $T = 2\pi \sqrt{I/mgd}$

Gravity acting at the center of mass provides a physical pendulum's restoring torque

The phase constant, and one more oscillator

Phase constant

In $x = A \cos(\omega t + \phi)$ the **phase constant** ϕ sets where the motion starts: $\phi = 0$ from the amplitude, $\phi = -\pi/2$ from the equilibrium moving positive.

Reading it off

Two initial conditions – $x(0)$ and $v(0)$ – fix A and ϕ together.

Torsion pendulum

A **torsion pendulum** twists a wire instead of stretching a spring: $\tau = -\kappa\theta$, so $\omega = \sqrt{\kappa/I}$.

The pattern

Every simple harmonic oscillator has the same form: a restoring quantity over an inertial one, square-rooted.

Identify the restoring constant and the inertia, and ω follows – k/m , κ/I , mgd/I for a physical pendulum.

Exercises 2.1·2.3

2.1 A simple pendulum has $L = 0.25$ m ($g = 9.8$ m s⁻²). Find its period. [2]

2.2 In $T = 2\pi\sqrt{I/(mgd)}$, what is d ? [1]

2.3 State whether making a pendulum's bob heavier changes its period. [1]

Exercises 2.1·2.3

2.1 A simple pendulum has $L = 0.25 \text{ m}$ ($g = 9.8 \text{ m s}^{-2}$). Find its period. [2]

2.1

- $T = 2\pi\sqrt{L/g} = 2\pi\sqrt{0.25/9.8} \approx 1.0 \text{ s}$

2.2 In $T = 2\pi\sqrt{I/(mgd)}$, what is d ? [1]

2.2

- The distance from the pivot to the centre of mass
-

2.3 State whether making a pendulum's bob heavier changes its period. [1]

2.3

- No (mass cancels)

Exercises 2.4·3.1

2.4 For a physical pendulum, about which axis must I be measured? [1]

3.1 The period of a simple pendulum increases if you [1]

☐

A increase the mass

B increase the length

C increase the amplitude

D paint it black

Exercises 2.4-3.1

2.4 For a physical pendulum, about which axis must I be measured? [1]

2.4

- About the pivot

3.1 The period of a simple pendulum increases if you [1]

☐

A increase the mass

B increase the length

C increase the amplitude

D paint it black

3.1 B

- increasing the length increases the period

Exercise 3.2

3.2 The simple-pendulum formula is a special case of the

[1]

☐

A mass-spring formula

B physical-pendulum formula

C ideal gas law

D work-energy theorem

Exercise 3.2

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A mass-spring formula

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3.2 B

- the physical-pendulum formula

Exercise 3.3 (a)

3.3 A uniform rod of length L is pivoted at one end, with $I = \frac{1}{3}mL^2$ and $d = L/2$.

(a) Write its period in terms of L and g .

[2]

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(a) Write its period in terms of L and g .

[2]

(a)

- $$T = 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg(L/2)}} = 2\pi\sqrt{\frac{2L}{3g}}$$

Exercise 3.3 (b)

3.3 A uniform rod of length L is pivoted at one end, with $I = \frac{1}{3}mL^2$ and $d = L/2$.

(b) State whether it swings faster or slower than a simple pendulum of the same length.^[1]

Exercise 3.3 (b)

3.3 A uniform rod of length L is pivoted at one end, with $I = \frac{1}{3}mL^2$ and $d = L/2$.

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(b)

- Slightly faster (shorter effective period)

Exercise 3.4 (a)

3.4 A pendulum has $L = 1.0 \text{ m}$ ($g = 9.8 \text{ m s}^{-2}$).

(a) Find its period.

[2]

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(a)

- $T = 2\pi\sqrt{1/9.8} \approx 2.0 \text{ s}$

Exercise 3.4 (b)

3.4 A pendulum has $L = 1.0 \text{ m}$ ($g = 9.8 \text{ m s}^{-2}$).

(b) State the effect of moving it to the Moon (smaller g).

[1]

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[1]

(b)

- The period gets longer (smaller g)