

Exercise walk-through

7.1

Defining Simple Harmonic Motion (SHM)

AP Physics C: Mechanics

SHM is a differential equation

A **restoring force** $F = -kx$
put into $\vec{F} = m\vec{a}$ gives the defining
law:

The SHM equation

$$\frac{d^2x}{dt^2} = -\omega^2 x, \quad \omega = \sqrt{\frac{k}{m}}$$

Any system whose acceleration
is $-\omega^2$ times its displacement
oscillates in SHM.



Foucault pendulum

What makes a motion simple harmonic

periodic motion

anything that repeats; SHM is one special case of it

condition 1

there is an **equilibrium** position the system returns to

condition 2

the restoring force is **proportional to the displacement** from it:
 $F = -kx$

the consequence

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$ – and every solution of that is a sinusoid

the payoff

the period is **independent of amplitude**: $T = 2\pi\sqrt{m/k}$

Whenever you can show a restoring force is proportional to displacement, you have proved the motion is SHM and may quote T immediately – that is the marked step.

Exercises 2.1-2.3

2.1 A spring with $k = 20 \text{ N m}^{-1}$ is stretched 0.15 m. Find the restoring force. [2]

2.2 State the meaning of the minus sign in $F = -kx$. [1]

2.3 For SHM with $\omega^2 = 25 \text{ s}^{-2}$ at $x = 0.2 \text{ m}$, find the acceleration magnitude. [2]

Exercises 2.1-2.3

2.1 A spring with $k = 20 \text{ N m}^{-1}$ is stretched 0.15 m. Find the restoring force. [2]

2.1

- $|F| = kx = 20 \times 0.15 = 3 \text{ N}$

2.2 State the meaning of the minus sign in $F = -kx$. [1]

2.2

- The force opposes the displacement (points back to equilibrium)

2.3 For SHM with $\omega^2 = 25 \text{ s}^{-2}$ at $x = 0.2 \text{ m}$, find the acceleration magnitude. [2]

2.3

- $|a| = \omega^2 x = 25 \times 0.2 = 5 \text{ m s}^{-2}$

Exercises 2.4·3.1

2.4 Where in the motion is the acceleration largest?

[1]

3.1 A motion is simple harmonic when the restoring force is

[1]

☐

A constant

B proportional to displacement and opposite in direction

C proportional to speed

D zero

Exercises 2.4·3.1

2.4 Where in the motion is the acceleration largest? [1]

2.4

- At the extremes (largest x)

3.1 A motion is simple harmonic when the restoring force is [1]

☐

A constant

B proportional to displacement and opposite in direction

C proportional to speed

D zero

3.1 B

- proportional to displacement and opposite in direction

Exercise 3.2

3.2 In SHM, the acceleration is zero at the

[1]

☐

A extremes

B equilibrium position

C turning points

D it is never zero

Exercise 3.2

3.2 In SHM, the acceleration is zero at the

[1]

☐

A extremes

B equilibrium position

C turning points

D it is never zero

3.2 B

- at equilibrium, $x = 0$ so $a = 0$

Exercise 3.3 (a)

3.3 A 0.5 kg mass on a spring ($k = 8 \text{ N m}^{-1}$) is pulled 0.1 m and released.

(a) Find the restoring force at release.

[2]

Exercise 3.3 (a)

3.3 A 0.5 kg mass on a spring ($k = 8 \text{ N m}^{-1}$) is pulled 0.1 m and released.

(a) Find the restoring force at release.

[2]

(a)

- $F = -kx = -8 \times 0.1 = -0.8 \text{ N}$ (magnitude 0.8 N)

Exercise 3.3 (b)

3.3 A 0.5 kg mass on a spring ($k = 8 \text{ N m}^{-1}$) is pulled 0.1 m and released.

(b) Find the acceleration at release.

[2]

Exercise 3.3 (b)

3.3 A 0.5 kg mass on a spring ($k = 8 \text{ N m}^{-1}$) is pulled 0.1 m and released.

(b) Find the acceleration at release.

[2]

(b)

- $\omega^2 = k/m = 16; a = -\omega^2 x = -1.6 \text{ m s}^{-2}$

Exercise 3.4 (a)

3.4 A pendulum swings with a small amplitude.

(a) State whether its motion is SHM.

[1]

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(a)

- Yes (approximately)

Exercise 3.4 (b)

3.4 A pendulum swings with a small amplitude.

(b) State the condition (angle) required for this to hold.

[1]

Exercise 3.4 (b)

3.4 A pendulum swings with a small amplitude.

(b) State the condition (angle) required for this to hold.

[1]

(b)

- Small angles (roughly under 15°)