

Exercise walk-through

6.6

Motion of Orbiting Satellites

AP Physics C: Mechanics

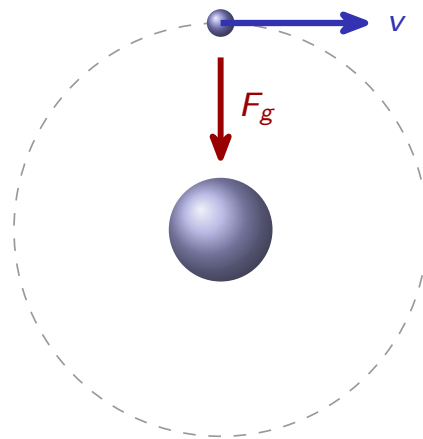
Orbiting satellites

Gravity supplies the centripetal force. Set them equal:

Orbital speed and Kepler

$$v = \sqrt{\frac{GM}{r}}, \quad T^2 \propto r^3$$

- energy $E = K + U$, with $U = -GMm/r$
- elliptical orbits conserve energy *and* angular momentum



A bigger orbit means
a **slower** speed, longer period.

The energy of an orbit is negative, and that matters

gravitational potential energy

$$U_g = -\frac{GMm}{r}, \text{ taking zero at infinity}$$

a circular orbit

$$K = -\frac{1}{2}U, \text{ so the total mechanical energy}$$
$$-\frac{GMm}{2r}$$

$$\text{is } E = K + U =$$

negative means bound

the satellite cannot escape; raising it to a larger orbit means *increasing* E toward zero

escape velocity

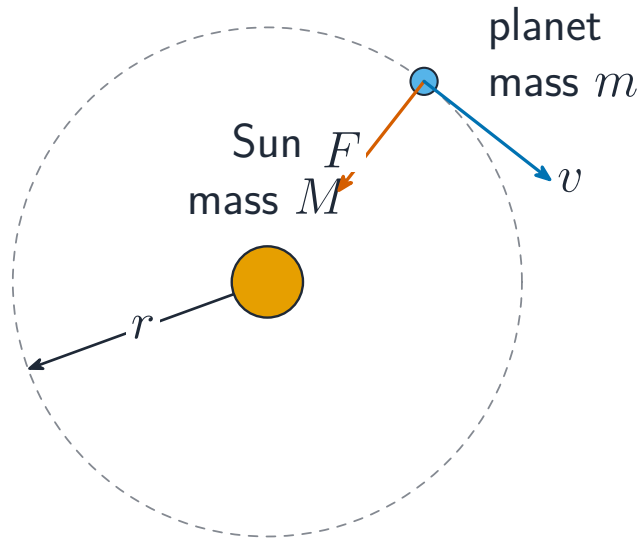
$$\text{set } E = 0: v_{\text{esc}} = \sqrt{\frac{2GM}{r}} = \sqrt{2} \times \text{the circular orbital speed}$$

Kepler's second law

equal areas in equal times – which is conservation of angular momentum in disguise

Larger orbits are **slower** ($v = \sqrt{GM/r}$) yet carry **more** energy. Speeding a satellite up moves it to a higher, slower orbit – the counter-intuitive result AP loves to test.

Motion of Orbiting Satellites



Gravity provides the centripetal force that keeps a satellite in orbit

Worked example – a satellite in low Earth orbit

A satellite orbits at $r = 7.0 \times 10^6$ m, with $GM = 4.0 \times 10^{14}$ m³/s². Find its speed and escape speed.

- $v = \sqrt{GM/r} = \sqrt{4.0 \times 10^{14} / 7.0 \times 10^6} = 7.6 \times 10^3$ m/s
- $v_{\text{esc}} = \sqrt{2} v = 1.1 \times 10^4$ m/s
- period $T = 2\pi r/v \approx 5.8 \times 10^3$ s, about 97 minutes

Exam skill. Orbit FRQs are energy-and-angular-momentum problems in disguise. Write $E = \frac{1}{2}mv^2 - \frac{GMm}{r}$ and $L = mvr \sin \theta$ at the **two points of interest** and solve the pair – never assume the circular-orbit formulae apply to an ellipse.

Exam tips

- Compute the **moment of inertia** $I = \int r^2 dm$ and shift axes with the **parallel-axis theorem** $I = I_{cm} + Md^2$.
- Rotational kinetic energy is $\frac{1}{2}I\omega^2$; a rolling body has both translational and rotational KE.
- Conserve **angular momentum** $L = I\omega$ when net external torque is zero (a spinning skater pulling in).
- Use energy conservation for rolling-without-slipping problems ($v = r\omega$ ties the two motions).
- Know standard I values (hoop, disk, rod, sphere) and where the axis is.

Exercises 2.1-2.3

2.1 What provides the centripetal force that holds a satellite in orbit? [1]

2.2 A satellite moves to a lower orbit. State whether its speed increases or decreases. [1]

2.3 Compute the orbital speed where $GM/r = 4.0 \times 10^7 \text{ m}^2 \text{ s}^{-2}$. [2]

Exercises 2.1-2.3

2.1 What provides the centripetal force that holds a satellite in orbit? [1]

2.1

- Gravity
-

2.2 A satellite moves to a lower orbit. State whether its speed increases or decreases. [1]

2.2

- It increases (lower orbits are faster)
-

2.3 Compute the orbital speed where $GM/r = 4.0 \times 10^7 \text{ m}^2 \text{ s}^{-2}$. [2]

2.3

- $v = \sqrt{4.0 \times 10^7} \approx 6300 \text{ m s}^{-1}$

Exercises 2.4-3.1

2.4 State whether a heavier satellite at the same radius orbits faster, slower, or the same. [1]

3.1 The orbital speed of a satellite depends on [1]

☐

- A** its own mass
- B** the central mass and the orbital radius
- C** its colour
- D** the launch angle

Exercises 2.4-3.1

2.4 State whether a heavier satellite at the same radius orbits faster, slower, or the same. [1]

2.4

- The same (mass cancels)

3.1 The orbital speed of a satellite depends on [1]

☐

A its own mass

B the central mass and the orbital radius

C its colour

D the launch angle

3.1 B

- the central mass and the orbital radius

Exercise 3.2

3.2 According to Kepler's third law, a larger orbit has a

[1]

☐

A shorter period

B longer period

C the same period

D zero period

Exercise 3.2

3.2 According to Kepler's third law, a larger orbit has a

[1]

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A shorter period

B longer period

C the same period

D zero period

3.2 B

- a larger orbit has a longer period

Exercise 3.3 (a)

3.3 Satellite B orbits at twice the radius of satellite A (same planet).

(a) State how B's orbital speed compares with A's (qualitatively).

[1]

Exercise 3.3 (a)

3.3 Satellite B orbits at twice the radius of satellite A (same planet).

(a) State how B's orbital speed compares with A's (qualitatively). [1]

(a)

- B is slower (larger r gives smaller v)

Exercise 3.3 (b)

3.3 Satellite B orbits at twice the radius of satellite A (same planet).

(b) Using $T^2 \propto r^3$, state how B's period compares with A's.

[2]

Exercise 3.3 (b)

3.3 Satellite B orbits at twice the radius of satellite A (same planet).

(b) Using $T^2 \propto r^3$, state how B's period compares with A's.

[2]

(b)

- $T^2 \propto r^3$, so $T_B/T_A = 2^{3/2} \approx 2.8$ times longer

Exercise 3.4 (a)

3.4 A satellite orbits where $GM/r = 2.5 \times 10^7 \text{ m}^2 \text{ s}^{-2}$.

(a) Find its orbital speed.

[2]

Exercise 3.4 (a)

3.4 A satellite orbits where $GM/r = 2.5 \times 10^7 \text{ m}^2 \text{ s}^{-2}$.

(a) Find its orbital speed.

[2]

(a)

- $v = \sqrt{2.5 \times 10^7} \approx 5000 \text{ m s}^{-1}$

Exercise 3.4 (b)

3.4 A satellite orbits where $GM/r = 2.5 \times 10^7 \text{ m}^2 \text{ s}^{-2}$.

(b) State whether doubling the satellite's mass changes this speed.

[1]

Exercise 3.4 (b)

3.4 A satellite orbits where $GM/r = 2.5 \times 10^7 \text{ m}^2 \text{ s}^{-2}$.

(b) State whether doubling the satellite's mass changes this speed.

[1]

(b)

- No – mass cancels, so the speed is unchanged