

Exercise walk-through

6.5

Rolling

AP Physics C: Mechanics

Rolling without slipping

Rolling without slipping

links

translation and rotation:

The condition

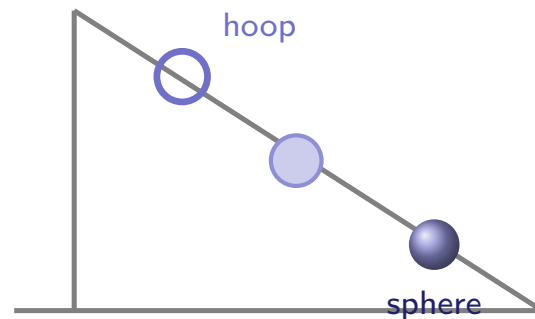
$$v_{cm} = \omega r, \quad a_{cm} = \alpha r$$

Static friction

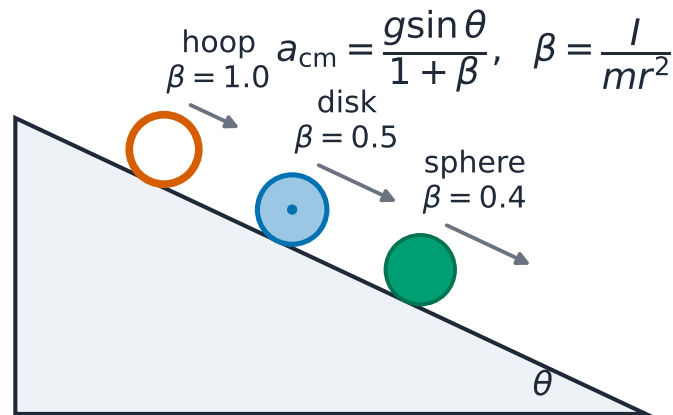
produces the rolling but

does **no work** (the contact point is momentarily at rest).

Down a ramp: the solid sphere wins,
the hoop last.



Rolling race: the shape with the smallest – one



smallest $\beta \Rightarrow$ largest a (sphere wins)
rolling race: smaller I reaches bottom first

Rolling race: the shape with the smallest I/mr^2 reaches the bottom first

Where the energy goes when a shape rolls

the split

a rolling body has **both** translational and rotational kinetic energy

write $I = \beta mr^2$

β is the shape factor: hoop 1, disc $\frac{1}{2}$, solid sphere $\frac{2}{5}$, spherical shell $\frac{2}{3}$

racing down an incline

$$a_{\text{cm}} = \frac{g \sin \theta}{1 + \beta}, \text{ from } mgh = \frac{1}{2}mv^2(1 + \beta)$$

so

$$v = \sqrt{\frac{2gh}{1 + \beta}} - \text{mass and radius both cancel}$$

the winner

the **smallest** β : solid sphere, then disc, then shell, then hoop

what β measures

how much **rotational inertia**

sits far from the axis

Mass and radius cancel, so the race is decided by shape alone – and $\Delta x_{\text{cm}} = r\Delta\theta$ ties the centre's displacement to the **angular displacement**.

The rolling race, decided by one number

write $I = \beta mr^2$

β is the shape factor: hoop 1, disc $\frac{1}{2}$, solid sphere $\frac{2}{5}$, shell $\frac{2}{3}$

energy conservation

$$mgh = \frac{1}{2}mv^2(1 + \beta), \text{ so } v = \sqrt{\frac{2gh}{1 + \beta}}$$

so the winner

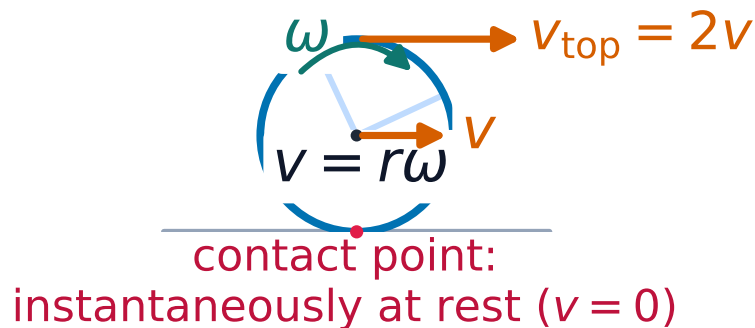
the **smallest** β – the solid sphere, then the disc, then the shell, then the hoop

and the mass?

cancels completely: size and mass do not matter, **only the shape**

β measures how much mass sits far from the axis. Mass at the rim has to be spun up as well as moved forward, and that energy is taken from the translation.

Rolling race: the shape with the smallest – two



$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

rolling no slip: $v = r\omega \quad \cdot \quad a = r\alpha$

In rolling without slipping the contact point is at rest, so $v = r\omega$

Exercises 2.1-2.3

2.1 A wheel of radius 0.4 m rolls without slipping at $\omega = 5 \text{ rad s}^{-1}$. Find its forward speed. [2]

2.2 State the speed of the point where a rolling wheel touches the ground. [1]

2.3 Write the two terms in the total kinetic energy of a rolling object. [2]

Exercises 2.1·2.3

2.1 A wheel of radius 0.4 m rolls without slipping at $\omega = 5 \text{ rad s}^{-1}$. Find its forward speed. [2]

2.1

- $v = \omega r = 5 \times 0.4 = 2 \text{ m s}^{-1}$
-

2.2 State the speed of the point where a rolling wheel touches the ground. [1]

2.2

- Zero (instantaneously at rest)
-

2.3 Write the two terms in the total kinetic energy of a rolling object. [2]

2.3

- $\frac{1}{2}mv^2$ (translational) and $\frac{1}{2}I\omega^2$ (rotational)

Exercises 2.4·3.1

2.4 A rolling ball of radius 0.1 m moves at 2 m s^{-1} . Find its angular velocity. [2]

3.1 For rolling without slipping, the link between speed and spin is [1]

☐

A $v = \omega/r$

B $v = \omega r$

C $v = r/\omega$

D $v = \omega^2 r$

Exercises 2.4·3.1

2.4 A rolling ball of radius 0.1 m moves at 2 m s^{-1} . Find its angular velocity. [2]

2.4

- $\omega = v/r = 2/0.1 = 20 \text{ rad s}^{-1}$

3.1 For rolling without slipping, the link between speed and spin is [1]

☐

A $v = \omega/r$

B $v = \omega r$

C $v = r/\omega$

D $v = \omega^2 r$

3.1 B

- $v = \omega r$

Exercise 3.2

3.2 Released together down the same ramp, which reaches the bottom first?

[1]

☐

A a hollow ring

B a solid cylinder

C a solid sphere

D they tie

Exercise 3.2

3.2 Released together down the same ramp, which reaches the bottom first?

[1]

☐

A a hollow ring

B a solid cylinder

C a solid sphere

D they tie

3.2 C

- the solid sphere (smallest rotational inertia per mr^2) wins

Exercise 3.3 (a)

3.3 A hoop and a solid disc (same mass and radius) roll from rest down the same ramp.

(a) State which arrives first.

[1]

Exercise 3.3 (a)

3.3 A hoop and a solid disc (same mass and radius) roll from rest down the same ramp.

(a) State which arrives first. [1]

(a)

- The solid disc

Exercise 3.3 (b)

3.3 A hoop and a solid disc (same mass and radius) roll from rest down the same ramp.

(b) Explain why, in terms of how the energy is shared. [2]

Exercise 3.3 (b)

3.3 A hoop and a solid disc (same mass and radius) roll from rest down the same ramp.

(b) Explain why, in terms of how the energy is shared. [2]

(b)

- The hoop puts more energy into spin (larger I), leaving less for speed, so it is slower

Exercise 3.4 (a)

3.4 A ball of radius 0.2 m rolls without slipping at 6 m s^{-1} .

(a) Find its angular velocity.

[2]

Exercise 3.4 (a)

3.4 A ball of radius 0.2 m rolls without slipping at 6 m s^{-1} .

(a) Find its angular velocity.

[2]

(a)

- $\omega = v/r = 6/0.2 = 30 \text{ rad s}^{-1}$

Exercise 3.4 (b)

3.4 A ball of radius 0.2 m rolls without slipping at 6 m s^{-1} .

(b) State what happens to v if it begins to skid (slip). [1]

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3.4 A ball of radius 0.2 m rolls without slipping at 6 m s^{-1} .

(b) State what happens to v if it begins to skid (slip). [1]

(b)

- $v = \omega r$ no longer holds – the speeds decouple