

Exercise walk-through

6.4

Conservation of Angular Momentum

AP Physics C: Mechanics

Angular momentum and impulse

Angular momentum is the rotational twin of momentum:

Point particle and rigid body

$$\vec{L} = \vec{r} \times \vec{p}, \quad L = I\omega$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

Angular impulse $\int \tau dt = \Delta L$.

Worked example

$L_i = 30$; $\tau = 5 \text{ N m}$ for 4 s:

$$\Delta L = 5 \times 4 = 20$$

$$L_f = 30 + 20 = 50 \text{ kg m}^2/\text{s}$$

Conservation of angular momentum

With **zero net external torque**, angular momentum is conserved – even as the shape changes:

Before = after

$$I_i \omega_i = I_f \omega_f$$

A skater pulls their arms in: I drops, so ω rises. (Kinetic energy rises too – paid by muscular work.)



Skater spin

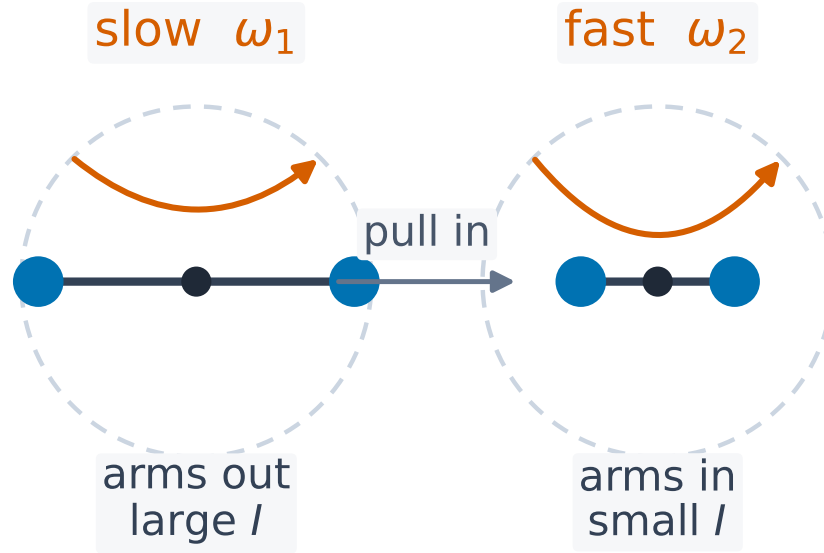
Worked example – a bullet strikes a pivoted rod

A 0.020 kg bullet at 300 m/s hits the tip of a uniform rod ($M = 1.5$ kg, $l = 0.60$ m) pivoted at its centre, and embeds. Find ω .

- angular momentum about the pivot is conserved; **linear momentum is not** (the pivot exerts a force)
- before: $L = mvr = (0.020)(300)(0.30) = 1.8 \text{ kg m}^2/\text{s}$
- after:
$$I = \frac{1}{12}Ml^2 + mr^2 = 0.045 + 0.0018 = 0.0468$$
- $\omega = L/I = 38 \text{ rad/s}$

Choose **angular** momentum whenever there is a pivot. The pivot's own force acts through the axis, so it exerts no torque and drops out.

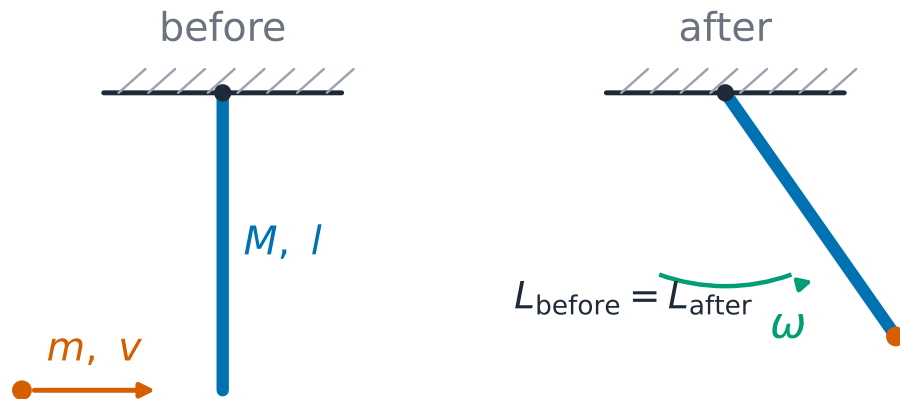
Pulling mass inward lowers I – one



$$L = I\omega \text{ conserved (no external torque): } I_1\omega_1 = I_2\omega_2$$

Pulling mass inward lowers I , so ω rises to conserve $L = I\omega$

Pulling mass inward lowers I – two



$$m v l = I_{\text{sys}} \omega \quad (\text{about pivot; } KE \text{ not conserved — inelastic})$$

Angular momentum about the pivot is conserved as the bullet embeds in the rod

Exercises 2.1·2.3

2.1 State the condition for angular momentum to be conserved.

[1]

2.2 A skater at $I_i = 6 \text{ kg m}^2$ spins at 2 rad s^{-1} , then pulls in to $I_f = 2 \text{ kg m}^2$. Find the new ω .

[2]

2.3 If a spinning body's rotational inertia halves (no external torque), what happens to ω ?

[1]

Exercises 2.1-2.3

2.1 State the condition for angular momentum to be conserved.

[1]

2.1

- When the net external torque is zero

2.2 A skater at $I_i = 6 \text{ kg m}^2$ spins at 2 rad s^{-1} , then pulls in to $I_f = 2 \text{ kg m}^2$. Find the new ω .

[2]

2.2

- $6(2) = 2\omega_f \Rightarrow \omega_f = 6 \text{ rad s}^{-1}$

2.3 If a spinning body's rotational inertia halves (no external torque), what happens to ω ?

[1]

2.3

- It doubles

Exercises 2.4·3.1

2.4 Is rotational kinetic energy always conserved when angular momentum is? Answer yes or no. [1]

3.1 Angular momentum is conserved when the net external torque is [1]

☐

- A maximum
- B zero
- C increasing
- D equal to $I\alpha$

Exercises 2.4-3.1

2.4 Is rotational kinetic energy always conserved when angular momentum is? Answer yes or no. [1]

2.4

- No

3.1 Angular momentum is conserved when the net external torque is [1]

☐

A maximum

B zero

C increasing

D equal to $I\alpha$

3.1 B

- zero net external torque

Exercise 3.2

3.2 A star collapses to one third of its rotational inertia. Its spin rate becomes [1]

☐

A one third

B unchanged

C three times faster

D nine times faster

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3.2 C

- $I \rightarrow I/3$ means $\omega \rightarrow 3\omega$

Exercise 3.3 (a)

3.3 A turntable ($I = 4 \text{ kg m}^2$) spins at 3 rad s^{-1} . A blob of clay is dropped on, raising I to 6 kg m^2 .

(a) Find the new angular speed.

[2]

Exercise 3.3 (a)

3.3 A turntable ($I = 4 \text{ kg m}^2$) spins at 3 rad s^{-1} . A blob of clay is dropped on, raising I to 6 kg m^2 .

(a) Find the new angular speed.

[2]

(a)

- $4(3) = 6\omega_f \Rightarrow \omega_f = 2 \text{ rad s}^{-1}$

Exercise 3.3 (b)

3.3 A turntable ($I = 4 \text{ kg m}^2$) spins at 3 rad s^{-1} . A blob of clay is dropped on, raising I to 6 kg m^2 .

(b) State whether angular momentum is conserved and why.

[1]

Exercise 3.3 (b)

3.3 A turntable ($I = 4 \text{ kg m}^2$) spins at 3 rad s^{-1} . A blob of clay is dropped on, raising I to 6 kg m^2 .

(b) State whether angular momentum is conserved and why. [1]

(b)

- Yes – no external torque acts on the turntable-plus-clay system

Exercise 3.4 (a)

3.4 A diver tucks to spin faster during a somersault.

(a) Explain, using $I\omega$, why tucking speeds the spin.

[2]

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3.4 A diver tucks to spin faster during a somersault.

(a) Explain, using $I\omega$, why tucking speeds the spin.

[2]

(a)

- Tucking lowers I ; since $I\omega$ is fixed, ω rises

Exercise 3.4 (b)

3.4 A diver tucks to spin faster during a somersault.

(b) State what provides no external torque here.

[1]

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3.4 A diver tucks to spin faster during a somersault.

(b) State what provides no external torque here.

[1]

(b)

- Gravity acts through the center of mass, giving no torque about it

Exercise 3.5

3.5 With negligible external torque, a rotor changes from $I_i = 6.0$ to $I_f = 2.0 \text{ kg m}^2$, starting at 2.0 rad s^{-1} . (a) Find its final angular speed. [2]

(b) Find the change in rotational kinetic energy and explain its source. [2]

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3.5

(a) $I_i \omega_i = I_f \omega_f$, so $\omega_f = 6 \times 2/2 = 6.0 \text{ rad s}^{-1}$.

(b) $\Delta K = \frac{1}{2}(2)(6^2) - \frac{1}{2}(6)(2^2) = 24 \text{ J}$; internal work supplies the increase.