

Exercise walk-through

6.3

Angular Momentum and Angular Impulse

AP Physics C: Mechanics

Angular momentum and impulse

Angular momentum is the rotational twin of momentum:

Point particle and rigid body

$$\vec{L} = \vec{r} \times \vec{p}, \quad L = I\omega$$

$$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$$

Angular impulse $\int \tau dt = \Delta L$.

Worked example

$L_i = 30$; $\tau = 5 \text{ N m}$ for 4 s:

$$\Delta L = 5 \times 4 = 20$$

$$L_f = 30 + 20 = 50 \text{ kg m}^2/\text{s}$$

Conservation of angular momentum

With **zero net external torque**, angular momentum is conserved – even as the shape changes:

Before = after

$$I_i \omega_i = I_f \omega_f$$

A skater pulls their arms in: I drops, so ω rises. (Kinetic energy rises too – paid by muscular work.)



Skater spin

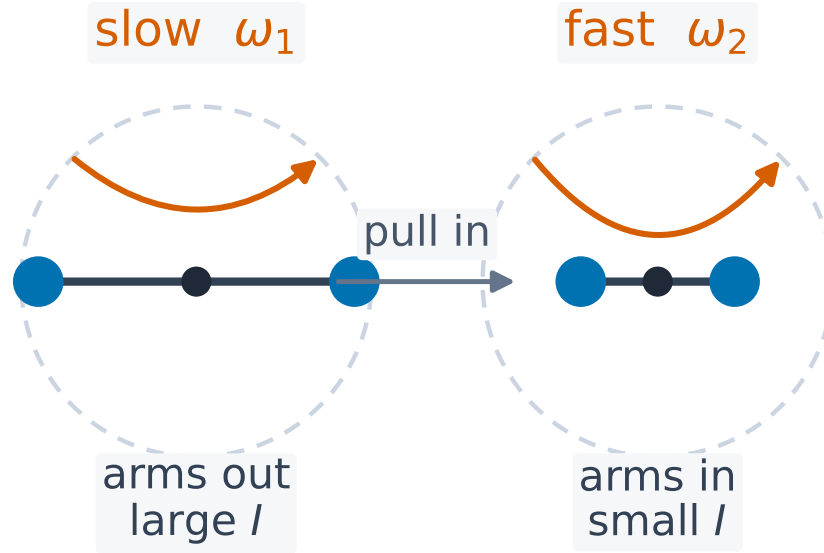
Worked example – a bullet strikes a pivoted rod

A 0.020 kg bullet at 300 m/s hits the tip of a uniform rod ($M = 1.5$ kg, $l = 0.60$ m) pivoted at its centre, and embeds. Find ω .

- angular momentum about the pivot is conserved; **linear momentum is not** (the pivot exerts a force)
- before: $L = mvr = (0.020)(300)(0.30) = 1.8 \text{ kg m}^2/\text{s}$
- after:
$$I = \frac{1}{12}Ml^2 + mr^2 = 0.045 + 0.0018 = 0.0468$$
- $\omega = L/I = 38 \text{ rad/s}$

Choose **angular** momentum whenever there is a pivot. The pivot's own force acts through the axis, so it exerts no torque and drops out.

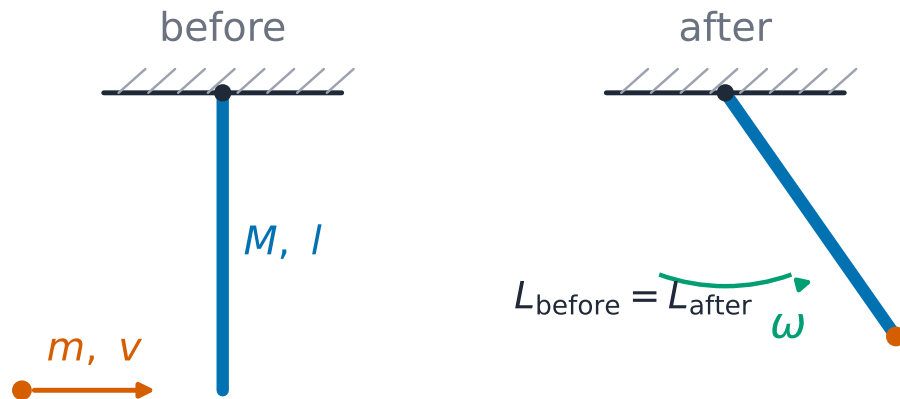
Pulling mass inward lowers I – one



$$L = I\omega \text{ conserved (no external torque): } I_1\omega_1 = I_2\omega_2$$

Pulling mass inward lowers I , so ω rises to conserve $L = I\omega$

Pulling mass inward lowers I – two



$$m v l = I_{\text{sys}} \omega \quad (\text{about pivot; } KE \text{ not conserved — inelastic})$$

Angular momentum about the pivot is conserved as the bullet embeds in the rod

Exercises 2.1-2.3

2.1 A flywheel with $I = 5 \text{ kg m}^2$ spins at 4 rad s^{-1} . Find its angular momentum. [2]

2.2 State the unit of angular momentum. [1]

2.3 A 3 N m torque acts for 5 s . Find the change in angular momentum. [2]

Exercises 2.1-2.3

2.1 A flywheel with $I = 5 \text{ kg m}^2$ spins at 4 rad s^{-1} . Find its angular momentum. [2]

2.1

- $L = I\omega = 5 \times 4 = 20 \text{ kg m}^2 \text{ s}^{-1}$

2.2 State the unit of angular momentum. [1]

2.2

- $\text{kg m}^2 \text{ s}^{-1}$

2.3 A 3 N m torque acts for 5 s . Find the change in angular momentum. [2]

2.3

- $\Delta L = \tau \Delta t = 3 \times 5 = 15 \text{ kg m}^2 \text{ s}^{-1}$

Exercises 2.4·3.1

2.4 A 2 kg ball moves at 3 m s^{-1} in a circle of radius 4 m. Find its angular momentum. [2]

3.1 Angular momentum is the rotational analogue of [1]

☐

A force

B momentum

C energy

D acceleration

Exercises 2.4·3.1

2.4 A 2 kg ball moves at 3 m s^{-1} in a circle of radius 4 m. Find its angular momentum. [2]

2.4

- $L = mvr = 2 \times 3 \times 4 = 24 \text{ kg m}^2 \text{ s}^{-1}$

3.1 Angular momentum is the rotational analogue of [1]

☐

A force

B momentum

C energy

D acceleration

3.1 B

- angular momentum is the rotational analogue of linear momentum

Exercise 3.2

3.2 For a rigid body, L equals

[1]

☐

A $I\alpha$

B $I\omega$

C $\frac{1}{2}I\omega^2$

D $\tau\theta$

Exercise 3.2

3.2 For a rigid body, L equals

[1]

☐

A $I\alpha$

B $I\omega$

C $\frac{1}{2}I\omega^2$

D $\tau\theta$

3.2 B

- $L = I\omega$

Exercise 3.3 (a)

3.3 A wheel with $I = 2 \text{ kg m}^2$ at $\omega = 3 \text{ rad s}^{-1}$ receives a steady 2 N m torque for 4 s .

(a) Find its initial angular momentum.

[1]

Exercise 3.3 (a)

3.3 A wheel with $I = 2 \text{ kg m}^2$ at $\omega = 3 \text{ rad s}^{-1}$ receives a steady 2 N m torque for 4 s .

(a) Find its initial angular momentum. [1]

(a)

- $L_i = 2 \times 3 = 6 \text{ kg m}^2 \text{ s}^{-1}$

Exercise 3.3 (b)

3.3 A wheel with $I = 2 \text{ kg m}^2$ at $\omega = 3 \text{ rad s}^{-1}$ receives a steady 2 N m torque for 4 s .

(b) Find its angular momentum after the torque acts. [2]

Exercise 3.3 (b)

3.3 A wheel with $I = 2 \text{ kg m}^2$ at $\omega = 3 \text{ rad s}^{-1}$ receives a steady 2 N m torque for 4 s .

(b) Find its angular momentum after the torque acts. [2]

(b)

- $\Delta L = 2 \times 4 = 8; L_f = 6 + 8 = 14 \text{ kg m}^2 \text{ s}^{-1}$

Exercise 3.4 (a)

3.4 A skater has $I = 4 \text{ kg m}^2$ spinning at 2 rad s^{-1} .

(a) Find her angular momentum.

[1]

Exercise 3.4 (a)

3.4 A skater has $I = 4 \text{ kg m}^2$ spinning at 2 rad s^{-1} .

(a) Find her angular momentum.

[1]

(a)

- $L = 4 \times 2 = 8 \text{ kg m}^2 \text{ s}^{-1}$

Exercise 3.4 (b)

3.4 A skater has $I = 4 \text{ kg m}^2$ spinning at 2 rad s^{-1} .

(b) State what quantity torque is the rate of change of.

[1]

Exercise 3.4 (b)

3.4 A skater has $I = 4 \text{ kg m}^2$ spinning at 2 rad s^{-1} .

(b) State what quantity torque is the rate of change of.

[1]

(b)

- Angular momentum ($\tau = dL/dt$)