

Exercise walk-through

5.6

Newton's Second Law in Rotational Form

AP Physics C: Mechanics

Net torque changes angular velocity

$$\sum \tau = I\alpha$$

For a rigid body with constant inertia about a fixed axis, the net torque component along that axis sets angular acceleration. The **net torque** can oppose the existing rotation.

If inertia changes, the constant- I model is insufficient; use the angular-momentum account developed in Unit 6.

Same torque, different inertia: original check

$$\alpha = \frac{\sum \tau}{I}$$

$$\alpha = \frac{6.0 \text{ N m}}{2.0 \text{ kg m}^2} = 3.0 \text{ rad/s}^2$$

At the same net torque, doubling I halves α .

Predict the motion if ω is initially negative but the net torque is positive. Explain why positive angular acceleration need not mean increasing angular speed.

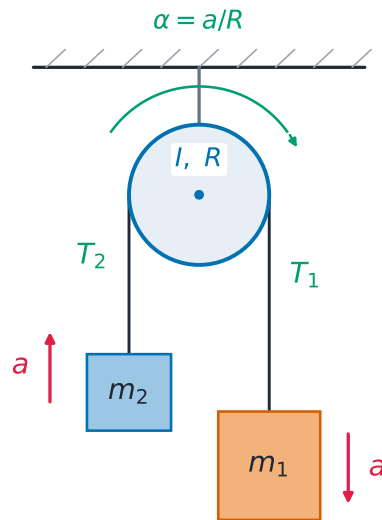
Massive pulley: original capstone attempt

A light taut inextensible rope passes over a pulley with negligible axle friction and no slip. The hanging masses are $m_1 = 4.0$ kg and $m_2 = 2.0$ kg. The pulley has $I = 0.50$ kg m² and $R = 0.20$ m. Use $g = 9.8$ m/s².

Find the acceleration and both tensions. Choose positive senses and draw the two mass diagrams and pulley diagram before forming equations.

Explain why the two tensions differ in this motion, and when a massive pulley could have equal tensions.

Pulley: connect translation and rotation



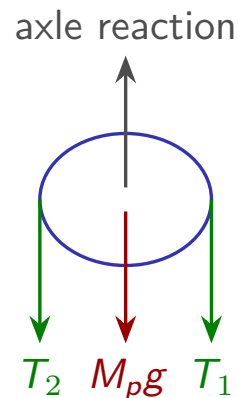
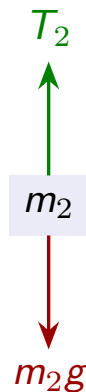
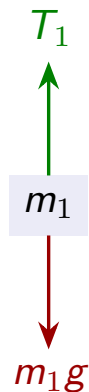
$$(T_1 - T_2)R = I\alpha$$

Ideal axle; taut rope without slip.

Unequal tensions when net torque is nonzero.

The rope does not slip: $a = R\alpha$. For this accelerating ideal-axle motion, the tension difference supplies the net torque.

Separate free-body diagrams



Forces of the rope on the pulley point downward, opposite to the tensions on the masses. Pulley weight and axle reaction pass through its centre, so give zero central torque. Diagrams are not to force scale.

Exercises 2.1-2.3

2.1 A torque of 12 N m acts on a body with $I = 3 \text{ kg m}^2$. Find the angular acceleration. [2]

2.2 State the rotational analogue of mass. [1]

2.3 What net torque gives a 5 kg m^2 wheel an angular acceleration of 2 rad s^{-2} ? [2]

Exercises 2.1-2.3

2.1 A torque of 12 N m acts on a body with $I = 3 \text{ kg m}^2$. Find the angular acceleration. [2]

2.1

- $\alpha = \tau/I = 12/3 = 4 \text{ rad s}^{-2}$
-

2.2 State the rotational analogue of mass. [1]

2.2

- Rotational inertia I
-

2.3 What net torque gives a 5 kg m^2 wheel an angular acceleration of 2 rad s^{-2} ? [2]

2.3

- $\tau = I\alpha = 5 \times 2 = 10 \text{ N m}$

Exercises 2.4·3.1

2.4 For a fixed torque, how does a larger rotational inertia affect α ? [1]

3.1 The rotational form of Newton's second law is [1]

☐

A $F = ma$

B $\tau = I\alpha$

C $p = mv$

D $W = \tau\theta$

Exercises 2.4·3.1

2.4 For a fixed torque, how does a larger rotational inertia affect α ? [1]

2.4

- It makes α smaller

3.1 The rotational form of Newton's second law is [1]

☐

A $F = ma$

B $\tau = I\alpha$

C $p = mv$

D $W = \tau\theta$

3.1 B

- $\tau = I\alpha$

Exercise 3.2

3.2 A 6 N m torque acts on a wheel with $I = 2 \text{ kg m}^2$. Its angular acceleration is [1]

☐

A 3 rad s^{-2}

B 12 rad s^{-2}

C 0.33 rad s^{-2}

D 8 rad s^{-2}

Exercise 3.2

3.2 A 6 N m torque acts on a wheel with $I = 2 \text{ kg m}^2$. Its angular acceleration is [1]

☐

A 3 rad s^{-2}

B 12 rad s^{-2}

C 0.33 rad s^{-2}

D 8 rad s^{-2}

3.2 A

- $\alpha = 6/2 = 3 \text{ rad s}^{-2}$

Exercise 3.3 (a)

3.3 A grindstone with $I = 0.5 \text{ kg m}^2$ is driven by a net torque of 4 N m .

(a) Find its angular acceleration.

[2]

Exercise 3.3 (a)

3.3 A grindstone with $I = 0.5 \text{ kg m}^2$ is driven by a net torque of 4 N m .

(a) Find its angular acceleration.

[2]

(a)

- $\alpha = 4/0.5 = 8 \text{ rad s}^{-2}$

Exercise 3.3 (b)

3.3 A grindstone with $I = 0.5 \text{ kg m}^2$ is driven by a net torque of 4 N m .

(b) State what happens to α if the torque is doubled.

[1]

Exercise 3.3 (b)

3.3 A grindstone with $I = 0.5 \text{ kg m}^2$ is driven by a net torque of 4 N m .

(b) State what happens to α if the torque is doubled.

[1]

(b)

- It doubles

Exercise 3.4 (a)

3.4 A wheel with $I = 4 \text{ kg m}^2$ speeds up from rest to 6 rad s^{-1} in 3 s.

(a) Find the angular acceleration.

[2]

Exercise 3.4 (a)

3.4 A wheel with $I = 4 \text{ kg m}^2$ speeds up from rest to 6 rad s^{-1} in 3 s.

(a) Find the angular acceleration.

[2]

(a)

- $\alpha = 6/3 = 2 \text{ rad s}^{-2}$

Exercise 3.4 (b)

3.4 A wheel with $I = 4 \text{ kg m}^2$ speeds up from rest to 6 rad s^{-1} in 3 s.

(b) Find the net torque needed.

[2]

Exercise 3.4 (b)

3.4 A wheel with $I = 4 \text{ kg m}^2$ speeds up from rest to 6 rad s^{-1} in 3 s.

(b) Find the net torque needed.

[2]

(b)

- $\tau = I\alpha = 4 \times 2 = 8 \text{ N m}$