

Exercise walk-through

5.4

Rotational Inertia

AP Physics C: Mechanics

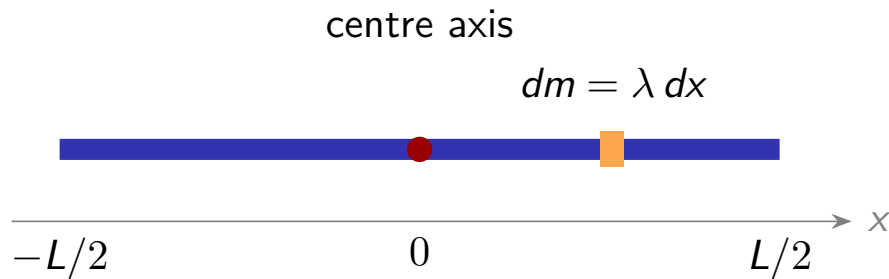
Inertia is defined relative to an axis

$$I = \sum_i m_i r_i^2, \quad I = \int r^2 dm$$

Rotational inertia measures resistance to angular acceleration. Use perpendicular distance from each mass element to the chosen axis; units are kg m^2 .

For the same point mass, doubling distance multiplies its contribution by four. For several bodies, add inertias about the same axis.

Uniform rod: original derivation task



A uniform rod has mass M and length L . Derive its inertia about a perpendicular axis through its centre. State density, mass element, distance and integration limits.

Uniform rod: build the integral

$$I = \int x^2 dm, \quad \lambda = \frac{M}{L}, \quad dm = \frac{M}{L} dx$$

$$I_{\text{cm}} = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx$$

$$I_{\text{cm}} = \frac{M}{L} \left[\frac{x^3}{3} \right]_{-L/2}^{L/2} = \frac{ML^2}{12}$$

Linear density
distance is squared.

has unit kg/m. Both halves contribute positively because

Nonuniform rod: original attempt

A thin rod extends from $x = 0$ to $x = L$. Its linear density is $\lambda(x) = cx$, with constant $c > 0$.

Find c in terms of total mass M and length L . Then derive inertia about a perpendicular axis at $x = 0$.

Compare with a uniform rod of the same mass and length about one end.

Nonuniform rod: normalise before integrating

$$M = \int_0^L \lambda(x) dx = \int_0^L cx dx = \frac{1}{2}cL^2$$

$$c = \frac{2M}{L^2}, \quad dm = cx dx$$

$$I = \int_0^L x^2(cx) dx = \frac{1}{4}cL^4 = \frac{1}{2}ML^2$$

This exceeds the uniform end-axis result $ML^2/3$: more mass lies far from the axis. The coefficient c has unit kg/m^2 .

Exercises 2.1-2.3

2.1 A 3 kg mass sits 2 m from the axis. Find its rotational inertia. [2]

2.2 State what rotational inertia measures. [1]

2.3 Two 1 kg masses sit at $r = 1$ m and $r = 2$ m. Find the total I . [2]

Exercises 2.1·2.3

2.1 A 3 kg mass sits 2 m from the axis. Find its rotational inertia. [2]

2.1

- $I = mr^2 = 3 \times 2^2 = 12 \text{ kg m}^2$

2.2 State what rotational inertia measures. [1]

2.2

- An object's resistance to a change in its rotation
-

2.3 Two 1 kg masses sit at $r = 1 \text{ m}$ and $r = 2 \text{ m}$. Find the total I . [2]

2.3

- $I = 1(1)^2 + 1(2)^2 = 1 + 4 = 5 \text{ kg m}^2$

Exercises 2.4·3.1

2.4 If a mass is moved twice as far from the axis, by what factor does its contribution to I change? [1]

3.1 Rotational inertia depends on [1]

☐

A only the total mass

B the mass and how it is distributed about the axis

C only the speed

D the colour of the object

Exercises 2.4-3.1

2.4 If a mass is moved twice as far from the axis, by what factor does its contribution to I change? [1]

2.4

- Four times (r is squared)

3.1 Rotational inertia depends on [1]

☐

A only the total mass

B the mass and how it is distributed about the axis

C only the speed

D the colour of the object

3.1 B

- mass and its distribution about the axis

Exercise 3.2

3.2 A point mass's contribution to rotational inertia is

[1]

☐

A mr

B mr^2

C $\frac{1}{2}mr^2$

D m/r

Exercise 3.2

3.2 A point mass's contribution to rotational inertia is

[1]

☐

A mr

B mr^2

C $\frac{1}{2}mr^2$

D m/r

3.2 B

- mr^2

Exercise 3.3 (a)

3.3 A 4 kg mass sits at $r = 1.5$ m and a 2 kg mass at $r = 3$ m on a light rod.

(a) Find the total rotational inertia.

[2]

Exercise 3.3 (a)

3.3 A 4 kg mass sits at $r = 1.5$ m and a 2 kg mass at $r = 3$ m on a light rod.

(a) Find the total rotational inertia.

[2]

(a)

- $I = 4(1.5)^2 + 2(3)^2 = 9 + 18 = 27 \text{ kg m}^2$

Exercise 3.3 (b)

3.3 A 4 kg mass sits at $r = 1.5$ m and a 2 kg mass at $r = 3$ m on a light rod.

(b) State which mass contributes more, and why.

[1]

Exercise 3.3 (b)

3.3 A 4 kg mass sits at $r = 1.5$ m and a 2 kg mass at $r = 3$ m on a light rod.

(b) State which mass contributes more, and why.

[1]

(b)

- The 2 kg mass, because it is farther out (r^2 dominates)

Exercise 3.4 (a)

3.4 A solid disc and a ring have the same mass and radius.

(a) State which has the larger rotational inertia.

[1]

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3.4 A solid disc and a ring have the same mass and radius.

(a) State which has the larger rotational inertia.

[1]

(a)

- The ring

Exercise 3.4 (b)

3.4 A solid disc and a ring have the same mass and radius.

(b) Explain why, in terms of mass distribution.

[2]

Exercise 3.4 (b)

3.4 A solid disc and a ring have the same mass and radius.

(b) Explain why, in terms of mass distribution.

[2]

(b)

- All the ring's mass is at the rim (maximum r), while the disc's mass is spread inward, lowering its I