

Exercise walk-through

5.2

Connecting Linear and Rotational Motion

AP Physics C: Mechanics

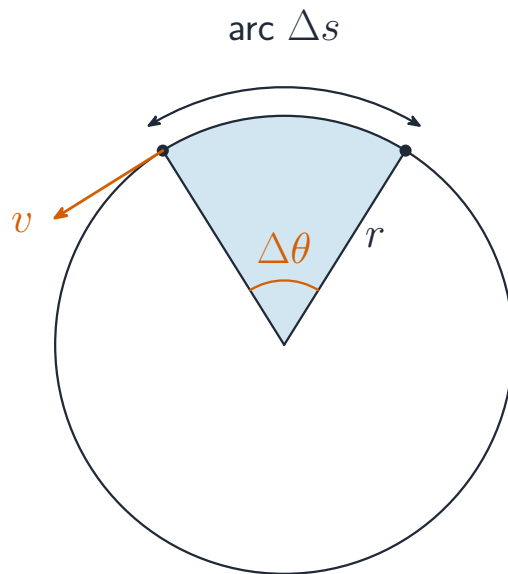
One angular motion, different linear motions

$$\Delta s = r\Delta\theta, \quad v = r|\omega|, \quad a_t = r\alpha, \quad a_c = r\omega^2$$

These relations describe points at perpendicular distance r from a fixed axis. All points of the rigid system share ω and α , but farther points have larger linear speed and acceleration components.

Tangential acceleration can oppose motion. **Centripetal acceleration** points inward, even at constant nonzero speed.

Angular speed and a point on the rim



Doubling radius at the same angular speed doubles linear speed; it does not double angular speed.

Slowing wheel: original attempt

A wheel rotates counterclockwise at $\omega = 4.0 \text{ rad/s}$ and has $\alpha = -2.0 \text{ rad/s}^2$. A marked point lies 0.50 m from the fixed axis.

Find its speed, signed tangential acceleration, inward acceleration and total acceleration magnitude.

At the rightmost position, draw the directions of velocity and both accelerations.

Wheel: calculate the components

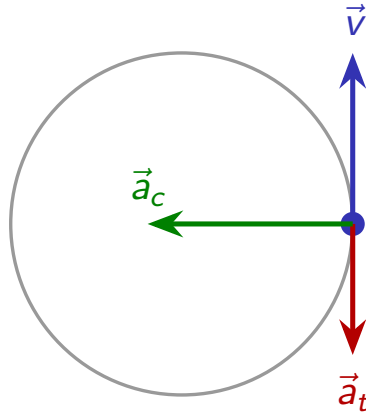
$$v = r|\omega| = (0.50 \text{ m})(4.0 \text{ rad/s}) = 2.0 \text{ m/s}$$

$$a_t = r\alpha = (0.50 \text{ m})(-2.0 \text{ rad/s}^2) = -1.0 \text{ m/s}^2$$

$$a_c = r\omega^2 = (0.50 \text{ m})(4.0 \text{ rad/s})^2 = 8.0 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(-1.0 \text{ m/s}^2)^2 + (8.0 \text{ m/s}^2)^2} \approx 8.1 \text{ m/s}^2$$

Wheel: directions at the rightmost point



Tangential acceleration opposes the upward velocity: the point is slowing. Inward acceleration turns the velocity. Arrow lengths are schematic, not a common numerical scale.

Exercises 2.1-2.3

2.1 A wheel of radius 0.5 m spins at 8 rad s^{-1} . Find the rim speed. [2]

2.2 State how the speed of a point on a spinning disc depends on its distance from the axis. [1]

2.3 A point at $r = 2 \text{ m}$ has angular acceleration 3 rad s^{-2} . Find its tangential acceleration. [2]

Exercises 2.1-2.3

2.1 A wheel of radius 0.5 m spins at 8 rad s^{-1} . Find the rim speed. [2]

2.1

- $v = \omega r = 8 \times 0.5 = 4 \text{ m s}^{-1}$

2.2 State how the speed of a point on a spinning disc depends on its distance from the axis. [1]

2.2

- It is proportional to the distance from the axis (farther is faster)

2.3 A point at $r = 2 \text{ m}$ has angular acceleration 3 rad s^{-2} . Find its tangential acceleration. [2]

2.3

- $a = \alpha r = 3 \times 2 = 6 \text{ m s}^{-2}$

Exercises 2.4·3.1

2.4 A rim moves at 6 m s^{-1} on a wheel of radius 0.2 m . Find the angular velocity. [2]

3.1 The relationship between rim speed and angular speed is [1]

☐

A $v = \omega/r$

B $v = \omega r$

C $v = r/\omega$

D $v = \omega r^2$

Exercises 2.4·3.1

2.4 A rim moves at 6 m s^{-1} on a wheel of radius 0.2 m . Find the angular velocity. [2]

2.4

- $\omega = v/r = 6/0.2 = 30 \text{ rad s}^{-1}$

3.1 The relationship between rim speed and angular speed is [1]

☐

A $v = \omega/r$

B $v = \omega r$

C $v = r/\omega$

D $v = \omega r^2$

3.1 B

- $v = \omega r$

Exercise 3.2

3.2 Two points on a spinning disc: A is near the centre, B is at the rim. Compared with A, point B moves [1]

☐

A slower

B faster

C at the same speed

D in the opposite direction

Exercise 3.2

3.2 Two points on a spinning disc: A is near the centre, B is at the rim. Compared with A, point B moves [1]

☐

A slower

B faster

C at the same speed

D in the opposite direction

3.2 B

- the rim (larger r) moves faster

Exercise 3.3 (a)

3.3 A bicycle wheel of radius 0.35 m rolls so its rim moves at 7 m s^{-1} .

(a) Find its angular velocity.

[2]

Exercise 3.3 (a)

3.3 A bicycle wheel of radius 0.35 m rolls so its rim moves at 7 m s^{-1} .

(a) Find its angular velocity.

[2]

(a)

- $\omega = v/r = 7/0.35 = 20 \text{ rad s}^{-1}$

Exercise 3.3 (b)

3.3 A bicycle wheel of radius 0.35 m rolls so its rim moves at 7 m s^{-1} .

(b) State the speed of the point where it touches the ground.

[1]

Exercise 3.3 (b)

3.3 A bicycle wheel of radius 0.35 m rolls so its rim moves at 7 m s^{-1} .

(b) State the speed of the point where it touches the ground.

[1]

(b)

- Zero – the contact point is instantaneously at rest

Exercise 3.4 (a)

3.4 A grinding wheel of radius 0.1 m has angular acceleration 20 rad s^{-2} .

(a) Find the tangential acceleration of a point on the rim.

[2]

Exercise 3.4 (a)

3.4 A grinding wheel of radius 0.1 m has angular acceleration 20 rad s^{-2} .

(a) Find the tangential acceleration of a point on the rim.

[2]

(a)

- $a = \alpha r = 20 \times 0.1 = 2 \text{ m s}^{-2}$

Exercise 3.4 (b)

3.4 A grinding wheel of radius 0.1 m has angular acceleration 20 rad s^{-2} .

(b) State whether a point halfway to the centre has more or less tangential acceleration.^[1]

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(b)

- Less (smaller r)