

Exercise walk-through

5.1

Rotational Kinematics

AP Physics C: Mechanics

Angular rates and signs

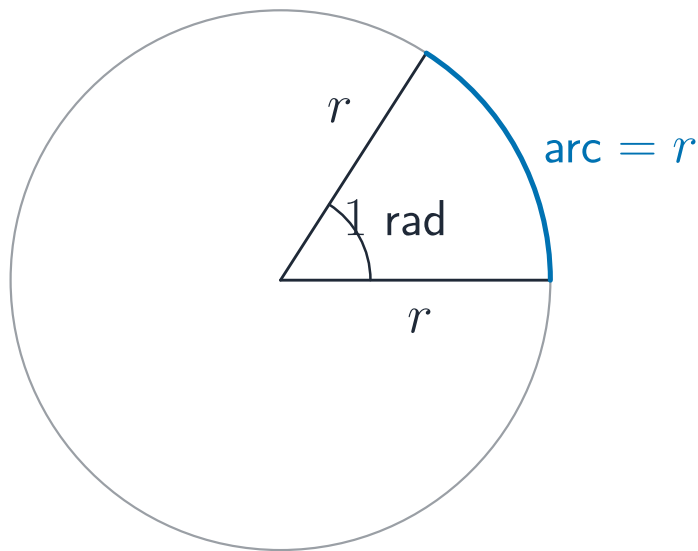
$$\Delta\theta = \theta - \theta_0, \quad \omega = \frac{d\theta}{dt}, \quad \alpha = \frac{d\omega}{dt}$$

Choose one fixed axis and a positive sense. A **rigid body** holds its shape; points share angular motion about that axis.

Angular displacement uses **radians** ; **angular velocity** and **angular acceleration** can have opposite signs while the body slows.

Kinematic directions here use clockwise/counterclockwise; torque-vector directions are considered later.

Radians connect angle and arc



one **radian**: the angle whose arc is exactly one radius long

$\pi \text{ rad} = 180^\circ$, so $1 \text{ rad} \approx 57.3^\circ$

degrees $\rightarrow$ radians: $\times \frac{\pi}{180}$

radians $\rightarrow$ degrees: $\times \frac{180}{\pi}$

One revolution is 2π radians. Use radians in the arc-length and linear-angular relations.

Constant angular acceleration: choose the model

$$\omega = \omega_0 + \alpha t$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

These equations require constant α . The final equation loses the sign of ω when squared: use the physical motion to choose a sign.

Stopping fan: original attempt

A fan rotates at 30 rad/s . It then slows with constant angular acceleration -6.0 rad/s^2 until it first stops.

Find the stopping time and angular displacement. Convert the displacement to revolutions.

Does the model establish what happens after rest?

Stopping fan: time and displacement

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{\omega - \omega_0}{\alpha}$$

$$t = \frac{0 - 30 \text{ rad/s}}{-6.0 \text{ rad/s}^2} = 5.0 \text{ s}$$

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\Delta\theta = (30 \text{ rad/s})(5.0 \text{ s}) + \frac{1}{2}(-6.0 \text{ rad/s}^2)(5.0 \text{ s})^2 = 75 \text{ rad}$$

About $75/(2\pi) = 12$ revolutions. The specified slowing phase ends at rest; subsequent motion needs a continued torque model.

Exercises 2.1-2.3

2.1 A disc spins at a constant 6 rad s^{-1} . State its angular acceleration. [1]

2.2 A wheel starts from rest with $\alpha = 2 \text{ rad s}^{-2}$. Find ω after 5 s. [2]

2.3 How many radians are in one full revolution? [1]

Exercises 2.1·2.3

2.1 A disc spins at a constant 6 rad s^{-1} . State its angular acceleration. [1]

2.1

- Zero (constant ω)
-

2.2 A wheel starts from rest with $\alpha = 2 \text{ rad s}^{-2}$. Find ω after 5 s. [2]

2.2

- $\omega = \omega_0 + \alpha t = 0 + 2(5) = 10 \text{ rad s}^{-1}$
-

2.3 How many radians are in one full revolution? [1]

2.3

- $2\pi \text{ rad}$

Exercises 2.4·3.1

2.4 A flywheel at 10 rad s^{-1} decelerates at 2 rad s^{-2} . Find ω after 3 s. [2]

3.1 The rotational analogue of linear acceleration is [1]

☐

A angular velocity

B angular acceleration

C torque

D angular momentum

Exercises 2.4·3.1

2.4 A flywheel at 10 rad s^{-1} decelerates at 2 rad s^{-2} . Find ω after 3 s. [2]

2.4

- $\omega = 10 + (-2)(3) = 4 \text{ rad s}^{-1}$

3.1 The rotational analogue of linear acceleration is [1]

☐

A angular velocity

B angular acceleration

C torque

D angular momentum

3.1 B

- angular acceleration

Exercise 3.2

3.2 A wheel starts from rest and reaches 20 rad s^{-1} in 4 s. Its angular acceleration is [1]

☐

A 5 rad s^{-2}

B 80 rad s^{-2}

C 16 rad s^{-2}

D 0.2 rad s^{-2}

Exercise 3.2

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☐

A 5 rad s^{-2}

B 80 rad s^{-2}

C 16 rad s^{-2}

D 0.2 rad s^{-2}

3.2 A

- $\alpha = 20/4 = 5 \text{ rad s}^{-2}$

Exercise 3.3 (a)

3.3 A turntable starts from rest with $\alpha = 4 \text{ rad s}^{-2}$.

(a) Find its angular velocity after 3 s.

[2]

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3.3 A turntable starts from rest with $\alpha = 4 \text{ rad s}^{-2}$.

(a) Find its angular velocity after 3 s.

[2]

(a)

- $\omega = 4(3) = 12 \text{ rad s}^{-1}$

Exercise 3.3 (b)

3.3 A turntable starts from rest with $\alpha = 4 \text{ rad s}^{-2}$.

(b) Find the angle turned in that time.

[2]

Exercise 3.3 (b)

3.3 A turntable starts from rest with $\alpha = 4 \text{ rad s}^{-2}$.

(b) Find the angle turned in that time.

[2]

(b)

- $\theta = \frac{1}{2}(4)(3)^2 = 18 \text{ rad}$

Exercise 3.4 (a)

3.4 A fan blade slows from 30 rad s^{-1} to rest in 6 s at constant deceleration.

(a) Find the angular acceleration.

[2]

Exercise 3.4 (a)

3.4 A fan blade slows from 30 rad s^{-1} to rest in 6 s at constant deceleration.

(a) Find the angular acceleration.

[2]

(a)

- $\alpha = (0 - 30)/6 = -5 \text{ rad s}^{-2}$

Exercise 3.4 (b)

3.4 A fan blade slows from 30 rad s^{-1} to rest in 6 s at constant deceleration.

(b) State its sign and what it means.

[1]

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3.4 A fan blade slows from 30 rad s^{-1} to rest in 6 s at constant deceleration.

(b) State its sign and what it means.

[1]

(b)

- Negative – it is decelerating (slowing the spin)