

Exercise walk-through

3.3

Potential Energy

AP Physics C: Mechanics

Potential belongs to an interaction

$$\Delta U = - \int_a^b \vec{F}_c \cdot d\vec{r}$$

Potential energy describes configuration. **Conservative forces** have path-independent work and an associated potential; their work around a closed path is zero.

Choose the zero of U for convenience. Adding a constant changes neither the force nor energy differences. For multiple interacting objects, count each pair once.

Three useful potentials

Interaction	Relation and assumption
Near-surface gravity	$\Delta U_g = mg\Delta y$ with approximately constant g .
Spherical masses	$U_g = -Gm_1m_2/r$ with zero at infinite separation.
Ideal spring	$U_s = \frac{1}{2}kx^2$, with x measured from natural length.
System choice	Include Earth or the spring when accounting for the corresponding interaction potential.

Force from a potential-energy graph

$$F_x = -\frac{dU}{dx}$$

A **derivative** gives slope: positive slope means negative force; negative slope means positive force.

Zero slope gives zero force at that position, not necessarily zero speed. A local minimum gives **stable equilibrium**; a local maximum gives **unstable equilibrium** in the usual smooth one-dimensional cases.

Cubic potential: original attempt

$$U(x) = ax^3 - bx$$

$a = 2.0 \text{ J/m}^3$, $b = 6.0 \text{ J/m}$. Derive the force, find equilibrium positions and classify their stability.

Find the force at $x = 0$. Why is the particle's speed not determined by the slope alone?

Cubic potential: differentiate and solve

$$F_x = -\frac{dU}{dx} = b - 3ax^2$$

$$F_x = 0 \quad \Rightarrow \quad x_{\text{eq}} = \pm \sqrt{\frac{b}{3a}}$$

$$x_{\text{eq}} = \pm \sqrt{\frac{6.0 \text{ J/m}}{3(2.0 \text{ J/m}^3)}} = \pm 1.0 \text{ m}$$

These positions give zero force. The energy and initial conditions still determine whether the particle passes them with nonzero speed.

Exercises 2.1-2.3

2.1 A 5 kg mass is at height 4 m ($g = 10 \text{ m s}^{-2}$). Find its gravitational PE. [2]

2.2 A spring ($k = 200 \text{ N m}^{-1}$) is stretched 0.2 m. Find its elastic PE. [2]

2.3 State what kind of energy a raised object stores. [1]

Exercises 2.1·2.3

2.1 A 5 kg mass is at height 4 m ($g = 10 \text{ m s}^{-2}$). Find its gravitational PE. [2]

2.1

- $U = mgh = 5(10)(4) = 200 \text{ J}$

2.2 A spring ($k = 200 \text{ N m}^{-1}$) is stretched 0.2 m. Find its elastic PE. [2]

2.2

- $U = \frac{1}{2}kx^2 = \frac{1}{2}(200)(0.2)^2 = 4 \text{ J}$

2.3 State what kind of energy a raised object stores. [1]

2.3

- Gravitational potential energy

Exercises 2.4·3.1

2.4 A 3 kg box is lifted from 1 m to 4 m ($g = 10 \text{ m s}^{-2}$). Find the change in PE. [2]

3.1 Gravitational potential energy near Earth's surface is [1]

☐

A $\frac{1}{2}mv^2$

B mgh

C $\frac{1}{2}kx^2$

D Fd

Exercises 2.4·3.1

2.4 A 3 kg box is lifted from 1 m to 4 m ($g = 10 \text{ m s}^{-2}$). Find the change in PE. [2]

2.4

- $\Delta U = mg(h_2 - h_1) = 3(10)(4 - 1) = 90 \text{ J}$

3.1 Gravitational potential energy near Earth's surface is [1]

☐

A $\frac{1}{2}mv^2$

B mgh

C $\frac{1}{2}kx^2$

D Fd

3.1 B

- $U = mgh$

Exercise 3.2

3.2 Doubling the compression of a spring changes its stored energy by a factor of [1]

☐

A 2

B 4

C $1/2$

D 1

Exercise 3.2

3.2 Doubling the compression of a spring changes its stored energy by a factor of [1]

☐

A 2

B 4

C $1/2$

D 1

3.2 B

- $U \propto x^2$, so doubling gives $2^2 = 4$

Exercise 3.3 (a)

3.3 A spring with $k = 500 \text{ N m}^{-1}$ is compressed 0.06 m.

(a) Find the elastic potential energy stored.

[2]

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(a) Find the elastic potential energy stored.

[2]

(a)

- $U = \frac{1}{2}(500)(0.06)^2 = 0.9 \text{ J}$

Exercise 3.3 (b)

3.3 A spring with $k = 500 \text{ N m}^{-1}$ is compressed 0.06 m.

(b) State what happens to this energy if the spring is released to push a block. [1]

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(b) State what happens to this energy if the spring is released to push a block. [1]

(b)

- It converts into the block's kinetic energy

Exercise 3.4 (a)

3.4 A 10 kg crate is raised 2.5 m ($g = 9.8 \text{ m s}^{-2}$).

(a) Find the gain in gravitational potential energy.

[2]

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(a) Find the gain in gravitational potential energy.

[2]

(a)

- $\Delta U = 10(9.8)(2.5) = 245 \text{ J}$

Exercise 3.4 (b)

3.4 A 10 kg crate is raised 2.5 m ($g = 9.8 \text{ m s}^{-2}$).

(b) State the minimum work needed to lift it.

[1]

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[1]

(b)

- 245 J