

Exercise walk-through

2.8

Spring Forces

AP Physics C: Mechanics

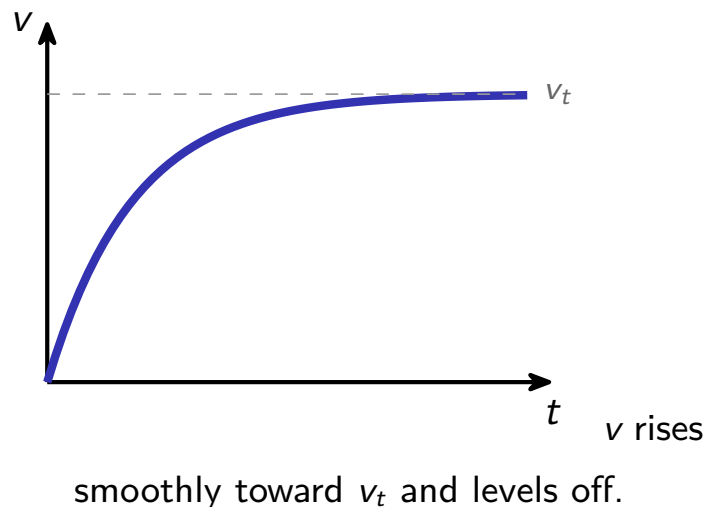
Resistive forces and terminal velocity

Drag grows with speed ($F = -bv$), so Newton's law becomes a **differential equation** :

Falling with drag

$$m \frac{dv}{dt} = mg - bv$$

At **terminal velocity** , $\frac{dv}{dt} = 0$, so $v_t = \frac{mg}{b}$.



Separating the variables, and why you must

the problem

with drag, $m \frac{dv}{dt} = mg - bv$ – the acceleration depends on v ,
so no SUVAT equation applies

separate the variables

$$\int_0^v \frac{dv'}{mg - bv'} = \int_0^t \frac{dt'}{m}$$

the result

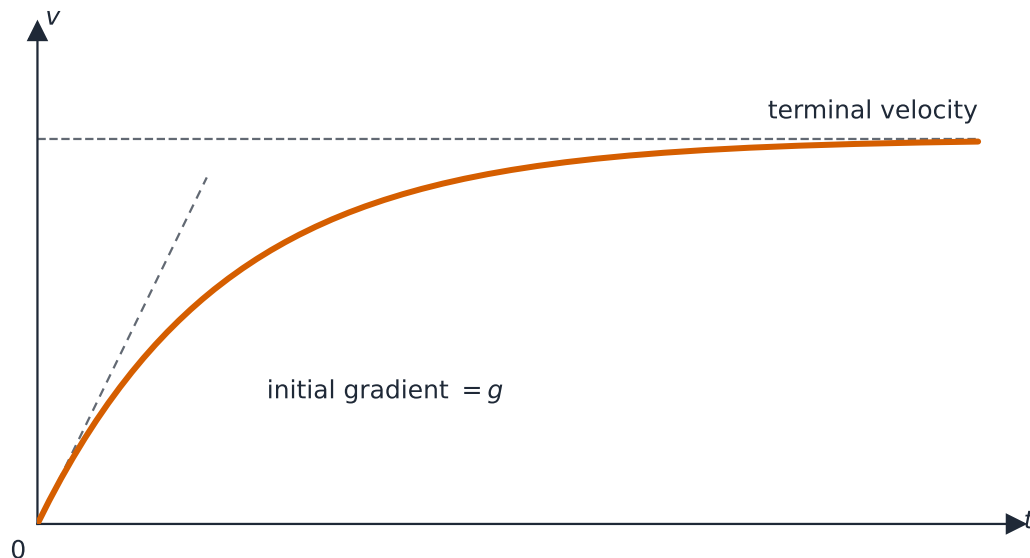
$$v(t) = \frac{mg}{b} (1 - e^{-bt/m})$$

terminal velocity

set $dv/dt = 0$: $v_T = \frac{mg}{b}$ – reachable without solving the equation at all

If a question asks only for the **terminal** value, set the acceleration to zero and stop. Integrate only when it asks for $v(t)$.

An object falling through a fluid speeds



An object falling through a fluid speeds up to a terminal velocity

Exercises 2.1-2.3

2.1 A spring with $k = 100 \text{ N m}^{-1}$ is stretched 0.2 m. Find the force. [2]

2.2 State the meaning of the minus sign in $F = -kx$. [1]

2.3 A force of 12 N stretches a spring 0.03 m. Find k . [2]

Exercises 2.1-2.3

2.1 A spring with $k = 100 \text{ N m}^{-1}$ is stretched 0.2 m. Find the force. [2]

2.1

- $F = kx = 100 \times 0.2 = 20 \text{ N}$

2.2 State the meaning of the minus sign in $F = -kx$. [1]

2.2

- The force opposes the displacement (points back toward the natural length)

2.3 A force of 12 N stretches a spring 0.03 m. Find k . [2]

2.3

- $k = F/x = 12/0.03 = 400 \text{ N m}^{-1}$

Exercises 2.4·3.1

2.4 Two springs stretch the same distance; spring A needs more force. Which has the larger k ? [1]

3.1 The magnitude of a spring's force is proportional to [1]

☐

A the mass on it

B its displacement from the natural length

C the square of the displacement

D time

Exercises 2.4-3.1

2.4 Two springs stretch the same distance; spring A needs more force. Which has the larger k ? [1]

2.4

- Spring A (more force for the same stretch means a larger k)

3.1 The magnitude of a spring's force is proportional to [1]

☐

A the mass on it

B its displacement from the natural length

C the square of the displacement

D time

3.1 B

- $F = kx$, proportional to displacement

Exercise 3.2

3.2 A spring force always points

[1]

☐

- A** in the direction of the stretch
- B** back toward the natural length
- C** downward
- D** perpendicular to the spring

Exercise 3.2

3.2 A spring force always points

[1]

☐

A in the direction of the stretch

B back toward the natural length

C downward

D perpendicular to the spring

3.2 B

- the restoring force points back toward the natural length

Exercise 3.3 (a)

3.3 A spring with $k = 500 \text{ N m}^{-1}$ is compressed 0.04 m.

(a) Find the size of the spring force.

[2]

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3.3 A spring with $k = 500 \text{ N m}^{-1}$ is compressed 0.04 m.

(a) Find the size of the spring force.

[2]

(a)

- $F = kx = 500 \times 0.04 = 20 \text{ N}$

Exercise 3.3 (b)

3.3 A spring with $k = 500 \text{ N m}^{-1}$ is compressed 0.04 m.

(b) State the direction of the force relative to the compression.

[1]

Exercise 3.3 (b)

3.3 A spring with $k = 500 \text{ N m}^{-1}$ is compressed 0.04 m.

(b) State the direction of the force relative to the compression.

[1]

(b)

- Opposite to the compression (pushing outward)

Exercise 3.4 (a)

3.4 A 2 kg mass hangs at rest from a spring, stretching it 0.1 m ($g = 10 \text{ m s}^{-2}$).

(a) State the spring force (it balances the weight).

[1]

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[1]

(a)

- $F = mg = 20 \text{ N}$

Exercise 3.4 (b)

3.4 A 2 kg mass hangs at rest from a spring, stretching it 0.1 m ($g = 10 \text{ m s}^{-2}$).

(b) Find the spring constant k .

[2]

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3.4 A 2 kg mass hangs at rest from a spring, stretching it 0.1 m ($g = 10 \text{ m s}^{-2}$).

(b) Find the spring constant k .

[2]

(b)

- $k = F/x = 20/0.1 = 200 \text{ N m}^{-1}$