HOLIDAY HOMEWORK 假期作业

AP Physics C: Mechanics

Exercise sheets and past-paper practice, topic by topic.
每个单元：练习卷 + 历年真题。

For class or self-study • no answer keys included.

Contents 目录

1 Describing motion and reading graphs – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
Kinematics	运动学			
scalar	标量			
magnitude	大小			
vector	矢量			
Distance	距离			
Displacement	位移			
Speed	速率			
Velocity	速度			
Acceleration	加速度			
deceleration	减速			
gradient	斜率			

Recall

You met motion long before AP. Bring it with you:

- You measured **distance** with a ruler and **time** with a clock, and found **speed** from speed = distance ÷ time.

- You described a moving car, a falling ball, a running friend – how fast each one goes.
- At AP we look more carefully at **direction** (not just how fast, but which way) and at how motion **changes**.

Nothing on this sheet is harder than what you already know. It just lines up the ideas the AP course starts with.

New ideas

Read these first. Each idea has a short worked example. Once you have read them you can do the Practice below.

1 Distance and displacement

Kinematics 运动学 is the study of how things move. It uses two kinds of quantity.

A **scalar** 标量 has only size (its **magnitude** 大小): distance, speed, time, mass. A **vector** 矢量 has size **and** direction: displacement, velocity, acceleration.

Distance 距离 is the whole path length you travel – it only grows. **Displacement** 位移 is the straight line from start to finish, together with a direction.

Worked example. Walk 3 m east, then 1 m back west. The distance is $3 + 1 = 4$ m, but the displacement is only 2 m east – how far you ended from where you began.

2 Speed and velocity

Speed 速率 tells you how fast, as a scalar. **Velocity** 速度 tells you how fast **and** which way, so it is a vector.

In one dimension, direction is just a **sign**. First choose which way is positive. Then a velocity of -5 m/s means 5 m/s in the negative direction – it does **not** mean "slow."

Worked example. A car covers 100 m in 5 s in the $+x$ direction, so its velocity is $\frac{100}{5} = +20$ m/s. A car going the other way at the same rate has velocity -20 m/s.

3 Acceleration

Acceleration 加速度 is how quickly velocity changes: $a = \frac{\Delta v}{\Delta t}$ (change in velocity ÷ time taken). Its unit is m/s^2.

An object speeds up when velocity and acceleration point the **same** way, and slows down (**deceleration** 减速) when they point **opposite** ways.

Worked example. A bike speeds up from 2 m/s to 8 m/s in 3 s: $a = \frac{8-2}{3} = 2$ m/s^2.

4 Position–time graphs

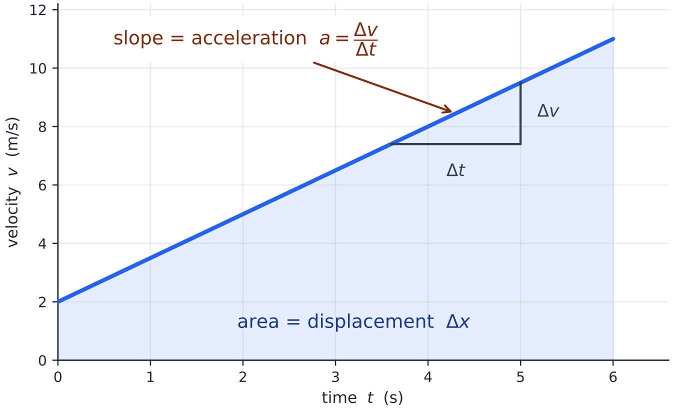
A graph shows motion at a glance. On a **position–time** graph, the **gradient** 斜率 (steepness) is the velocity.

- A flat line means the object is not moving.

- A straight slope means constant velocity (steeper = faster).
- A curve means the velocity is changing.

5 Velocity–time graphs

On a **velocity–time** graph, two useful things are hidden in the shape:



- The gradient (slope) is the acceleration.
- The **area** under the line is the displacement.

Worked example. A car holds 6 m/s for 4 s. On the velocity–time graph this is a rectangle, so the displacement is $6 \times 4 = 24$ m.

Watch out: a negative velocity does not mean slow – it means moving in the negative direction. Speed is the size of the velocity, so it is always positive.

Practice

Try these in order. They get a little harder. The methods are all above.

2.1 A car travels 6 km north, then 2 km south. Find (a) the distance travelled, (b) the displacement. [2]

2.2 State whether each quantity is a scalar or a vector: (a) speed, (b) velocity, (c) distance, (d) acceleration. [2]

2.3 A runner covers 200 m in 25 s. Calculate the average speed. [1]

2.4 A velocity is written as −8 m/s. Explain what the minus sign tells you. [1]

2.5 A train speeds up from 5 m/s to 20 m/s in 6 s. Calculate its acceleration. [2]

2.6 A velocity–time graph shows a constant 12 m/s for 8 s. Find the displacement. [2]

2.7 On a position–time graph, describe the motion shown by (a) a flat horizontal line, (b) a straight upward slope. [2]

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \mathrm{m/s^2}$, $L = 0.25\ \mathrm{m/s^4}$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

(b) Calculate the speed of the rocket at $t = 6.0$ s.

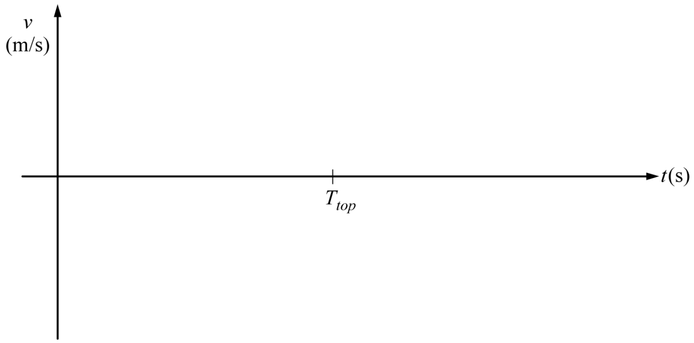
(c)

i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

(d) Calculate the maximum height reached by the rocket relative to its launching point.

(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



1 Equations of motion, free fall, and 2D motion – Part 2

Name: ________ Class: ________ Date: ________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
air resistance	空气阻力			
acceleration due to gravity	重力加速度			
free fall	自由落体			
components	分量			
projectile	抛体			

Recall

In Part A you learned to describe motion and read graphs. Now bring two more tools:

- When velocity is constant, velocity = displacement ÷ time.
- When something speeds up steadily, you need equations that include **acceleration** – that is this sheet.
- From maths you know right-angled triangles (Pythagoras, sine, cosine). You will use them to split motion into two directions.

Take it slowly. Each idea has a worked example before the Practice.

New ideas

1 The constant-acceleration equations

When acceleration is **constant** (steady), four equations link five quantities: start velocity v_0, final velocity v, acceleration a, time t, and displacement Δx.

$$v = v_0 + at, \qquad \Delta x = v_0 t + \tfrac{1}{2}at^2, \qquad v^2 = v_0^2 + 2a\,\Delta x, \qquad \Delta x = \tfrac{1}{2}(v_0 + v)t.$$

The trick: list what you know, then pick the equation that holds your three knowns plus the one you want, so only one unknown is left.

Worked example. A car starts from rest ($v_0 = 0$) and accelerates at 2.0 m/s^2 for 6.0 s. Final velocity: $v = v_0 + at = 0 + 2.0 \times 6.0 = 12$ m/s. Distance: $\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.0 \times 6.0^2 = 36$ m.

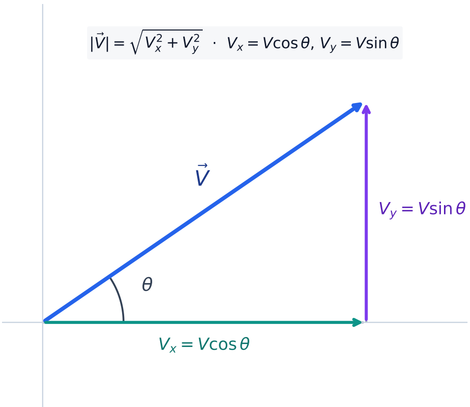
2 Free fall

Near Earth, if we ignore **air resistance** 空气阻力, everything falls with the same **acceleration due to gravity** 重力加速度 $g = 9.8$ m/s^2, pointing down. This is **free fall** 自由落体.

Worked example. A stone is dropped ($v_0 = 0$) and falls for 2.0 s. Taking down as positive: speed $v = gt = 9.8 \times 2.0 = 20$ m/s, and distance $\Delta x = \frac{1}{2}gt^2 = \frac{1}{2} \times 9.8 \times 2.0^2 = 20$ m.

3 Motion in two directions – components

A vector at an angle splits into two **components** 分量 at right angles: a horizontal part and a vertical part.



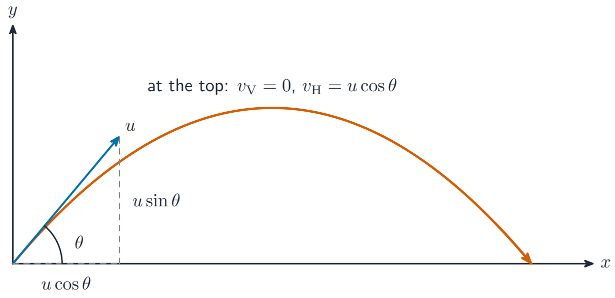
For a velocity v at angle θ above the horizontal:

$$v_x = v\cos\theta, \qquad v_y = v\sin\theta.$$

Worked example. A ball leaves at 10 m/s, 30° above the horizontal: $v_x = 10\cos 30° = 8.7$ m/s, and $v_y = 10\sin 30° = 5.0$ m/s.

4 Projectile motion

A **projectile** 抛体 moves under gravity alone. The big idea: its horizontal and vertical motions are **independent** and share only the **time**.



- Horizontal: no acceleration, so the horizontal velocity stays constant.
- Vertical: acceleration g downward, exactly like free fall.

So a ball dropped and a ball thrown sideways from the same height reach the ground **at the same moment** – their vertical motions are identical.

Watch out: the two directions share only the time. Never mix a horizontal distance with a vertical velocity in one equation – solve each direction on its own, then join them through t.

Practice

Try these in order. The methods are all above.

2.1 A car accelerates from rest at 3.0 m/s^2 for 5.0 s. Find its final velocity. [2]

2.2 Using the same car, find the distance it travels in those 5.0 s. [2]

2.3 A ball is dropped from rest. Taking $g = 9.8\ \text{m/s}^2$ and down as positive, find its speed after 1.5 s. [2]

2.4 A velocity of 12 m/s points 60° above the horizontal. Find its horizontal and vertical components. [2]

2.5 A stone is thrown horizontally from a cliff at the same moment another is dropped. Explain why they hit the ground at the same time. [2]

2.6 A motorbike slows from 20 m/s to 8 m/s while covering 42 m. Use $v^2 = v_0^2 + 2a\,\Delta x$ to find its acceleration. [3]

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

(b) Calculate the speed of the rocket at $t = 6.0$ s.

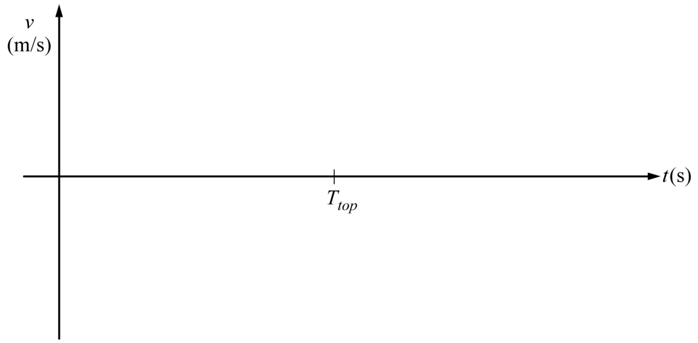
(c)

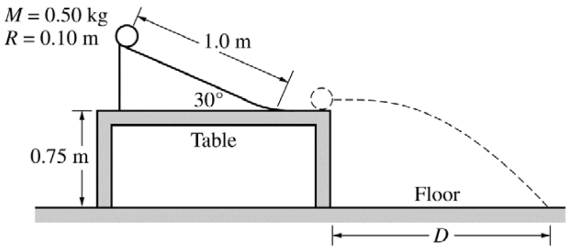
i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

(d) Calculate the maximum height reached by the rocket relative to its launching point.

(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.





3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM

2 Forces and Newton's laws – Part 1

Name: __________ Class: ________ Date: _________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
force	力	
free-body diagram	受力图	
Weight	重力	
Newton's first law	牛顿第一定律	
inertia	惯性	
net force	合力	
equilibrium	平衡	
Newton's second law	牛顿第二定律	
Newton's third law	牛顿第三定律	

Recall

You have met forces before. Bring these ideas with you:

- A **force** is a push or a pull, measured in newtons (N).
- Gravity pulls things down; a table pushes up; a rope pulls along its length.
- You know that a heavier object is harder to get moving. At AP we make this exact with **Newton's laws**.

Nothing here is harder than what you already know – we just line it up carefully and draw good diagrams.

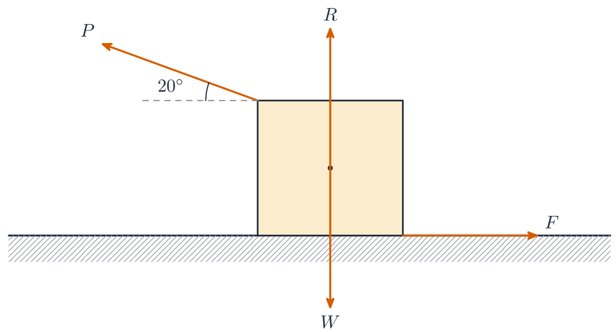
New ideas

Read these first. Each idea has a short worked example. Then do the Practice below.

1 Force and the free-body diagram

A **force** 力 is a push or pull. It is a vector, so it has a size **and** a direction.

To solve any force problem, first draw a **free-body diagram** 受力图: show the object as a dot, then draw an arrow for **every** force acting **on** it – weight down, the surface's push, any rope or hand.



Two rules keep it honest: draw only forces acting **on** your object (not forces it pushes on other things), and draw only **real** forces (a rope, a surface, gravity) – never an "ma" arrow.

2 Weight

Weight 重力 is the force of gravity on an object: $F_g = mg$, pointing down. Here m is the mass and $g = 9.8\ \text{m/s}^2$ near Earth.

Worked example. A 2.0 kg bag has weight $F_g = mg = 2.0 \times 9.8 = 19.6$ N. On the Moon ($g = 1.6\ \text{m/s}^2$) the same bag weighs only $2.0 \times 1.6 = 3.2$ N – but its **mass** is still 2.0 kg.

3 Newton's first law

Newton's first law 牛顿第一定律 (the law of **inertia** 惯性): an object keeps a constant velocity unless a **net force** 合力 acts on it. So if the forces balance to zero, the object stays at rest or keeps moving at steady speed – it is in **equilibrium** 平衡.

Worked example. A car moves at a steady 25 m/s. Because the velocity is constant, the net force is zero – the engine's push exactly balances friction and air drag.

■ 4 Newton's second law

When the net force is **not** zero, the object accelerates: **Newton's second law** 牛顿第二定律 says

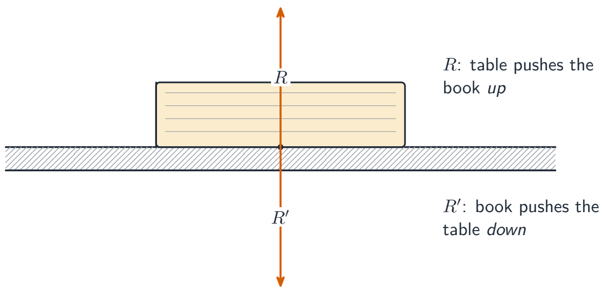
$$a = \frac{\sum F}{m}, \qquad \text{so} \qquad \sum F = ma.$$

The acceleration points the same way as the net force.

Worked example. A 4.0 kg box is pulled with 18 N while friction pulls back 6.0 N. The net force is $18 - 6.0 = 12$ N, so $a = \frac{12}{4.0} = 3.0\ \text{m/s}^2$.

■ 5 Newton's third law

Newton's third law 牛顿第三定律: if A pushes on B, then B pushes back on A with an **equal and opposite** force.



The two forces act on **different** objects, so they never cancel.

Watch out: a book's weight and the table's push on the book are **not** a third-law pair – they act on the **same** object (the book). A third-law pair is always "A on B" with "B on A."

Practice

Try these in order. The methods are all above.

2.1 State the size and direction of the weight of a 3.0 kg book ($g = 9.8\ \text{m/s}^2$). [2]

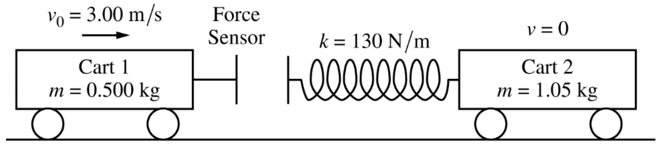
2.2 A box rests on a table. Name the two forces on the box and state their directions. [2]

2.3 A 6.0 kg trolley is pushed with a net force of 18 N. Find its acceleration. [2]

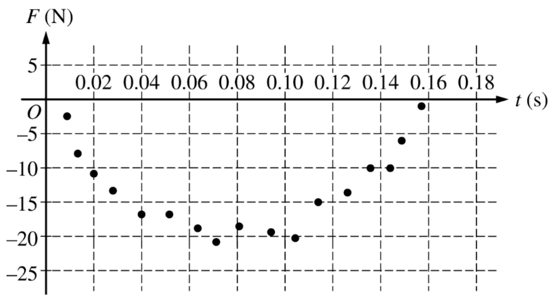
2.4 A lift moves upward at a **constant** speed. What is the net force on it? Explain. [2]

2.5 A 2.0 kg ball is pulled by a 10 N force while a 4.0 N friction force opposes it. Find the acceleration. [2]

2.6 A student says "the ground pushes up on my feet, and my feet push down on the ground – these cancel out, so I feel nothing." State what is wrong with this reasoning. [2]



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

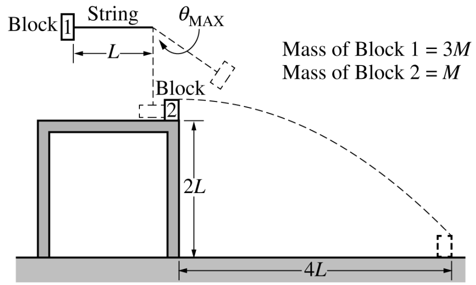
i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

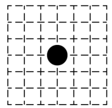


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

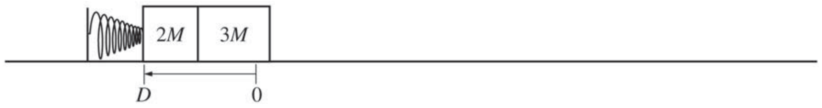
(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$ Block of Mass $3M$



The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

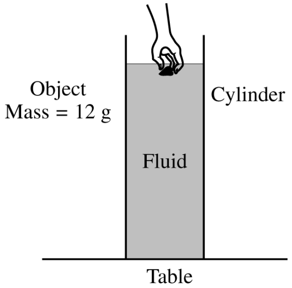
(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

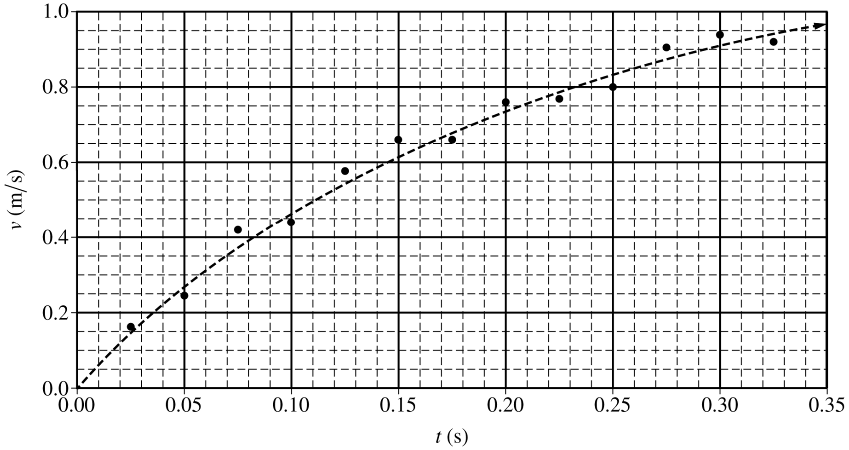
Justify your answer.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of $m = 12$ g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at $t = 0.20$ s.

The students use the equation $v = A\left(1 - e^{-Bt}\right)$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5\ \text{s}^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement $y(t)$ of the falling object as a function of time t .

ii. Derive an expression for the magnitude of the net force $F(t)$ exerted on the object as it falls through the fluid as a function of time t .

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.

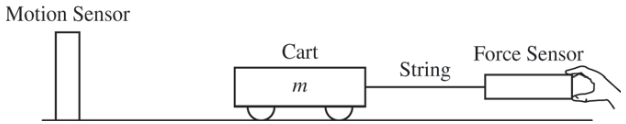
Justify your answer.

ii. Determine the force exerted by the fluid on the object at this time.

Justify your answer.

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech.1.

A cart of mass m is pulled along a level dynamics track as shown above. A force sensor is attached to the cart with a string and used to measure the horizontal force exerted on the cart to the right. A motion sensor is used to measure the acceleration of the cart with the positive direction toward the right. Friction is not negligible.

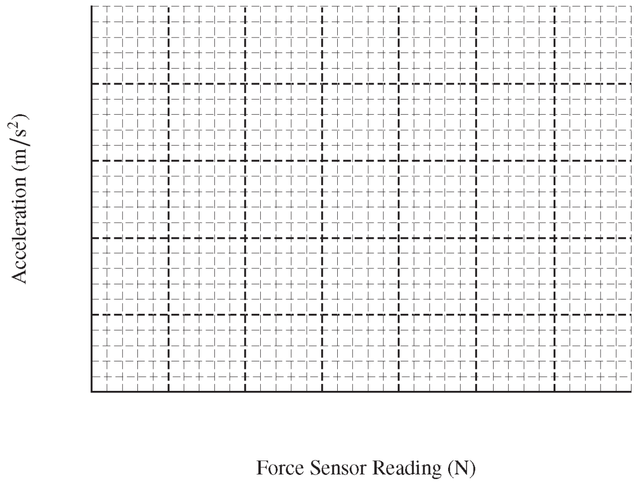
(a) On the dot below, which represents the cart, draw and label the forces (not components) that act on the cart. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

A student pulls the force sensor with a constant force, and the cart accelerates. This is repeated for several trials, with a different constant force used for each trial. The data are recorded in the table below.

Trial	1	2	3	4	5
Force sensor reading (N)	0.32	0.38	0.44	0.50	0.60
Acceleration (m/s^2)	0.12	0.22	0.33	0.50	0.70

(b)

i. On the grid below, plot data points for the acceleration of the cart as a function of the force sensor reading. Clearly scale all axes. Draw a straight line that best represents the data.



ii. Using the straight line from the graph, calculate the mass of the cart.

iii. Using the straight line from the graph, determine the magnitude of the force of friction.

The above experiment is repeated by using a constant force sensor reading of 0.45 N. The cart starts from rest at time $t = 0$ s and is pulled for a time of 2.0 s along the dynamics track.

(c)

i. Determine the acceleration of the cart.

ii. The string breaks at time $t = 2.0$ s. Calculate the time it takes for the cart to stop after the string breaks.

The experiment and analysis in parts (a) and (b) are repeated with a cart that has the same mass but a greater force of friction.

(d)

i. Will the slope of your new line be greater than, less than, or equal to the slope of your line in part (b)i?

____ Greater than ____ Less than ____ Equal to

ii. Will the horizontal intercept of your new line be greater than, less than, or equal to the horizontal intercept of your line in part (b)i?

____ Greater than ____ Less than ____ Equal to

2 Friction, springs, and circular motion – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
normal force	法向力	
Friction	摩擦力	
Static friction	静摩擦	
Kinetic friction	动摩擦	
coefficient of friction	摩擦系数	
Hooke's law	胡克定律	
spring constant	弹簧常数	
centripetal acceleration	向心加速度	
centripetal force	向心力	

Recall

In Part A you drew free-body diagrams and used $\sum F = ma$. Now meet three everyday forces that appear again and again:

- **Friction** slows sliding – you feel it when you drag a heavy box.
- A **spring** pulls back when you stretch it.
- Anything moving in a **circle** needs a force pulling it inward.

Each idea below has a worked example before the Practice.

New ideas

1 The normal force

When an object rests on a surface, the surface pushes back at right angles to it. This is the **normal force** 法向力 N. On a flat floor it balances the weight, so $N = mg$.

Worked example. A 5.0 kg box on a level floor has $N = mg = 5.0 \times 9.8 = 49$ N.

2 Friction

Friction 摩擦力 acts along a surface and opposes sliding. It comes in two kinds:

- **Static friction** 静摩擦 (before sliding) adjusts itself up to a maximum $f_s \le \mu_s N$ to stop motion starting.
- **Kinetic friction** 动摩擦 (while sliding) is $f_k = \mu_k N$.

Here μ is the **coefficient of friction** 摩擦系数 (a number for how rough the contact is).

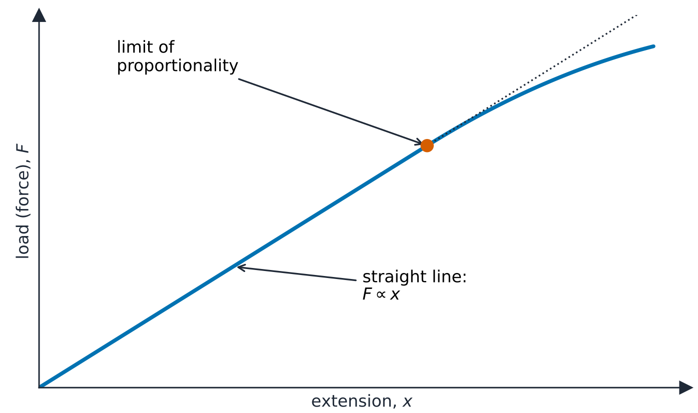
Worked example. A crate with $N = 49$ N and $\mu_s = 0.40$ needs a push above $f_{s,\max} = \mu_s N = 0.40 \times 49 = 19.6$ N to start moving. A 15 N push is too small, so it stays still.

3 Springs and Hooke's law

A spring pulls (or pushes) back toward its natural length. **Hooke's law** 胡克定律 says the force is proportional to the stretch x:

$$F_s = kx,$$

where k is the **spring constant** 弹簧常数 (its stiffness).

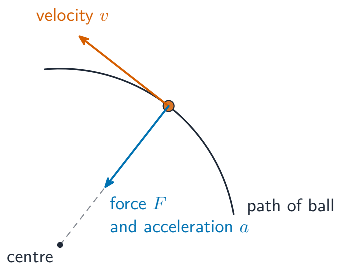


Worked example. A spring with $k = 200$ N/m holds a 0.50 kg mass at rest. The spring force balances the weight, $kx = mg$, so $x = \dfrac{mg}{k} = \dfrac{0.50 \times 9.8}{200} = 0.025$ m $= 2.5$ cm.

■ 4 Circular motion

An object moving in a circle at steady speed is still **accelerating**, because its direction keeps changing. This **centripetal acceleration** 向心加速度 points to the centre:

$$a_c = \frac{v^2}{r}.$$



It is caused by a real inward **centripetal force** 向心力 $F_c = \dfrac{mv^2}{r}$ – supplied by tension, gravity, or friction.

Watch out: there is no outward "centrifugal" force pulling you out of the circle. The only force is the real one pulling you **inward**; you feel pushed out only because your body wants to keep going straight.

Practice

Try these in order. The methods are all above.

2.1 A 4.0 kg block sits on a flat floor. Find the normal force on it ($g = 9.8$ m/s^2). [2]

2.2 Using that block, the coefficient of static friction is $\mu_s = 0.30$. Find the largest static friction force before it slides. [2]

2.3 A spring stretches 0.040 m when a 12 N force pulls it. Find the spring constant k. [2]

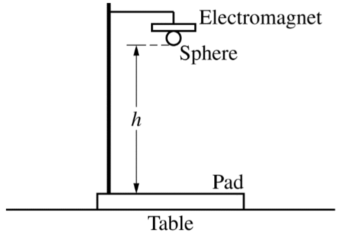
2.4 Explain why an object going around a circle at constant speed is still accelerating. [2]

2.5 A 0.30 kg ball is whirled in a circle of radius 0.80 m at 4.0 m/s. Find the centripetal force needed. [3]

2.6 A student says a car turning a corner is pushed **outward** by a force. Correct this statement. [2]

PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. A student wants to determine the value of the acceleration due to gravity g for a specific location and sets up the following experiment. A solid sphere is held vertically a distance h above a pad by an electromagnet, as shown in the figure above. The experimental equipment is designed to release the sphere when the electromagnet is turned off. A timer also starts when the electromagnet is turned off, and the timer stops when the sphere lands on the pad.

(a) While taking the first data point, the student notices that the electromagnet actually releases the sphere after the timer begins. Would the value of g calculated from this one measurement be greater than, less than, or equal to the actual value of g at the student's location?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

The electromagnet is replaced so that the timer begins when the sphere is released. The student varies the distance h. The student measures and records the time Δt of the fall for each particular height, resulting in the following data table.

h (m)	0.10	0.20	0.60	0.80	1.00
Δt (s)	0.105	0.213	0.342	0.401	0.451

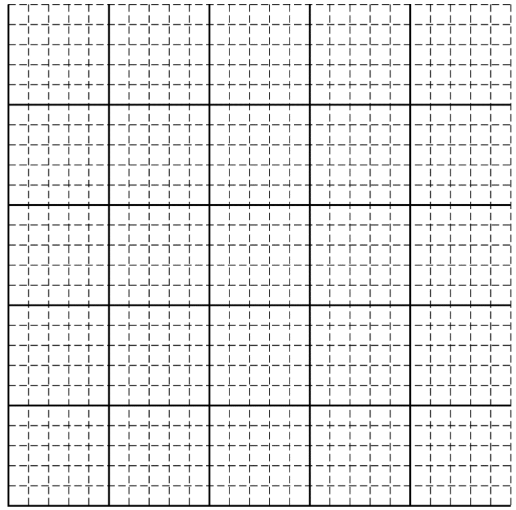
(b) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for g.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining rows in the table above, as needed, to record any quantities that you indicated that are not given in the table. Label each row you use and include units.

(c) Plot the data points for the quantities indicated in part (b) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(d) Using the straight line, calculate an experimental value for g.

Another student fits the data in the table to a quadratic equation. The student's equation for the distance fallen y as a function of time t is $y = At^2 + Bt + C$, where $A = 5.75\ \text{m/s}^2$, $B = -0.524\ \text{m/s}$, and $C = +0.080\ \text{m}$. Vertically down is the positive direction.

(e) Using the student's equation above, do the following.

i. Derive an expression for the velocity of the sphere as a function of time.

ii. Calculate the new experimental value for g.

iii. Using $9.81\ \text{m/s}^2$ as the accepted value for g at this location, calculate the percent error for the value found in part (e)ii.

iv. Assuming the sphere is at a height of 1.40 m at $t = 0$, calculate the velocity of the sphere just before it strikes the pad.

Question 4

4. A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

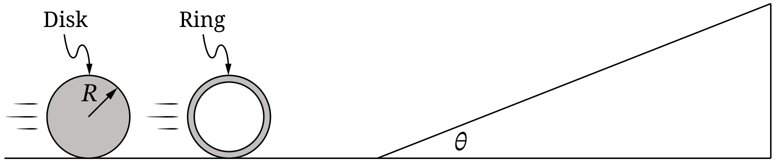


Figure 1

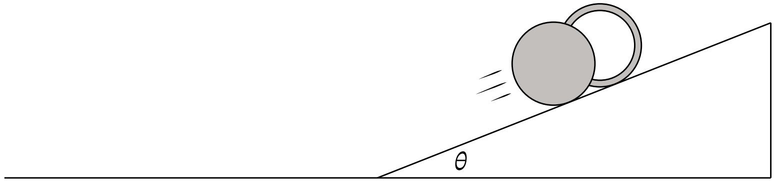


Figure 2

A. While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_D and f_R, respectively.

Indicate whether f_D is greater than, less than, or equal to f_R by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

Justify your answer using qualitative reasoning beyond referencing equations.

B. A cylinder has mass M, radius R, and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.

Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M, R, I, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp.

Indicate whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping.

Briefly **justify** your answer.

STOP

END OF EXAM

Mech.2.

A block of mass $2M$ rests on a horizontal, frictionless table and is attached to a relaxed spring, as shown in the figure above. The spring is nonlinear and exerts a force $F(x) = -Bx^3$, where B is a positive constant and x is the displacement from equilibrium for the spring. A block of mass $3M$ and initial speed v_0 is moving to the left as shown.

(a) On the dots below, which represent the blocks of mass $2M$ and $3M$, draw and label the forces (not components) that act on each block before they collide. Each force must be represented by a distinct arrow starting on, and pointing away from, the appropriate dot.

Block of Mass $2M$ Block of Mass $3M$

The two blocks collide and stick to each other. The two-block system then compresses the spring a maximum distance D, as shown above. Express your answers to parts (b), (c), and (d) in terms of M, B, v_0, and physical constants, as appropriate.

(b) Derive an expression for the speed of the blocks immediately after the collision.

(c) Determine an expression for the kinetic energy of the two-block system immediately after the collision.

(d) Derive an expression for the maximum distance D that the spring is compressed.

(e)

i. In which direction is the net force, if any, on the block of mass $2M$ when the spring is at maximum compression?

____ Left ____ Right ____ The net force on the block of mass $2M$ is zero.

Justify your answer.

ii. Which of the following correctly describes the magnitude of the net force on each of the two blocks when the spring is at maximum compression?

____ The magnitude of the net force is greater on the block of mass $2M$.

____ The magnitude of the net force is greater on the block of mass $3M$.

____ The magnitude of the net force on each block has the same nonzero value.

____ The magnitude of the net force on each block is zero.

Justify your answer.

(f) Do the two blocks, which remain stuck together and attached to the spring, exhibit simple harmonic motion after the collision?

____ Yes ____ No

Justify your answer.

Note: Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

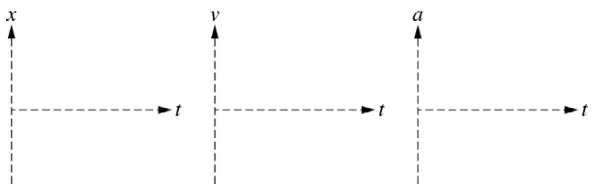
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of $\mathrm{kg/m}$.

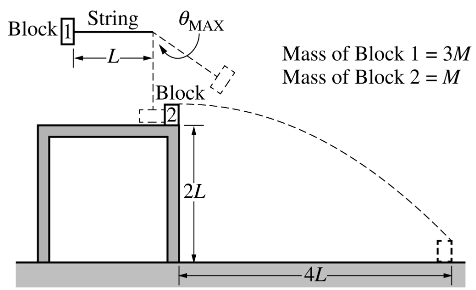
(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



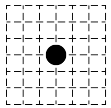


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.

3 Work, kinetic energy, and potential energy – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
Energy	能量			
joules	焦耳			
kinetic energy	动能			
Work	功			
work–energy theorem	动能定理			
Potential energy	势能			
Gravitational potential energy	重力势能			
Elastic potential energy	弹性势能			

Recall

You already know a lot about energy:

- Energy comes in forms – movement, height, heat, light – and can change from one form to another.
- You cannot make energy from nothing or destroy it; it is only moved around.
- You met the idea that a fast or heavy object carries more energy. At AP we make this exact with formulas.

Each idea below has a worked example. Then do the Practice.

New ideas

1 Kinetic energy

Energy 能量 is the capacity to do work, measured in **joules** 焦耳 (J). A moving object has **kinetic energy** 动能:

$$K = \tfrac{1}{2}mv^2.$$

It depends on the **square** of the speed, so doubling the speed makes the kinetic energy **four times** larger.

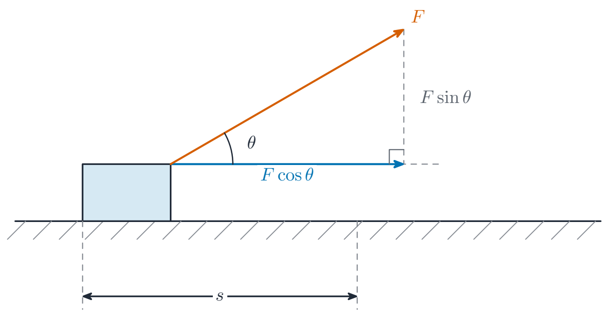
Worked example. A 1500 kg car at 20 m/s has $K = \frac{1}{2} \times 1500 \times 20^2 = 3.0 \times 10^5$ J. At 40 m/s it is four times as much.

2 Work

Work 功 is energy transferred when a force pushes something through a distance:

$$W = Fd\cos\theta,$$

where θ is the angle between the force and the movement. Work is **positive** if the force helps the motion, **negative** if it opposes it, and **zero** if the force is at right angles to the motion.



Worked example. A 20 N force pushes a box 3.0 m in its own direction: $W = Fd = 20 \times 3.0 = 60$ J.

3 The work–energy theorem

The net work done on an object equals the change in its kinetic energy – the **work–energy theorem** 动能定理:

$$W_{\text{net}} = \Delta K.$$

Worked example. If 60 J of net work is done on a resting 3.0 kg trolley, then $60 = \frac{1}{2} \times 3.0 \times v^2$, giving $v^2 = 40$ and $v = 6.3$ m/s.

■ 4 Potential energy

Potential energy 势能 is stored energy that depends on position:

- **Gravitational potential energy** 重力势能 (from height h): $U_g = mgh$.
- **Elastic potential energy** 弹性势能 (in a stretched spring): $U_s = \frac{1}{2}kx^2$.

Worked example. Lifting a 2.0 kg book 1.5 m stores $U_g = mgh = 2.0 \times 9.8 \times 1.5 = 29.4$ J.

Watch out: carrying a bag horizontally at steady height does **no** work on the bag – the lifting force is vertical but the movement is horizontal, so $\cos 90° = 0$.

Practice

Try these in order. The methods are all above.

2.1 Find the kinetic energy of a 0.50 kg ball moving at 6.0 m/s. [2]

2.2 A 40 N force pushes a crate 5.0 m in its own direction. Find the work done. [2]

2.3 State whether the work is positive, negative, or zero when (a) friction acts on a sliding box, (b) you carry a box horizontally at constant height. [2]

2.4 A 3.0 kg mass is lifted 2.0 m. Find the gain in gravitational potential energy ($g = 9.8$ m/s^2). [2]

2.5 A 2.0 kg trolley starts at rest. A net work of 36 J is done on it. Use the work–energy theorem to find its final speed. [3]

2.6 A car's speed triples. By what factor does its kinetic energy increase? [1]

2. A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

(b) Calculate the speed of the rocket at $t = 6.0$ s.

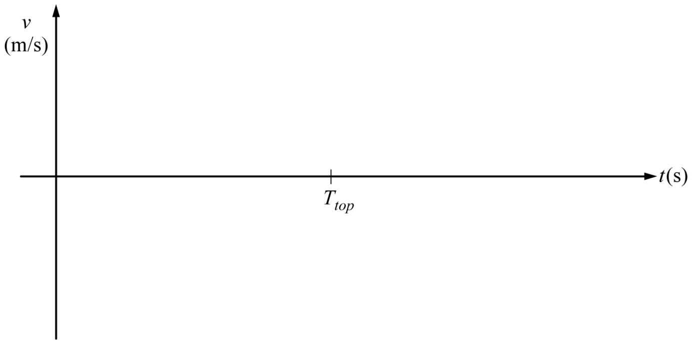
(c)

i. Calculate the kinetic energy of the rocket at $t = 6.0$ s.

ii. Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

(d) Calculate the maximum height reached by the rocket relative to its launching point.

(e) On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



Begin your response to **QUESTION 1** on this page.

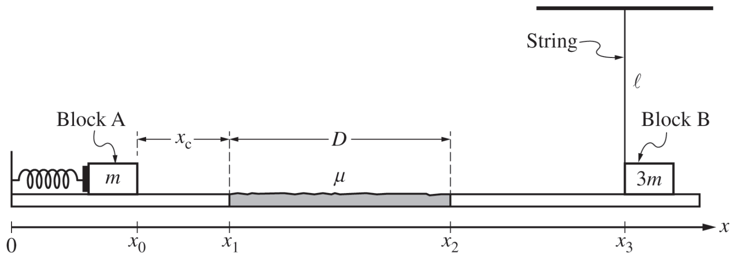
PHYSICS C: MECHANICS

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Note: Figure not drawn to scale.

Figure 1

1. Block A and Block B of masses m and $3m$, respectively, are arranged in a setup consisting of an ideal spring with spring constant k and a horizontal surface. Friction between the surface and the blocks is negligible except in a region of length D, where the coefficient of kinetic friction between Block A and the surface is μ. Block B is attached to a string of length ℓ and negligible mass, as shown in Figure 1. Block A is held against the spring, compressing the spring a distance x_c.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

At time $t = 0$, Block A is located at position $x = x_0$ and is released from rest. After the block is released, the following occurs.

- At time $t = t_1$, Block A is at $x = x_1$ after traveling a distance x_c. Block A moves with speed v, and the spring is at its equilibrium position.
- At time $t = t_2$, the left side of Block A is at $x = x_2$ after passing through a distance D across the region with nonnegligible friction.
- At time $t = t_3$, Block A is at $x = x_3$ and Block A collides with and sticks to Block B.

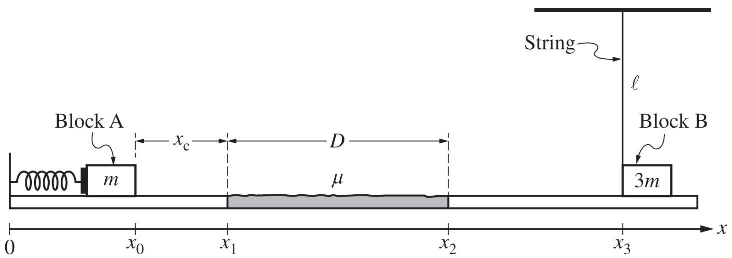
(a) For parts (a)(i) and (a)(ii), express your answer in terms of m, k, D, μ, x_c, and physical constants, as appropriate.

i. **Derive** an expression for the speed v of Block A at time t_1.

ii. **Derive** an expression for the speed $v_{A,B}$ of the two-block system immediately after the collision at time t_3.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

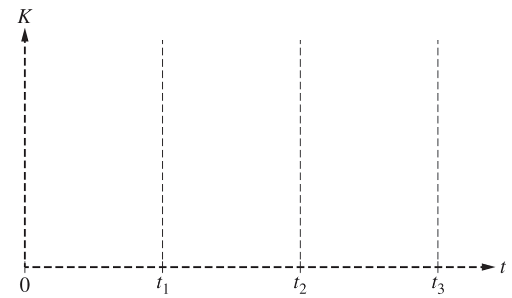


Note: Figure not drawn to scale.

Figure 1

(b)

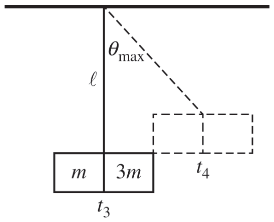
i. On the following axes, **sketch** a graph of the kinetic energy K of Block A as a function of time t from time $t = 0$ to time t_3.



ii. Use principles of work and energy to **justify** the graph drawn in part (b)(i) for the time interval $t = 0$ to $t = t_1$. Explicitly reference features of the shape of the graph you drew in part (b)(i).

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.



Note: Figure not drawn to scale.

Figure 2

After the collision, the two-block system instantaneously comes to rest at time t_4, which occurs when the string makes a small angle θ_{max} with the vertical, as shown in Figure 2. For times $t > t_4$, the system oscillates with frequency f_ℓ. The support holding the string is raised, and the procedure is then repeated using a new string of length 2ℓ.

(c) **Indicate** how the new frequency of oscillation $f_{2\ell}$ of the system on the new string of length 2ℓ will compare to the frequency of oscillation f_ℓ from the original procedure.

____ $f_{2\ell} > f_\ell$ ____ $f_{2\ell} < f_\ell$ ____ $f_{2\ell} = f_\ell$

Briefly **justify** your answer.

GO ON TO THE NEXT PAGE.

3 Conservation of energy and power – Part 2

Name: ______ Class: ______ Date: ______

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
mechanical energy	机械能			
conserved	守恒			
thermal energy	热能			
Power	功率			
watts	瓦特			
efficiency	效率			

Recall

In Part A you met kinetic energy $K = \frac{1}{2}mv^2$ and potential energy $U = mgh$. Now put them together:

- Energy is never lost – it only changes form.
- A falling ball speeds up (gains K) as it drops (loses U).
- Doing the same job faster needs more **power** – that is the last idea on this sheet.

Each idea has a worked example before the Practice.

New ideas

1 Mechanical energy

The **mechanical energy** 机械能 of an object is its kinetic plus potential energy:

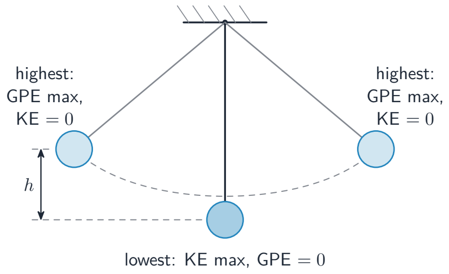
$$E = K + U.$$

2 Conservation of energy

When only gravity and springs act (no friction), the mechanical energy is **conserved** 守恒 – it stays the same:

$$K_1 + U_1 = K_2 + U_2.$$

So energy lost from one form appears in another.



Worked example. A ball is released from rest at the top of a smooth ramp 5.0 m high. All its potential energy becomes kinetic: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5.0} = 9.9$ m/s. The mass cancels, so every object reaches the same speed.

3 When friction acts

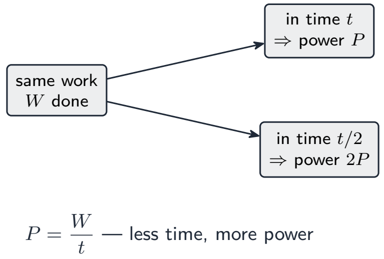
Friction turns mechanical energy into **thermal energy** 热能 (heat). Then energy is still conserved overall, but some has left the "useful" $K + U$ pool.

Worked example. If the ramp above loses 30 J to friction, you subtract it: $mgh - 30 = \frac{1}{2}mv^2$.

4 Power

Power 功率 is the **rate** of transferring energy, measured in **watts** 瓦特 (W):

$$P = \frac{W}{\Delta t}, \qquad \text{and at steady speed} \qquad P = Fv.$$



Worked example. A motor lifts a 50 kg load at a steady 2.0 m/s. The force equals the weight, so $P = Fv = mgv = 50 \times 9.8 \times 2.0 = 980$ W.

5 Efficiency

Real machines waste some energy, so we quote **efficiency** 效率 – useful output divided by total input:

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}}.$$

Worked example. A motor that draws 1400 W but delivers 980 W of useful lifting is $\frac{980}{1400} = 0.70 = 70\%$ efficient.

Watch out: at constant speed the net force is zero, but power is **not** zero – the motor still does work against gravity or friction. "Balanced forces" does not mean "no energy used."

Practice

Try these in order. The methods are all above.

2.1 A 0.20 kg ball is dropped from 1.8 m. Using energy conservation, find its speed just before it lands ($g = 9.8$ m/s^2). [3]

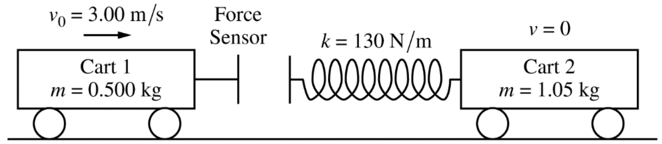
2.2 State the energy change as a pendulum swings from its highest point down to its lowest point. [2]

2.3 A crane does 6000 J of work in 4.0 s. Find its power output. [2]

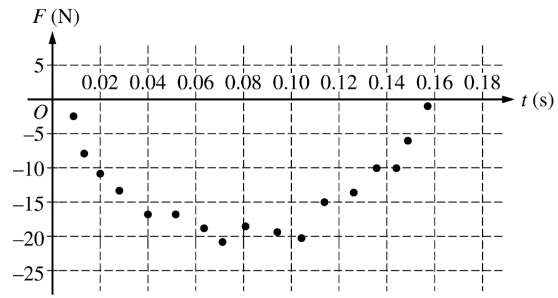
2.4 A machine takes in 500 W and delivers 300 W of useful power. Find its efficiency as a percentage. [2]

2.5 A ball rolls down a smooth slope 2.0 m high but 15 J is lost to friction. If the ball's mass is 1.0 kg, find its speed at the bottom. [3]

2.6 Explain why a car moving at constant speed still uses fuel (energy), even though the net force on it is zero. [2]



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200 \text{ N/s}^2\right)t^2 - (500 \text{ N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

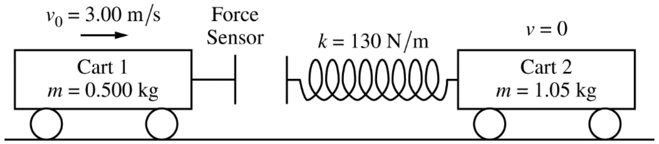
i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

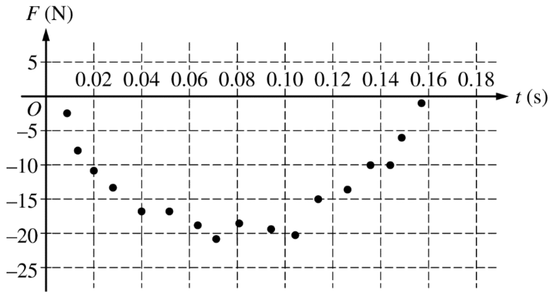
(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

AP PHYSICS C: MECHANICS • HOLIDAY HOMEWORK 假期作业

4 Momentum and impulse – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
Linear momentum	动量	
impulse	冲量	

Recall

You already have a feel for momentum, even without the word:

- A heavy truck is harder to stop than a bicycle at the same speed.
- A fast ball hurts more than a slow one when you catch it.
- Catching a ball gently (moving your hands back) hurts less. This sheet explains why, exactly.

Each idea has a worked example before the Practice.

New ideas

1 Momentum

Linear momentum 动量 is mass times velocity – a vector pointing the same way as the velocity:

$$p = mv.$$

It measures how hard something is to stop. Its unit is kg m/s.

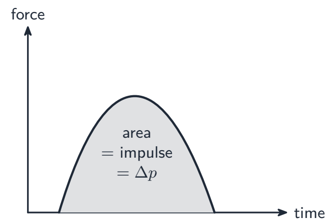
Worked example. A 0.40 kg ball moving at 5.0 m/s has momentum $p = mv = 0.40 \times 5.0 = 2.0$ kg m/s.

2 Impulse

A force acting for a time changes the momentum. The **impulse** 冲量 is

$$J = F\,\Delta t,$$

and it equals the change in momentum: $J = \Delta p$. This is the **impulse–momentum theorem**.



Worked example. A 12 N force pushes a trolley for 3.0 s: impulse $= F\Delta t = 12 \times 3.0 = 36$ kg m/s, so the momentum increases by 36 kg m/s.

3 Why time matters

Because $F = \dfrac{\Delta p}{\Delta t}$, spreading the same momentum change over a **longer time** makes the force **smaller**. That is exactly how airbags, crash mats and bending your knees on landing protect you.

Worked example. A 0.15 kg ball hits a wall at 20 m/s and bounces back at 15 m/s; contact lasts 0.020 s. Taking the rebound direction as positive, $\Delta p = m(v - u) = 0.15(15 - (-20)) = 5.25$ kg m/s, so $F = \dfrac{\Delta p}{\Delta t} = \dfrac{5.25}{0.020} = 260$ N.

Watch out: when a ball **bounces back**, its velocity **reverses sign**, so the change in momentum is large. Forgetting the sign flip is the most common mistake here.

Practice

Try these in order. The methods are all above.

2.1 Find the momentum of a 2.0 kg ball moving at 6.0 m/s. [2]

2.2 A 5.0 N force acts on a cart for 4.0 s. Find the impulse. [2]

2.3 The cart in 2.2 started at rest with mass 2.0 kg. Find its final speed. [2]

2.4 Explain, using impulse, why bending your knees when landing from a jump reduces the force on your legs. [2]

2.5 A 0.20 kg ball moving at 8.0 m/s is caught and stopped in 0.50 s. Find the average force on it. [3]

2.6 A ball hits a wall at 10 m/s and rebounds at 10 m/s. Its mass is 0.25 kg. Find the size of the change in momentum. [2]

Question 1: Version J

1. Two blocks, 1 and 2, slide toward each other on a horizontal surface. Block 1 has mass m and slides in the $+x$-direction with constant speed $2v_0$. Block 2 has mass $6m$ and slides in the $-x$-direction with constant speed v_0, as shown in Figure 1. The blocks then collide and stick together. The collision occurs from time $t = 0$ to $t = t_c$. After the collision, where $t > t_c$, the blocks move together with the same constant speed.

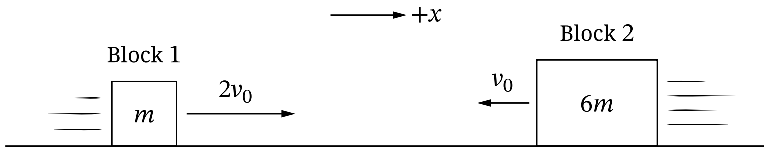


Figure 1

A. The diagrams in Figure 2 can be used to represent the momentum of blocks 1 and 2 before and after the collision. The momentum vector diagram for Block 1 before the collision is shown.

i. **Draw** arrows on the grids to represent the momentum vectors of Block 2 before the collision and the two-block system before and after the collision.

- Arrows should start at the zero-momentum line.
- The length of the arrows should be proportional to the relative magnitudes of the vectors.
- Represent an arrow of zero length by drawing a dot at zero.

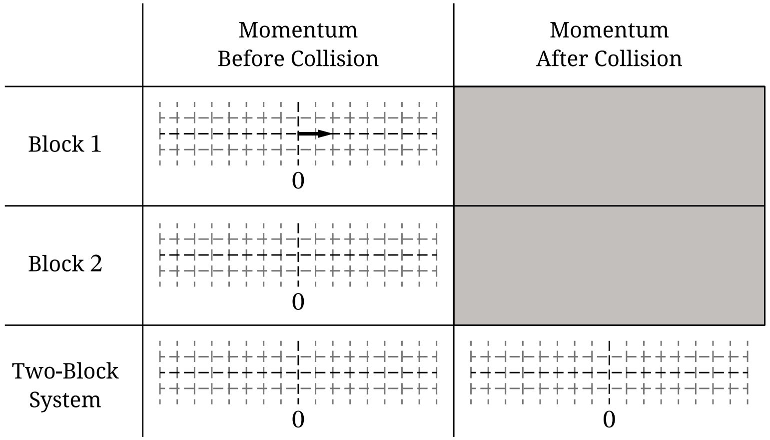


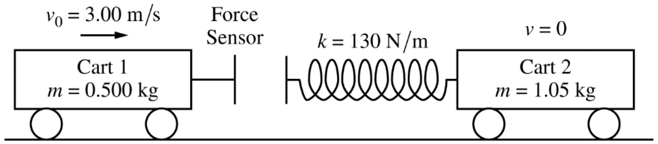
Figure 2

ii. During the time interval $0 \le t \le t_c$, a force F is exerted on Block 2 by Block 1 along the x-direction as a function of t that is modeled by $F(t) = F_{max}\sin(At)$, where A is a positive constant and F_{max} is the magnitude of the maximum force exerted on Block 2 by Block 1 during the collision.

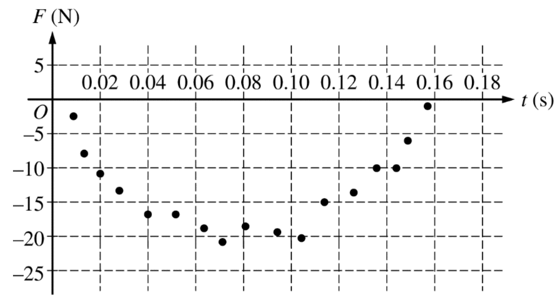
Derive an expression for F_{max}. Express your answer in terms of m, v_0, A, t_c, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

B. Consider a new scenario where Block 1 initially slides in the $+x$-direction with a new constant speed v_1 and Block 2 again initially slides in the $-x$-direction with constant speed v_0. The blocks collide and stick together. In this new scenario, the two-block system has constant speed v_0 after the collision.

Derive an expression for v_1 in terms of v_0. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = (3200\ \text{N/s}^2)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

4 Conservation of momentum and collisions – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
conserved	守恒			
perfectly inelastic collision	非弹性碰撞			
elastic collision	弹性碰撞			
recoils	反冲			

Recall

In Part A you met momentum $p = mv$ and impulse. Now the big payoff:

- When two things push on each other, their pushes are equal and opposite (Newton's third law).
- So in a crash or an explosion, the **total** momentum does not change.
- This one idea solves collisions that would be hard to do with forces.

Each idea has a worked example before the Practice.

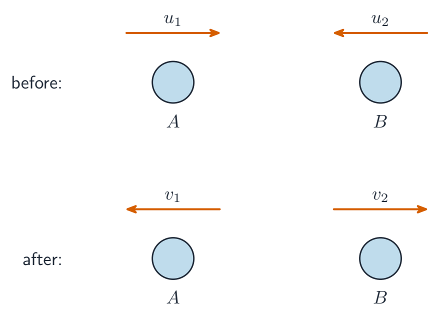
New ideas

1 Conservation of momentum

If no outside force acts on a system, its total momentum is **conserved** 守恒 – the total before equals the total after:

$$\sum p_{\text{before}} = \sum p_{\text{after}}.$$

The internal pushes come in equal-and-opposite pairs, so they cancel and cannot change the total.



2 Collisions that stick

In a **perfectly inelastic collision** 非弹性碰撞 the objects **stick together** and move off with one shared velocity.

Worked example. A 1000 kg car at 20 m/s hits a stationary 1500 kg car and they lock together. Momentum is conserved:

$$1000 \times 20 = (1000 + 1500)\,v \;\Rightarrow\; v = \frac{20000}{2500} = 8.0 \text{ m/s}.$$

3 Elastic and inelastic

Momentum is conserved in **every** collision, but kinetic energy is not:

- In an **elastic collision** 弹性碰撞, kinetic energy is also conserved (a clean bounce).
- In an **inelastic collision**, some kinetic energy becomes heat and deformation (a crash).

4 Recoil

When one part of a still system is pushed one way, the rest **recoils** 反冲 the other way so the total momentum stays zero.

Worked example. A 60 kg skater at rest throws a 2.0 kg ball at 8.0 m/s. Starting from zero total momentum: $0 = (60)v + (2.0)(8.0)$, so $v = -\frac{16}{60} = -0.27$ m/s – she slides back at 0.27 m/s. This is how rockets work.

Watch out: momentum is a **vector**. Give one direction a + sign and the opposite a − sign before you add – two equal speeds in opposite directions add to **zero** momentum, not double.

Practice

Try these in order. The methods are all above.

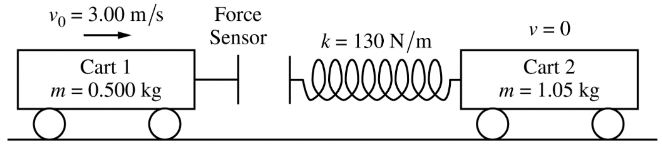
2.1 A 2.0 kg trolley at 3.0 m/s hits a stationary 1.0 kg trolley and they stick together. Find their common speed. [3]

2.2 State what is always conserved in a collision, and what is conserved only if the collision is elastic. [2]

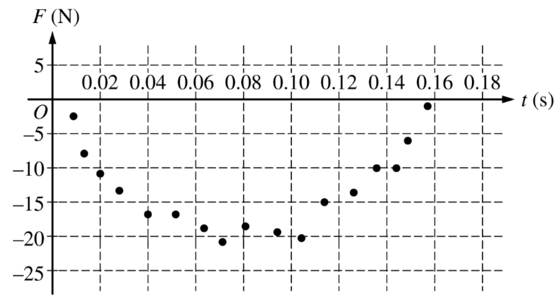
2.3 A 50 kg student on frictionless ice throws a 1.0 kg ball at 6.0 m/s. Find the student's recoil speed. [3]

2.4 Two identical balls roll toward each other at the same speed and stick together. Explain why they stop. [2]

2.5 A 0.50 kg ball at 4.0 m/s collides head-on with a stationary 0.50 kg ball. After an **elastic** collision the first stops. State the second ball's speed and give your reason. [2]



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00$ m/s toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130$ N/m. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

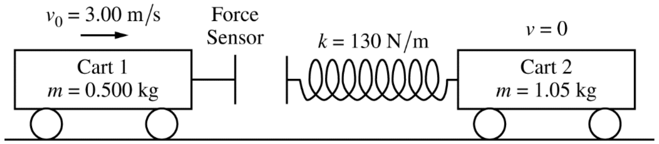
i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

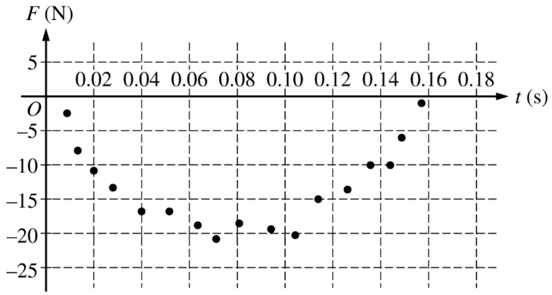
(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.



2. Two carts are on a horizontal, level track of negligible friction. Cart 1 has a sensor that measures the force exerted on it during a collision with cart 2, which has a spring attached. Cart 1 is moving with a speed of $v_0 = 3.00\ \text{m/s}$ toward cart 2, which is at rest, as shown in the figure above. The total mass of cart 1 and the force sensor is 0.500 kg, the mass of cart 2 is 1.05 kg, and the spring has negligible mass. The spring has a spring constant of $k = 130\ \text{N/m}$. The data for the force the spring exerts on cart 1 are shown in the graph below. A student models the data as the quadratic fit $F = \left(3200\ \text{N/s}^2\right)t^2 - (500\ \text{N/s})t$.



(a) Using integral calculus, calculate the total impulse delivered to cart 1 during the collision.

(b)

i. Calculate the speed of cart 1 after the collision.

ii. In which direction does cart 1 move after the collision?

____ Left ____ Right

____ The direction is undefined, because the speed of cart 1 is zero after the collision.

(c)

i. Calculate the speed of cart 2 after the collision.

ii. Show that the collision between the two carts is elastic.

(d)

i. Calculate the speed of the center of mass of the two-cart–spring system.

ii. Calculate the maximum elastic potential energy stored in the spring.

5 Rotational motion and torque – Part 1

Name: __________ Class: ________ Date: _________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
angular displacement	角位移			
radians	弧度			
angular velocity	角速度			
Torque	力矩			
moment arm	力臂			

Recall

You already know spinning things – wheels, gears, a spinning top:

- A wheel turns through an angle; you have measured angles in degrees.
- A point on the edge of a big wheel moves faster than one near the middle.
- You have used a **seesaw** and a spanner: pushing farther from the pivot turns things more easily.

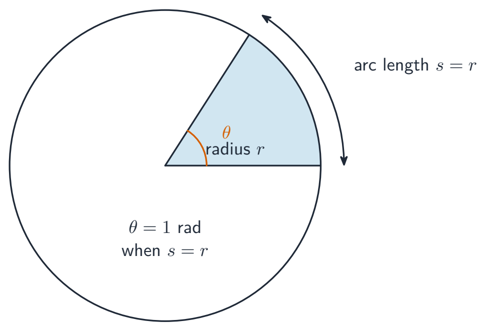
This sheet gives these ideas exact names and formulas. Each idea has a worked example.

New ideas

1 Angular quantities

Rotation copies straight-line motion, with new names:

- **angular displacement** 角位移 θ – the angle turned, measured in **radians** 弧度 (one radian is the angle whose arc length equals the radius).
- **angular velocity** 角速度 $\omega = \dfrac{\Delta\theta}{\Delta t}$ – how fast it spins.



Worked example. A wheel turns 20 rad in 4.0 s, so $\omega = \dfrac{20}{4.0} = 5.0$ rad/s.

2 Linking spin to straight-line speed

A point at distance r from the axis moves at speed

$$v = r\omega.$$

Points farther from the axis move faster.

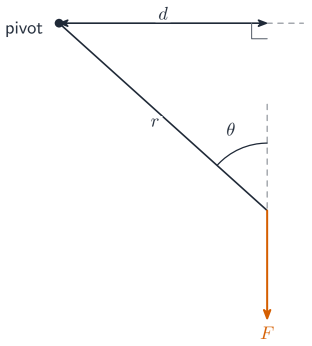
Worked example. A bicycle wheel of radius 0.35 m spins at 12 rad/s, so the rim (and the bike) moves at $v = r\omega = 0.35 \times 12 = 4.2$ m/s.

3 Torque – the turning effect

Torque 力矩 (also called the moment) is how strongly a force turns something about a pivot:

$$\tau = F \times r_{\perp},$$

where $r_{\perp}$ is the **moment arm** 力臂 – the perpendicular distance from the pivot to the line of the force.



Worked example. Pushing with 20 N at the end of a 0.30 m spanner, at right angles, gives $\tau = Fr = 20 \times 0.30 = 6.0$ N m.

Watch out: a force pointing straight **through** the pivot has zero moment arm, so it makes **no** torque. Push at a right angle, as far out as you can, for the biggest turning effect.

Practice

Try these in order. The methods are all above.

2.1 A wheel turns through 18 rad in 3.0 s. Find its angular velocity. [2]

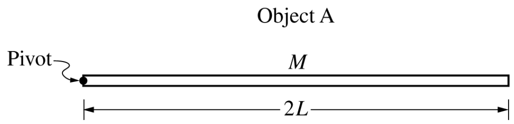
2.2 A point on a rim of radius 0.20 m has the wheel spinning at 10 rad/s. Find the speed of the point. [2]

2.3 A force of 15 N is applied at right angles 0.40 m from a pivot. Find the torque. [2]

2.4 Explain why a door handle is placed far from the hinges, not near them. [2]

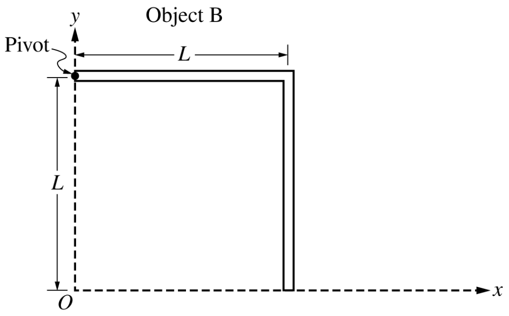
2.5 Two points sit on a spinning disc, one at the rim and one halfway to the centre. State which moves faster and why. [2]

2.6 A spanner is 0.25 m long. You push with 30 N at right angles to it. Find the torque, then state what happens to the torque if you push at the same force but only halfway along. [3]



2. Object A is a long, thin, uniform rod of mass M and length $2L$ that is free to rotate about a pivot of negligible friction at its left end, as shown above.

(a) Using integral calculus, derive an expression to show that the rotational inertia I_A of object A about the pivot is given by $\frac{4}{3}ML^2$.



Object B of total mass M is formed by attaching two thin, uniform, identical rods of length L at a right angle to each other. Object B is held in place, as shown above. Express your answers in part (b) in terms of L .

(b) Determine the following for the given coordinate system shown in the figure.

i. The x-coordinate of the center of mass of object B

ii. The y-coordinate of the center of mass of object B

GO ON TO THE NEXT PAGE.

Object B has a rotational inertia of I_B about its pivot.

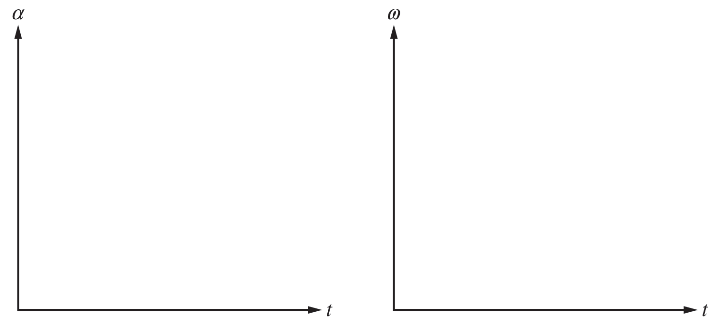
(c) Is the value of I_B greater than, less than, or equal to I_A ?

_____ Greater than   _____ Less than   _____ Equal to

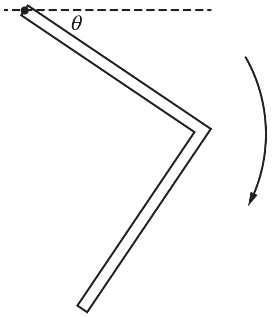
Justify your answer.

Object B is released from rest and begins to rotate about its pivot.

(d) On the axes below, sketch graphs of the magnitude of the angular acceleration α and the angular speed ω of object B as functions of time t from the time it is released to the time its center of mass reaches its lowest point.



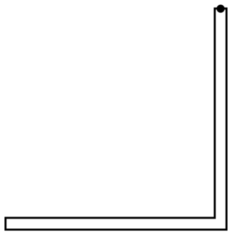
GO ON TO THE NEXT PAGE.



(e) While object B rotates from the horizontal position down through the angle θ shown above, is the magnitude of its angular acceleration increasing, decreasing, or not changing?

_____ Increasing   _____ Decreasing   _____ Not changing

Justify your answer.

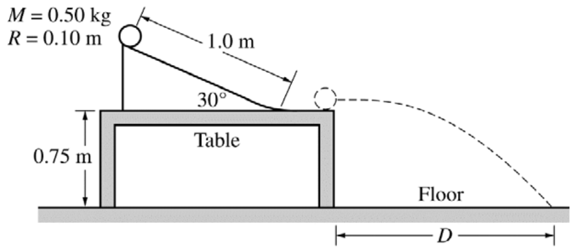


Object B rotates through the position shown above.

(f) Derive an expression for the angular speed of object B when it is in the position shown above. Express your answer in terms of M, L, I_B, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

Use a pencil or pen with black or dark blue ink only. Do NOT write your name. Do NOT write outside the box.



3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than   ____ Less than   ____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than   ____ Less than   ____ Equal to

Justify your answer.

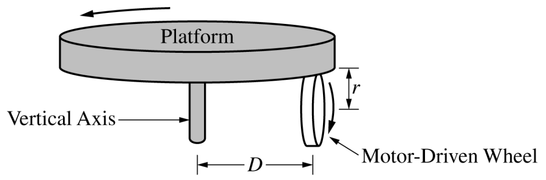
iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than   ____ Less than   ____ Equal to

Justify your answer.

STOP

END OF EXAM

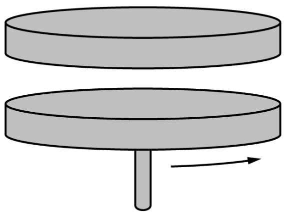


3. A horizontal circular platform with rotational inertia I_P rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_P after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

(a) Derive an expression for the angular speed ω_P of the platform. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_P. Express your answer in terms of I_P, r, D, F, Δt, and physical constants, as appropriate.

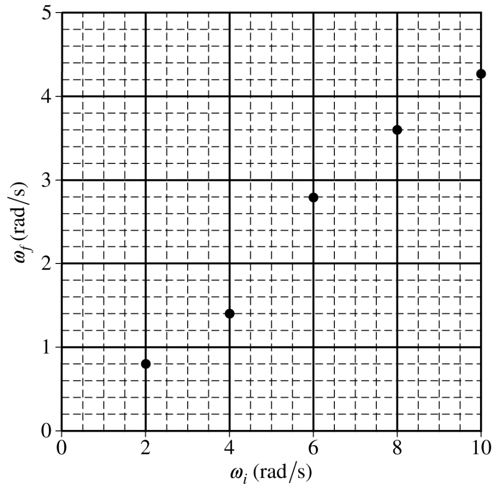
(c) Derive an expression for the angular speed of the wheel ω_W when the platform has reached angular speed ω_P. Express your answer in terms of D, r, ω_P, and physical constants, as appropriate.



When the platform is spinning at angular speed ω_P, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_P as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f.

(d) Derive an expression for ω_f. Express your answer in terms of ω_P, I_P, and physical constants, as appropriate.

A student now uses the rotating platform $\left(I_P = 3.1\ \text{kg}\cdot\text{m}^2\right)$ to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed ω_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed ω_f. Trials are repeated for different values of ω_i. A graph of ω_f as a function of ω_i is shown on the axes below.



(e)

i. On the graph on the previous page, draw a best-fit line for the data.

ii. Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.

(f) The kinetic energy of the spinning platform before the object is dropped on it is K_i. The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

____ $K_f < K_i$ ____ $K_f = K_i$ ____ $K_f > K_i$

Justify your answer.

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM

5 Rotational inertia, balance, and torque = I alpha – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
Rotational inertia	转动惯量	________ ________ ________
rotational equilibrium	转动平衡	________ ________ ________

Recall

In Part A you met torque – the turning effect of a force. Now use it two ways:

- to **balance** things (a seesaw, a shelf), which you have done before;
- to find how fast something starts to spin, the rotational copy of $F = ma$.

Each idea has a worked example before the Practice.

New ideas

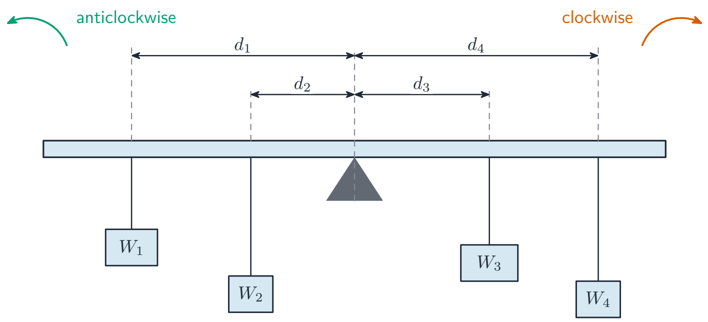
1 Rotational inertia

Rotational inertia 转动惯量 (symbol I) measures how hard it is to change something's spin – the rotational version of mass. It depends on the mass **and how far** that mass sits from the axis: mass spread farther out makes I larger.

This is why a spinning skater speeds up when she pulls her arms **in** – she brings her mass closer to the axis, making I smaller.

2 Rotational equilibrium

An object is in **rotational equilibrium** 转动平衡 when the total turning effect is zero – the clockwise torques balance the anticlockwise torques. Then it does not start to spin.



Worked example. A 30 kg child sits 2.0 m from the pivot of a seesaw. Where must a 40 kg child sit to balance it? Set the two torques equal (the g cancels):

$$30 \times 2.0 = 40 \times d \;\Rightarrow\; d = \frac{60}{40} = 1.5 \text{ m}.$$

The heavier child sits closer to the pivot.

■ 3 The rotational Newton's second law

Just as a net force gives linear acceleration, a **net torque** gives angular acceleration:

$$\sum \tau = I\alpha,$$

the exact twin of $\sum F = ma$ (with $\tau \leftrightarrow F$, $I \leftrightarrow m$, $\alpha \leftrightarrow a$).

Worked example. A net torque of 12 N m acts on a wheel with $I = 3.0$ kg m^2. Its angular acceleration is $\alpha = \frac{\tau}{I} = \frac{12}{3.0} = 4.0$ rad/s^2.

Watch out: the same mass can have different rotational inertia depending on where it sits – a ring is harder to spin than a solid disc of the same mass, because its mass is all at the edge.

Practice

Try these in order. The methods are all above.

2.1 State what two things the rotational inertia of an object depends on. [2]

2.2 A 20 kg child sits 1.5 m from a seesaw pivot. A 25 kg child sits on the other side. How far from the pivot must the second child sit to balance? [3]

2.3 A net torque of 8.0 N m acts on a wheel of rotational inertia 2.0 kg m^2. Find the angular acceleration. [2]

2.4 Explain why a spinning ice skater speeds up when she pulls her arms in. [2]

2.5 A shelf is held by a bracket. State the condition on the torques for the shelf to stay balanced (not rotate). [1]

2.6 A wheel with $I = 5.0$ kg m^2 must reach $\alpha = 3.0$ rad/s^2. Find the net torque needed. [2]

3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

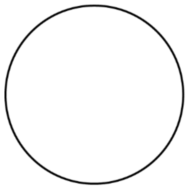


Figure 1

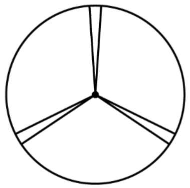


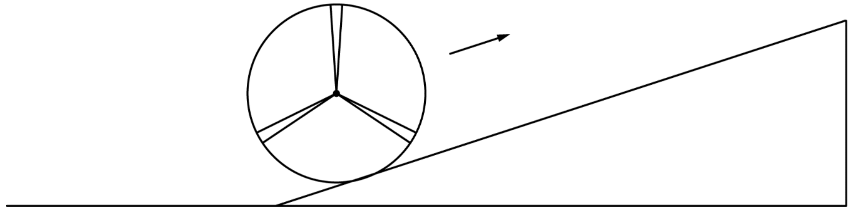
Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

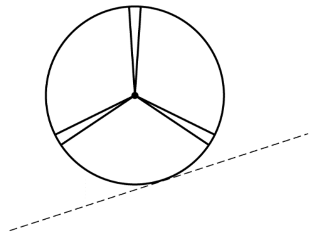
(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

STOP

END OF EXAM

Begin your response to **QUESTION 3** on this page.

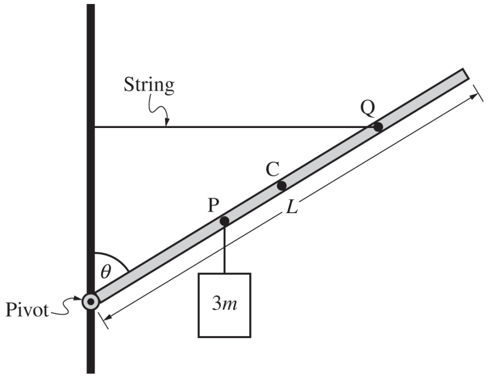
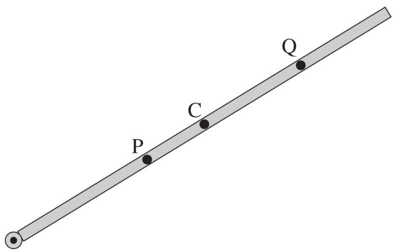


Figure 1

Note: Figure not drawn to scale.

3. A uniform rod of length L and mass m is attached to a pivot on a vertical pole, as shown in Figure 1. There is negligible friction between the rod and the pivot. A horizontal string connects Point Q on the rod to the pole. The rod makes an angle θ with the pole. A block of mass $3m$ hangs from the rod at Point P. The center of mass of the rod is located at Point C.

(a) On the following representation of the rod, **draw** and **label** the forces (not components) that are exerted on the rod. Each force must be represented by a distinct arrow that starts on and points away from the point at which the force is exerted on the rod.



GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

(b) In Figure 1, Point P is located $\frac{3}{8}L$ from the pivot and Point Q is located $\frac{6}{8}L$ from the pivot. **Derive** an equation for the tension F_T in the horizontal string in terms of L, m, θ, and physical constants, as appropriate.

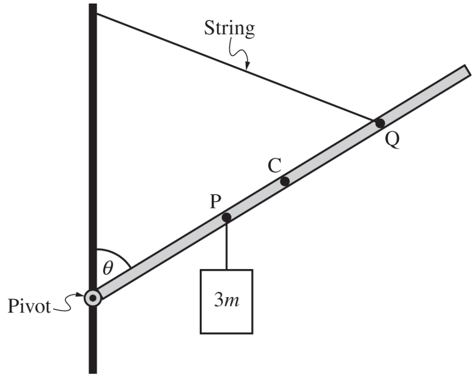


Figure 2

Note: Figure not drawn to scale.

(c) The original string is replaced with a longer string that connects Point Q to a higher location on the vertical pole, as shown in Figure 2. The angle θ remains the same. How does the new tension $F_{T,\text{ new}}$ compare with the original tension F_T from part (b) ? **Justify** your reasoning.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 3** on this page.

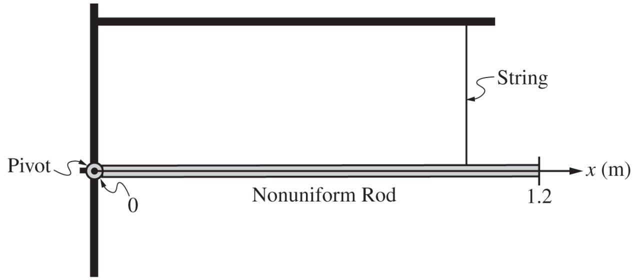


Figure 3

Note: Figure not drawn to scale.

(d) A nonuniform rod is now attached to the pivot, as shown in Figure 3. There is negligible friction between the nonuniform rod and the pivot. The rod has a length of 1.2 m and a linear mass density $\lambda(x) = A + Bx$, where x is the distance from the pivot, $A = 6.0\text{ kg/m}$, and $B = 10.0\text{ kg/m}^2$.

i. **Calculate** the mass of the rod.

ii. **Calculate** the rotational inertia of the rod about the pivot.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM

Question 4

4. A uniform disk and ring, each of mass M and radius R, roll without slipping along a horizontal surface, as shown in Figure 1. The outer edges of the disk and ring are made of the same material. The center of mass of the disk and the center of mass of the ring each initially move with the same constant speed v.

The disk and the ring then smoothly transition to a ramp that is inclined at an angle θ above the horizontal. Both the disk and the ring continue to roll without slipping as they move up the ramp, as shown in Figure 2.

The ring travels a greater distance along the ramp than the disk travels before each momentarily comes to rest.

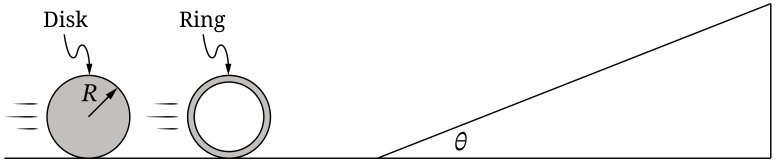


Figure 1

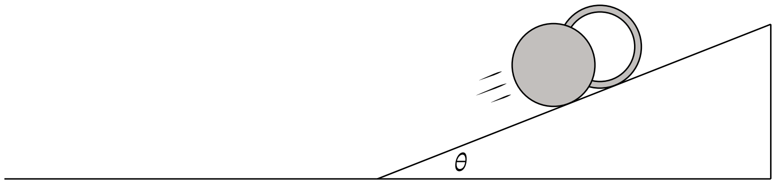


Figure 2

A. While the disk and the ring are rolling on the ramp without slipping, the magnitudes of the static frictional force exerted on the disk and on the ring by the ramp are f_D and f_R, respectively.

Indicate whether f_D is greater than, less than, or equal to f_R by writing one of the following.

- $f_D > f_R$
- $f_D < f_R$
- $f_D = f_R$

Justify your answer using qualitative reasoning beyond referencing equations.

B. A cylinder has mass M, radius R, and rotational inertia I about its central axis. The cylinder rolls without slipping up a ramp that is inclined at an angle θ above the horizontal.

Derive an expression for the magnitude of the static frictional force f exerted on the cylinder by the ramp. Express your answer in terms of M, R, I, θ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

C. In a different scenario, the centers of mass of the original disk and ring each have the same initial speed v as they did in the original scenario. The ramp is replaced by a new ramp on which the disk and the ring initially slip as they roll up the new ramp.

Indicate whether the magnitude of the kinetic frictional force exerted on the disk by the new ramp is greater than, less than, or equal to the magnitude of the kinetic frictional force exerted on the ring by the new ramp while both are slipping.

Briefly **justify** your answer.

STOP

END OF EXAM

6 Rotational energy and angular momentum – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
rotational kinetic energy	转动动能			
Angular momentum	角动量			
conserved	守恒			

Recall

In the last topic you met angular velocity ω and rotational inertia I. Now spinning things get **energy** and **momentum**, just like moving things:

- A moving object has kinetic energy $\frac{1}{2}mv^2$ and momentum mv.
- A spinning object has rotational copies of both.
- A spinning skater who pulls her arms in speeds up – this sheet explains exactly why.

Each idea has a worked example before the Practice.

New ideas

1 Rotational kinetic energy

A spinning object stores **rotational kinetic energy** 转动动能:

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2,$$

the twin of $\frac{1}{2}mv^2$. A rolling ball has **both**: $\frac{1}{2}mv^2$ from moving plus $\frac{1}{2}I\omega^2$ from spinning.

Worked example. A wheel with $I = 2.0$ kg m^2 spins at $\omega = 3.0$ rad/s: $K_{\text{rot}} = \frac{1}{2} \times 2.0 \times 3.0^2 = 9.0$ J.

2 Work done by a torque

A torque turning through an angle does work: $W = \tau\,\Delta\theta$ (the twin of $W = Fd$).

Worked example. A motor gives a steady torque of 8.0 N m while a flywheel turns 10 rad: $W = \tau\,\Delta\theta = 8.0 \times 10 = 80$ J.

3 Angular momentum

Angular momentum 角动量 is the rotational version of momentum:

$$L = I\omega.$$

4 Conservation of angular momentum

If no outside torque acts, angular momentum is **conserved** 守恒:

$$I_1\omega_1 = I_2\omega_2.$$

So if I goes down, ω goes up to keep L the same.

Worked example. A skater spins at 2.0 rev/s with $I_1 = 4.0$ kg m^2. She pulls her arms in, dropping I to $I_2 = 1.6$ kg m^2: $\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{4.0}{1.6} \times 2.0 = 5.0$ rev/s.

Watch out: pulling the arms in raises **both** the spin rate and the kinetic energy – the extra energy comes from the work her muscles do pulling inward. Angular momentum stays the same; energy does not.

Practice

Try these in order. The methods are all above.

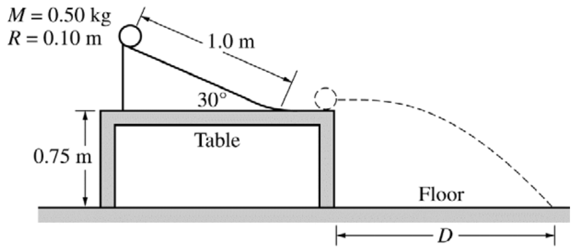
2.1 A flywheel with $I = 3.0$ kg m^2 spins at 4.0 rad/s. Find its rotational kinetic energy. [2]

2.2 A torque of 6.0 N m turns a wheel through 5.0 rad. Find the work done. [2]

2.3 Find the angular momentum of a disc with $I = 0.50$ kg m^2 spinning at 8.0 rad/s. [2]

2.4 A spinning student on a stool has $I_1 = 6.0$ kg m^2 at $\omega_1 = 1.0$ rad/s. He pulls weights in so $I_2 = 2.0$ kg m^2. Find his new angular velocity. [3]

2.5 State what stays the same and what changes when the student in 2.4 pulls the weights in. [2]



3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of 30° with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM

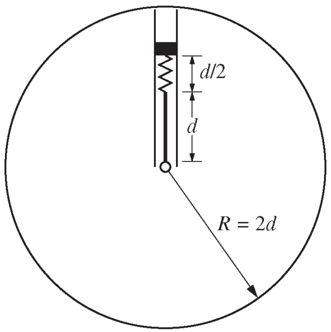


Figure 1

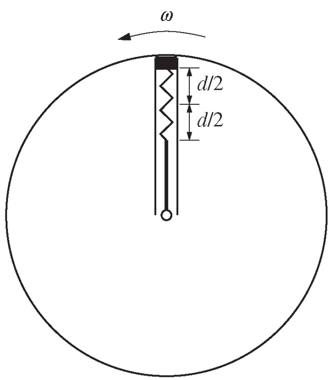


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

Name: AP Physics C: **Mechanics: 2016**

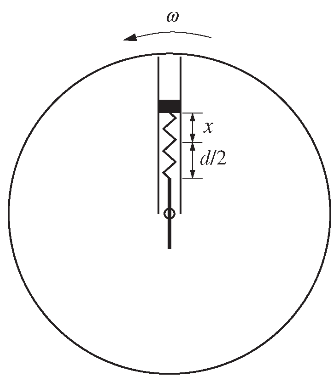


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

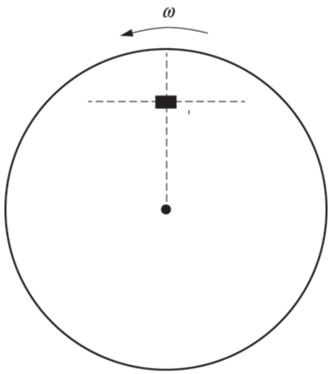


Figure 4

STOP

END OF EXAM

6 Rolling and orbiting satellites – Part 2

Name: ________ Class: ________ Date: ________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	**中文**	**Write it 3 times** (English)
rolls without slipping	纯滚动	
satellite	卫星	

Recall

In Part A you met rotational energy and angular momentum. Two everyday examples bring them together:

- A wheel that **rolls** is moving and spinning at the same time.
- A **satellite** stays in orbit because gravity keeps pulling it toward the planet.

Each idea has a worked example before the Practice.

New ideas

1 Rolling without slipping

When a wheel **rolls without slipping** 纯滚动, the point touching the ground is momentarily still, and the speed of the centre links to the spin:

$$v = r\omega.$$

Worked example. A wheel of radius 0.30 m rolls so its centre moves at 6.0 m/s. Its spin rate is $\omega = \dfrac{v}{r} = \dfrac{6.0}{0.30} = 20$ rad/s.

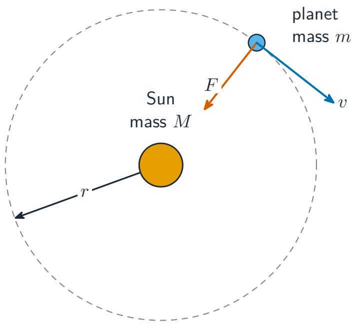
2 Rolling shares the energy

A rolling object splits its kinetic energy between **moving** ($\frac{1}{2}mv^2$) and **spinning** ($\frac{1}{2}I\omega^2$). Because some energy goes into spin, a ball rolling down a ramp speeds up **more slowly** than a block that slides down without friction.

3 Satellites and orbits

A **satellite** 卫星 in orbit is really in free fall: gravity provides exactly the **centripetal force** needed to bend its path into a circle. Setting gravity equal to the centripetal requirement gives the orbit speed

$$v = \sqrt{\frac{GM}{r}}.$$



Worked example. For a low orbit, $r = 6.4 \times 10^6$ m and $GM = 4.0 \times 10^{14}$ m^3/s^2: $v = \sqrt{\dfrac{4.0 \times 10^{14}}{6.4 \times 10^6}} \approx 7.9 \times 10^3$ m/s, about 7.9 km/s.

4 Bigger orbit, slower speed

Because $v = \sqrt{GM/r}$, a satellite farther out (*larger* r) moves **slower**. The satellite's own mass cancels, so all satellites in the same orbit move at the same speed.

Watch out: a satellite is not "beyond gravity." Gravity is exactly what holds it in orbit – it is falling toward the planet all the time, but moving sideways fast enough to keep missing it.

Practice

Try these in order. The methods are all above.

2.1 A wheel of radius 0.25 m rolls without slipping; its centre moves at 5.0 m/s. Find its angular velocity. [2]

2.2 Explain why a ball rolling down a slope accelerates more slowly than a block sliding down the same frictionless slope. [2]

2.3 What force provides the centripetal force that keeps a satellite in orbit? [1]

2.4 Two satellites orbit the same planet, one close and one far. State which moves faster, and why. [2]

2.5 A wheel of radius 0.40 m spins at $\omega = 15$ rad/s while rolling. Find the speed of its centre. [2]

3. A triangular rod, shown above, has length L, mass M, and a nonuniform linear mass density given by the equation $\lambda = \frac{2M}{L^2}x$, where x is the distance from one end of the rod.

(a) Using integral calculus, show that the rotational inertia of the rod about its left end is $ML^2/2$.

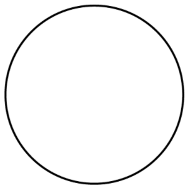


Figure 1

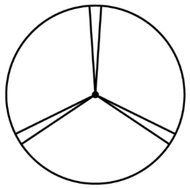


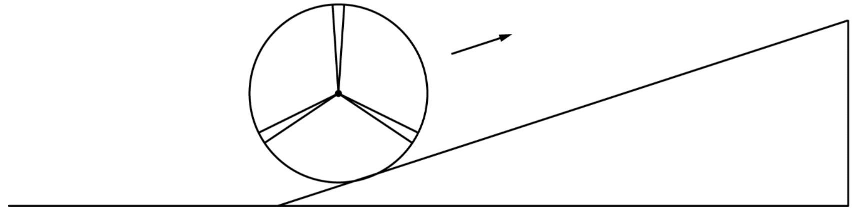
Figure 2

The thin hoop shown above in Figure 1 has a mass M, radius L, and a rotational inertia around its center of ML^2. Three rods identical to the rod from part (a) are now fastened to the thin hoop, as shown in Figure 2 above.

(b) Derive an expression for the rotational inertia I_{tot} of the hoop-rods system about the center of the hoop. Express your answer in terms of M, L, and physical constants, as appropriate.

The hoop-rods system is initially at rest and held in place but is free to rotate around its center. A constant force F is exerted tangent to the hoop for a time Δt.

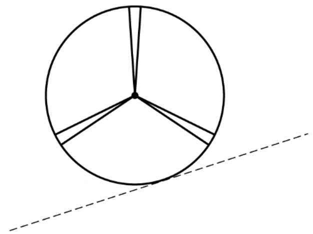
(c) Derive an expression for the final angular speed ω of the hoop-rods system. Express your answer in terms of M, L, F, Δt, and physical constants, as appropriate.



The hoop-rods system is rolling without slipping along a level horizontal surface with the angular speed ω found in part (c). At time $t = 0$, the system begins rolling without slipping up a ramp, as shown in the figure above.

(d)

i. On the figure of the hoop-rods system below, draw and label the forces (not components) that act on the system. Each force must be represented by a distinct arrow starting at, and pointing away from, the point at which the force is exerted on the system.



ii. Justify your choice for the direction of each of the forces drawn in part (d)i.

(e) Derive an expression for the change in height of the center of the hoop from the moment it reaches the bottom of the ramp until the moment it reaches its maximum height. Express your answer in terms of M, L, I_{tot}, ω, and physical constants, as appropriate.

STOP

END OF EXAM

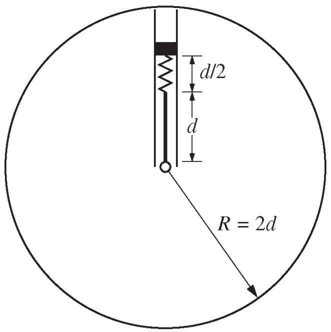


Figure 1

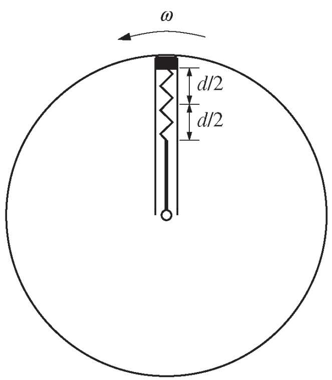


Figure 2

Mech.3.

A uniform rod of length d has one end fixed to the central axis of a horizontal, frictionless circular platform of radius $R = 2d$. Fixed at the other end of the rod is an ideal spring of negligible mass to which a block is attached. The block is set in frictionless grooves so that it can only move along a radius of the platform, as shown in Figure 1 above. The equilibrium length of the spring is $d/2$. Below is a table showing the mass of the block and the masses and rotational inertias of the rod and platform.

	Mass	Rotational Inertia
Block	m	
Rod	$m_R = 3m$	$\frac{m_R d^2}{3}$ (about the end of the rod)
Platform	$m_P = 5m$	$\frac{m_P R^2}{2}$ (about the central axis)

A motor begins to slowly rotate the platform counterclockwise as viewed from above until the platform reaches a constant angular speed ω. Under these conditions, the spring has stretched by an additional length $d/2$, as shown in Figure 2.

Answer the following questions for the platform rotating at constant angular speed ω. Express all algebraic answers in terms of m, d, ω, and physical constants, as appropriate.

(a) Derive an expression for the spring constant of the spring.

(b)

i. Determine an expression for the rotational inertia of the block around the axis of the platform.

ii. Derive an expression for the rotational inertia of the entire system about the axis of the platform.

(c) Determine an expression for the angular momentum of the entire system about the axis of the platform.

Name: AP Physics C: **Mechanics: 2016**

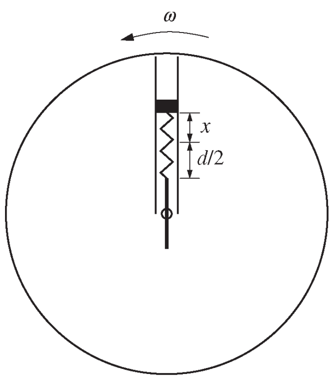


Figure 3

While the system continues to rotate, a small mechanism in the pivot moves the rod slowly until the center of the rod is positioned on the axis, as shown in Figure 3 above. The same constant angular speed ω is maintained by the motor driving the platform.

(d) Derive an expression for the distance x that the spring is stretched when the rod reaches the position shown in Figure 3 above.

For parts (e), (f), and (g), assume the center of the rod is still moving toward the axis of the platform.

(e) Is the angular momentum of the entire system increasing, decreasing, or staying the same?

_____ Increasing _____ Decreasing _____ Staying the same

Justify your answer.

(f) In order to keep the system rotating with constant angular speed ω, is the motor doing positive work, negative work, or no work on the rotating system?

_____ Positive _____ Negative _____ No work

Justify your answer.

(g) On the block in Figure 4 below, draw a single vector representing the direction of the acceleration of the block. Draw the vector so that it is starting on, and pointing away from, the block.

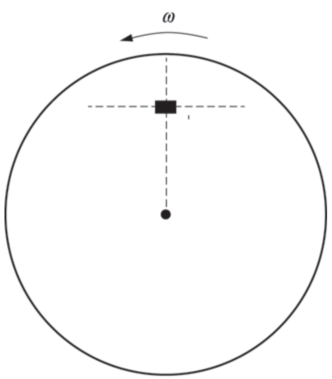
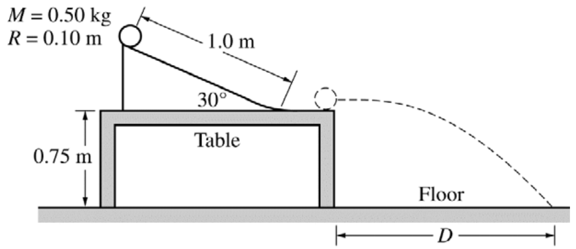


Figure 4

STOP

END OF EXAM



3. A uniform solid cylinder of mass $M = 0.50$ kg and radius $R = 0.10$ m is released from rest, rolls without slipping down a 1.0 m long inclined plane, and is launched horizontally from a horizontal table of height 0.75 m. The inclined plane makes an angle of $30°$ with the horizontal. The cylinder lands on the floor a distance D away from the edge of the table, as shown in the figure above. There is a smooth transition from the inclined plane to the horizontal table, and the motion occurs with no frictional energy losses. The rotational inertia of a cylinder around its center is $MR^2/2$.

(a) Calculate the total kinetic energy of the cylinder as it reaches the horizontal table.

(b) Calculate the angular velocity of the cylinder around its axis at the moment it reaches the floor.

(c) Calculate the ratio of the rotational kinetic energy to the total kinetic energy for the cylinder at the moment it reaches the floor.

(d) Calculate the horizontal distance D.

A sphere of the same mass and radius is now rolled down the same inclined plane. The rotational inertia of a sphere around its center is $\frac{2}{5}MR^2$.

(e)

i. Is the total kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the total kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

ii. Is the rotational kinetic energy of the sphere at the moment it reaches the floor greater than, less than, or equal to the rotational kinetic energy of the cylinder at the moment it reaches the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

iii. Is the horizontal distance the sphere travels from the table to where it hits the floor greater than, less than, or equal to the horizontal distance the cylinder travels from the table to where it hits the floor?

____ Greater than ____ Less than ____ Equal to

Justify your answer.

STOP

END OF EXAM

7 Simple harmonic motion, period, and frequency – Part 1

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
Simple harmonic motion	简谐运动	
oscillation	振动	
restoring force	回复力	
period	周期	
frequency	频率	

Recall

You have watched things swing and bounce back and forth:

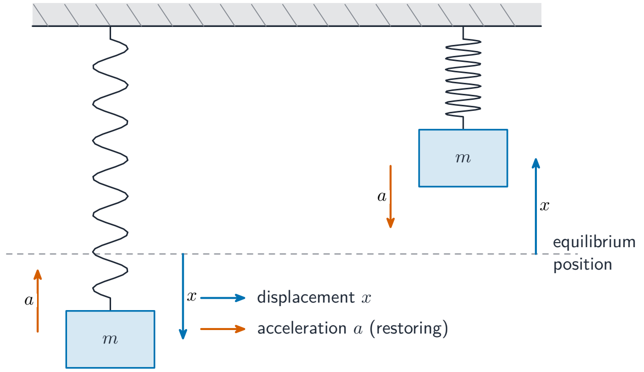
- A pendulum swings side to side; a mass on a spring bounces up and down.
- The motion repeats over and over, each cycle taking about the same time.
- A stiffer spring bounces faster; a longer pendulum swings slower.

This sheet gives the repeating motion a name – simple harmonic motion – and simple formulas. Each idea has a worked example.

New ideas

1 Simple harmonic motion

Simple harmonic motion 简谐运动 (SHM) is a back-and-forth **oscillation** 振动 caused by a **restoring force** 回复力 that always pulls the object back toward the middle, and grows with the distance from the middle ($F = -kx$).



A mass on a spring, and a pendulum swinging through **small** angles, are the standard examples.

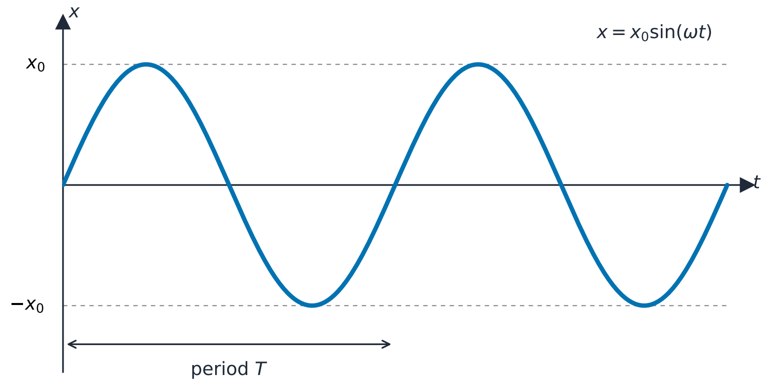
2 Period and frequency

- The **period** 周期 T is the time for one full cycle.
- The **frequency** 频率 $f = \dfrac{1}{T}$ is the number of cycles per second (in hertz, Hz).

3 The period formulas

For SHM the period depends only on the system, **not** on how big the swing is:

$$T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}}, \qquad T_{\text{pendulum}} = 2\pi\sqrt{\frac{L}{g}}.$$



Worked example. A 0.25 kg mass hangs on a spring of stiffness $k = 100$ N/m:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.25}{100}} = 0.31 \text{ s}, \qquad f = \frac{1}{T} = 3.2 \text{ Hz}.$$

Watch out: the period does **not** depend on the amplitude – a wide swing and a narrow swing take the same time. That is exactly what makes a pendulum a good clock.

Practice

Try these in order. The methods are all above.

2.1 A pendulum completes one full swing (there and back) in 2.0 s. State its period and find its frequency. [2]

2.2 Name the two conditions a force must meet to cause simple harmonic motion. [2]

2.3 A 0.40 kg mass sits on a spring of stiffness $k = 160$ N/m. Find the period of its oscillation. [3]

2.4 State what happens to a spring's period if the mass is increased. [1]

2.5 A pendulum has period 1.0 s. A student makes the swing wider (larger amplitude). State the effect on the period, and explain. [2]

<u>Note:</u> Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

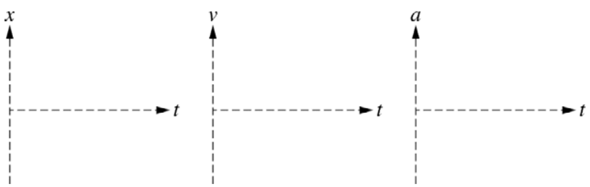
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



Name: AP Physics C: **Mechanics: 2017**

<u>Note:</u> Figure not drawn to scale.

2. A block of mass m starts at rest at the top of an inclined plane of height h, as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant k. There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of m, h, k, and physical constants, as appropriate.

(a)

i. Derive an expression for the speed of the block just before it collides with the spring.

ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

____ Greater than ______ Less than ____ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

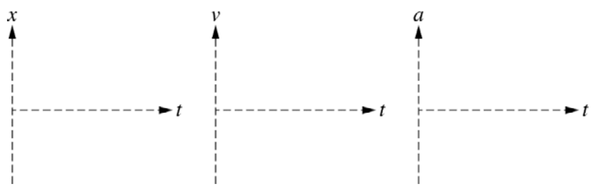
The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed v_0. Now suppose the block experiences a resistive force as it slides on the horizontal surface. The magnitude of the resistive force F is given as a function of speed v by $F = \beta v^2$, where β is a positive constant with units of kg/m.

(d)

i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time t before it reaches the spring. Express your answer in terms of m, h, k, β, v, and physical constants, as appropriate.

ii. Using the differential equation from part (d)i, show that the speed of the block $v(t)$ as a function of time t can be written in the form $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$, where v_0 is the speed at $t = 0$.

(e) Sketch graphs of position x as a function of time t, velocity v as a function of time t, and acceleration a as a function of time t for the block as it is moving on the horizontal surface before it reaches the spring.



7 The energy of oscillations – Part 2

Name: ____________ Class: ________ Date: __________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
equilibrium position	平衡位置	
amplitude	振幅	

Recall

In Part A you met simple harmonic motion and its period. Now follow the **energy** and **speed** through one cycle:

- At the far ends of a swing the object stops for an instant, then turns back.
- At the middle it is moving fastest.
- The energy keeps swapping between two forms while the total stays fixed.

Each idea has a worked example before the Practice.

New ideas

1 Speed and acceleration through the cycle

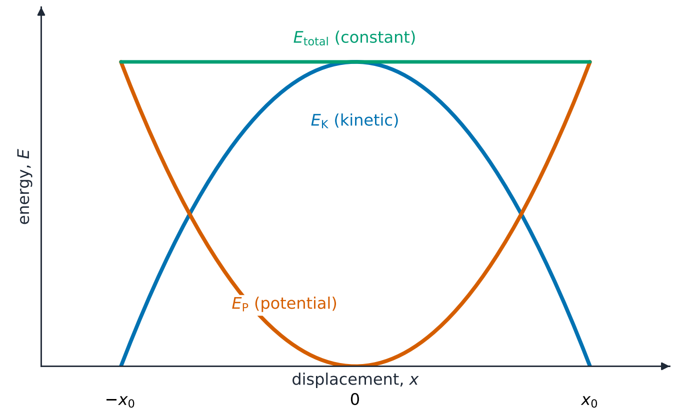
As the object oscillates about its **equilibrium position** 平衡位置 (the middle):

- At the **ends** (maximum displacement, called the **amplitude** 振幅 A): the speed is **zero** and the acceleration is largest.
- At the **middle** ($x = 0$): the acceleration is zero and the **speed is largest**.

2 Energy swaps between two forms

The energy slides back and forth between kinetic and elastic potential, while the **total** stays constant (no friction):

$$E = \tfrac{1}{2}kA^2.$$



- At the ends: all **potential** (stretched spring, not moving).
- At the middle: all **kinetic** (moving fastest, spring relaxed).

Worked example. A 0.50 kg mass on a spring ($k = 200$ N/m) has amplitude $A = 0.10$ m. All the energy is kinetic at the middle, so $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$, giving $v_{max} = A\sqrt{\frac{k}{m}} = 0.10 \times \sqrt{\frac{200}{0.50}} = 2.0$ m/s.

3 Amplitude and energy

Because $E = \frac{1}{2}kA^2$, the energy depends on the **square** of the amplitude – double the amplitude and the energy becomes **four times** larger.

Watch out: the fastest point is the **middle**, not the ends. At the ends the object is momentarily at rest, even though the force on it is largest there.

Practice

Try these in order. The methods are all above.

2.1 State where in its swing an oscillating mass has (a) maximum speed, (b) zero speed. [2]

2.2 At which point of the motion is all the energy kinetic? [1]

2.3 A spring ($k = 300$ N/m) oscillates with amplitude $A = 0.20$ m. Find the total energy $E = \frac{1}{2}kA^2$. [2]

2.4 A 0.20 kg mass on a spring ($k = 80$ N/m) has amplitude 0.15 m. Find its maximum speed. [3]

2.5 The amplitude of an oscillation is doubled. State the factor by which the total energy changes. [1]

2.6 Explain why the acceleration is largest at the ends of the swing, even though the speed there is zero. [2]

Question 2: Version J

2. In Scenario 1, a system composed of two springs, A and B, and a block of mass m is at rest on a horizontal surface. Friction between the block and the surface is negligible. Each spring is attached to a fixed wall and the block, as shown in Figure 1. Spring A has a spring constant k and Spring B has a spring constant $2k$. Each spring is at its relaxed length when the block is at position $x = 0$, as shown.

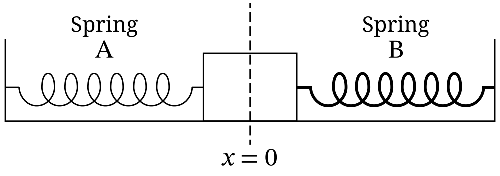


Figure 1

The block is moved to $x = x_1$ and held at rest, as shown in Figure 2.

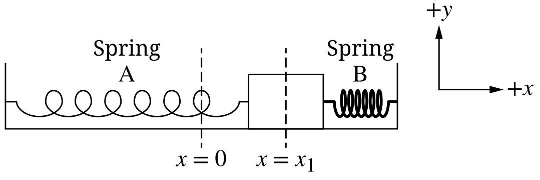


Figure 2

A. An energy bar chart can be used to represent the elastic potential energy U_A of Spring A, the elastic potential energy U_B of Spring B, and the kinetic energy K_{block} of the block. On the energy bar chart in Figure 3, **draw** shaded bars to represent the energy of the system for when the block is at $x = x_1$.

- The height of the shaded bars should be proportional to the relative values of U_A, U_B, and K_{block}.
- Any energy that is equal to zero should be represented by a distinct line on the zero-energy line.

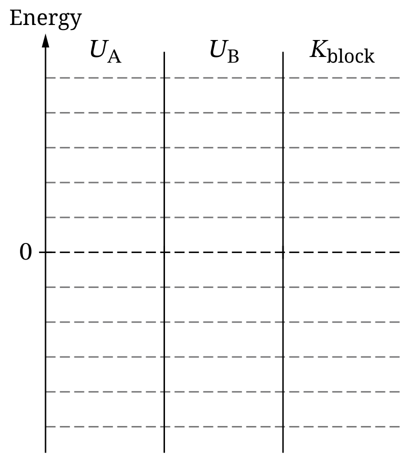
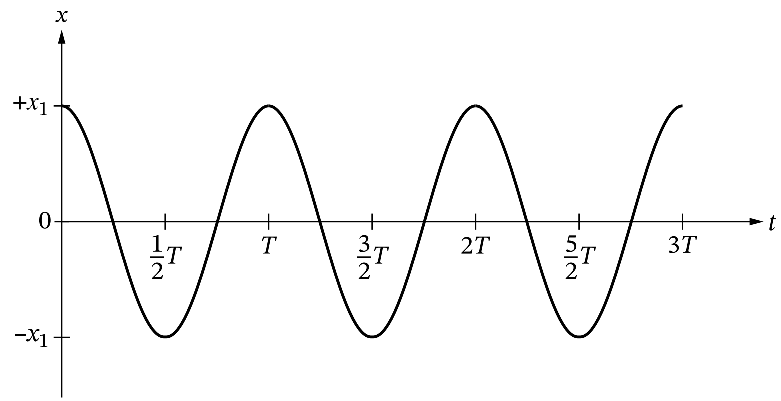


Figure 3

B. The block is released from rest at $x = x_1$ and begins to oscillate. **Derive** an expression for the speed v of the block as the block passes through $x = \frac{1}{2}x_1$. Express your answer in terms of m, k, x_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Name: AP Physics C: **Mechanics: 2025**

C. In Scenario 1, the block oscillates with period T. The position x of the block in Scenario 1 as a function of time t is shown in Figure 4.

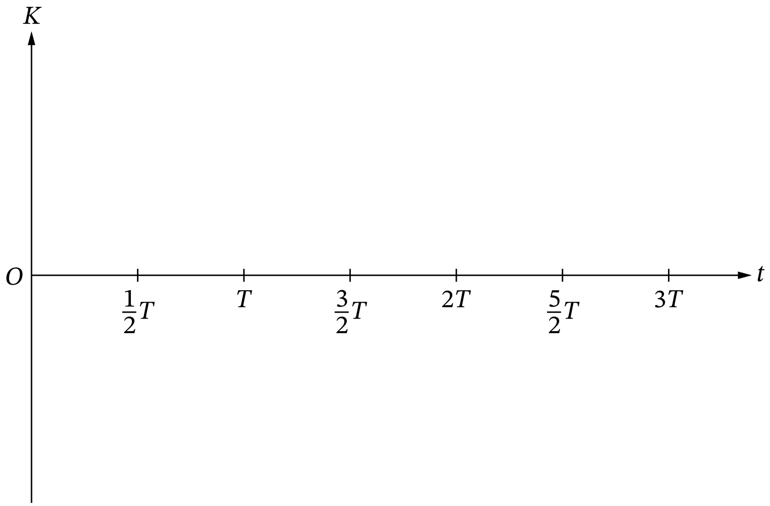


Scenario 1

Figure 4

In Scenario 2, the block-springs system is placed on a new surface. There is friction between the block and the new surface. The block is again moved to the same position $x = x_1$ and released from rest. The block completes multiple oscillations with the same period as in Scenario 1 before coming to rest.

On the axes shown in Figure 5, **sketch** a graph of the kinetic energy K of the block as a function of t for Scenario 2.



Scenario 2

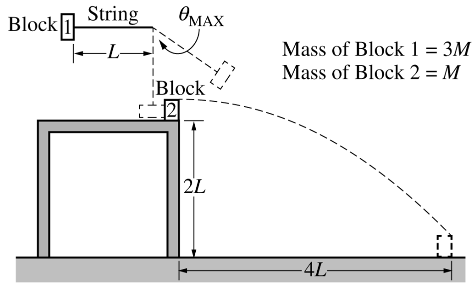
Figure 5

D. In Scenario 3, the block is replaced with a new block of larger mass. The coefficient of kinetic friction between the new block and the surface in Scenario 3 is the same as the coefficient of kinetic friction between the original block and the surface in Scenario 2.

The new block is moved to position $x = x_1$ and released from rest. The kinetic energy of the new block is plotted as a function of time.

Describe how one feature of the graph of K as a function of t in Scenario 3 would differ from the graph you drew in Figure 5 for Scenario 2.

Briefly **justify** your answer.

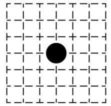


Note: Figure not drawn to scale.

2. A pendulum of length L consists of block 1 of mass $3M$ attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height $2L$. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance $4L$ from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{MAX} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

For parts (d)–(g), the value for the length of the pendulum is $L = 75$ cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

(e) Calculate the speed of block 2 as it leaves the table.

(f) Calculate the speed of block 1 just after it collides with block 2.

(g) Calculate the angle θ_{MAX} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.