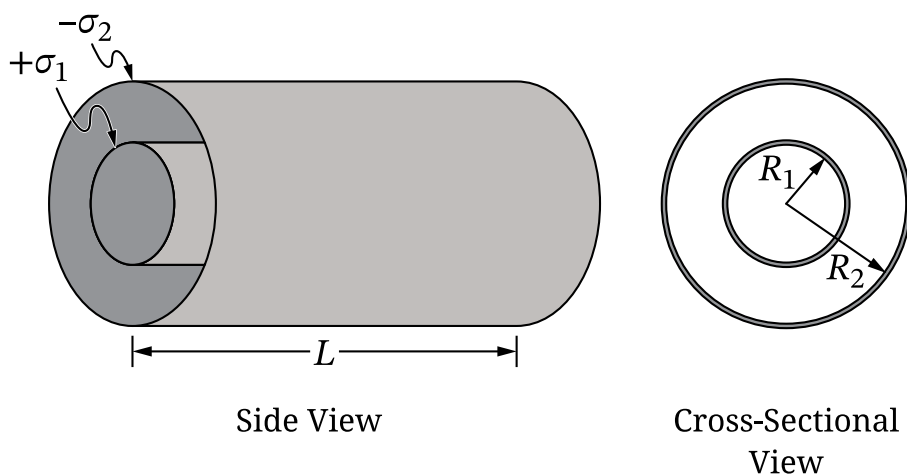


Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L . The inner shell has radius R_1 and the outer shell has radius R_2 , as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

- i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1 , σ_1 , r , and physical constants, as appropriate.
- ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1 , R_2 , σ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

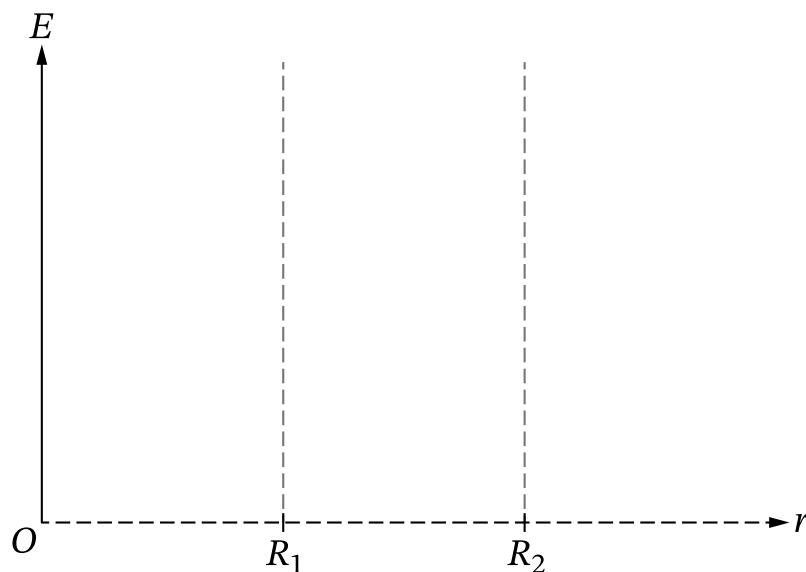
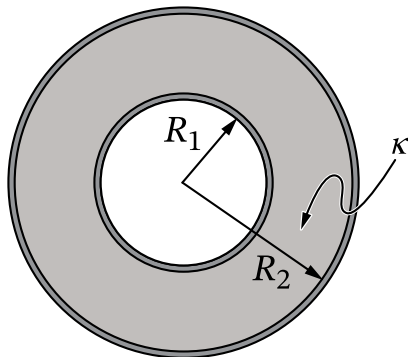


Figure 2

- B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L , R_1 , R_2 , κ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.**PHYSICS C: ELECTRICITY AND MAGNETISM****SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

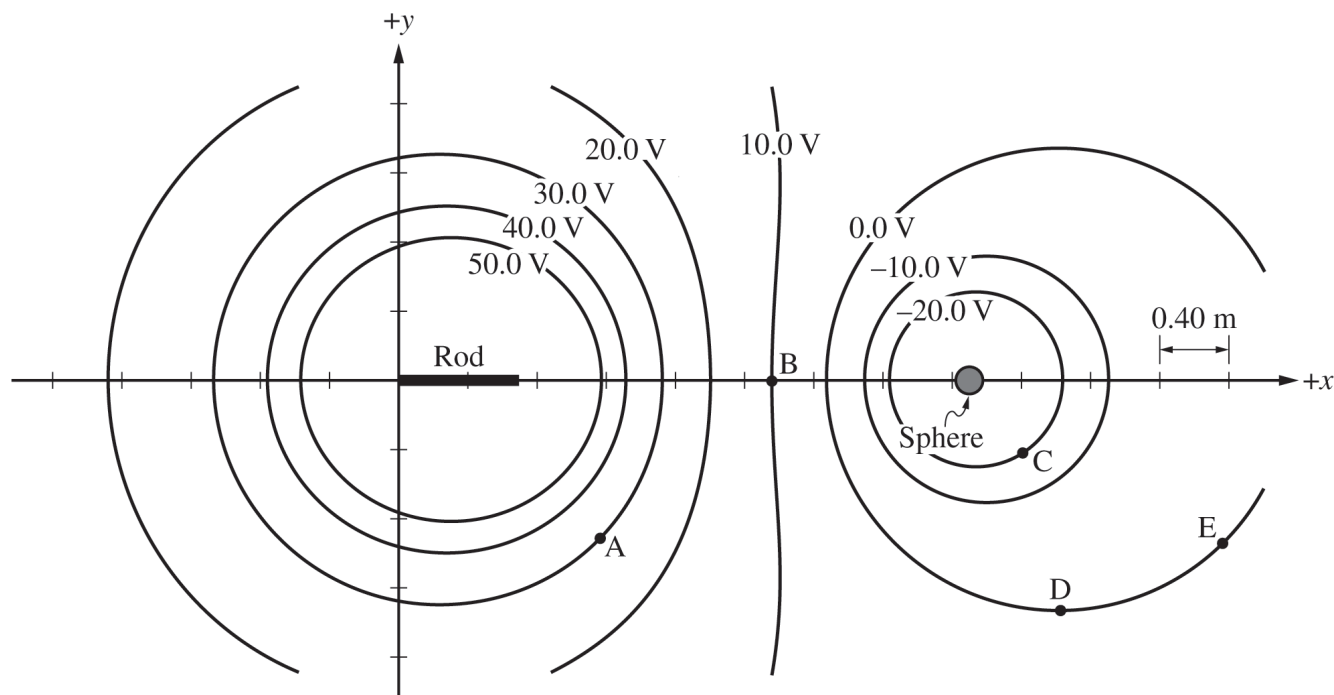


Figure 1

1. A nonconducting rod of uniform positive linear charge density is near a sphere with charge -2.0 nC . The rod and sphere are held at rest on the x -axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x -axis and the y -axis are 0.40 m apart.
- (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the -20.0 V equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of $C \rightarrow E \rightarrow D \rightarrow A$. The test charge is momentarily held at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE} . If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a "0" in the corresponding columns, as appropriate.

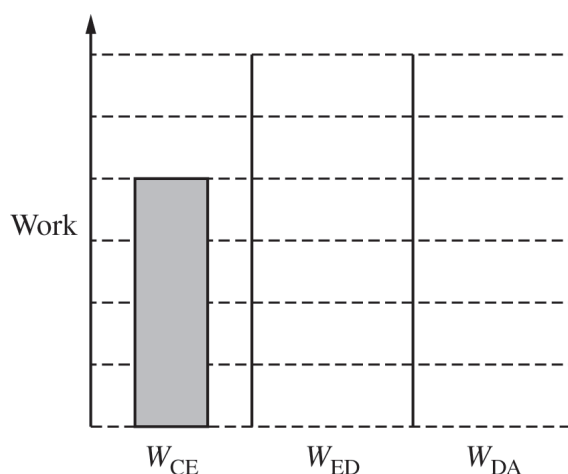


Figure 2

ii. **Calculate** the approximate magnitude of the x -component of the electric field at Position B.

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Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position D. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

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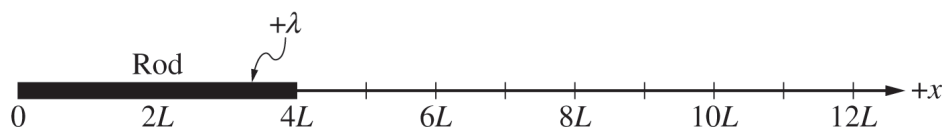
Continue your response to **QUESTION 1** on this page.

Figure 3

The sphere and the test charge are removed. The rod has length $4L$ and uniform positive linear charge density $+\lambda$. The rod is held at rest on the x -axis in the orientation shown in Figure 3. Position P (not shown) is located on the x -axis a distance x_P from the origin, where $x_P > 4L$.

(d) The electric potential V_P at x_P is $V_P = k\lambda \ln\left(\frac{x_P}{x_P - 4L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- ii. On Figure 4, **sketch** a graph of the x -component E_x of the electric field from the rod as a function of x in the region $4L < x < 12L$.

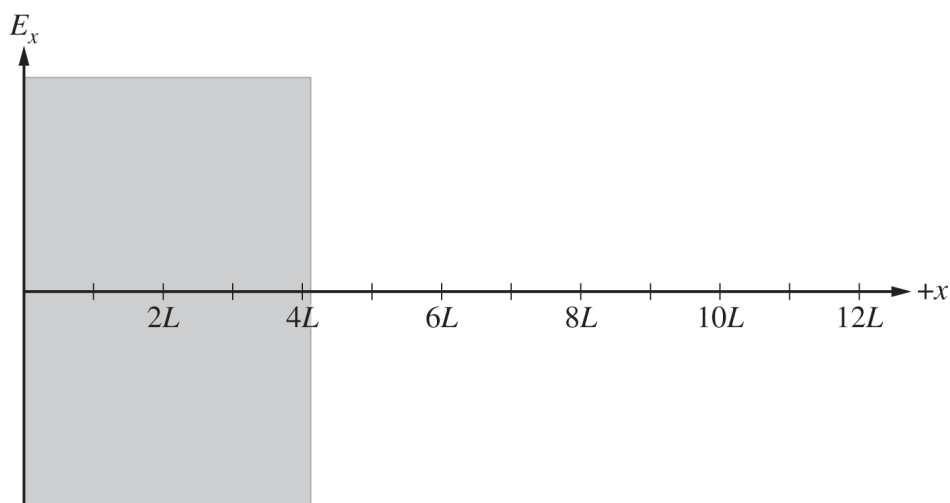
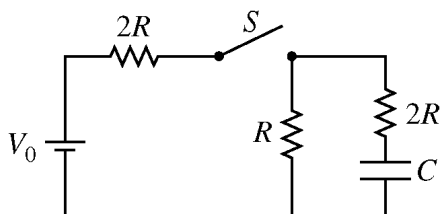


Figure 4

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. The circuit represented above is composed of three resistors with the resistances shown, a battery of voltage V_0 , a capacitor of capacitance C , and a switch S . The switch is closed, and after a long time, the circuit reaches steady-state conditions. Answer the following questions in terms of V_0 , R , C , and fundamental constants, as appropriate.
- (a) Derive an expression for the steady-state current supplied by the battery.
 - (b) Derive an expression for the charge on the capacitor.
 - (c) Derive an expression for the energy stored in the capacitor.

Now the switch is opened at time $t = 0$.

- (d) Write, but do NOT solve, a differential equation that could be used to solve for the charge $q(t)$ on the capacitor as a function of the time t after the switch is opened.

(e)

- i. Calculate the current in resistor R immediately after the switch is opened.
- ii. On the axes below, sketch the current in the circuit as a function of time from time $t = 0$ to a long time after the switch is opened. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



- (f) Is the total amount of energy dissipated in the resistors after the switch is opened greater than, less than, or equal to the amount of energy stored in the capacitor calculated in part (c) ?

____ Greater than ____ Less than ____ Equal to

Justify your answer.