Question 1: Mathematical Routines (MR) — 10 points

A (i) For a multistep derivation that includes the equation $\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\varepsilon_0}$ — **Point A1**

Scoring Note: Vector notation is not required for this point to be earned.

For a correct substitution of the area of an appropriate Gaussian surface with nonzero flux for the region $R_1 < r < R_2$ (e.g., $2\pi r\ell$) — **Point A2**

For a correct expression for the enclosed charge (e.g., $\sigma_1(2\pi R_1\ell)$) — **Point A3**

Example Response

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\pi r\ell) = \frac{\sigma_1(2\pi R_1\ell)}{\varepsilon_0}$$

$$E = \frac{\sigma_1 R_1}{\varepsilon_0 r}$$

(ii) For substituting the expression for E from part A (i) into $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}$ — **Point A4**

Scoring Notes:

- Vector notation is not required for this point to be earned.
- The sign of ΔV is not considered for this point to be earned.

For an attempt to solve the integral for $|\Delta V|$ that includes correct limits (e.g., $|\Delta V| = \dfrac{\sigma_1 R_1}{\varepsilon_0}\displaystyle\int_{R_1}^{R_2} \frac{1}{r}dr$) — **Point A5**

Scoring Note: This point may be earned regardless of the order of the limits of integration; for example, $\dfrac{\sigma_1 R_1}{\varepsilon_0}\displaystyle\int_{R_2}^{R_1} \frac{1}{r}dr$.

Example Response

$$|\Delta V| = \left|-\int_a^b \vec{E} \cdot d\vec{r}\right|$$

$$|\Delta V| = \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{r\varepsilon_0}dr = \frac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r}dr$$

$$|\Delta V| = \frac{\sigma_1 R_1}{\varepsilon_0}\ln(r)\Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$$

(iii) For sketching a graph that is zero for both $0 < r < R_1$ and $r > R_2$ — **Point A6**

For sketching a graph that is decreasing and concave up for $R_1 < r < R_2$ — **Point A7**

Scoring Note: The curve does not have to intersect the vertical dashed lines for this point to be earned.

Example Response

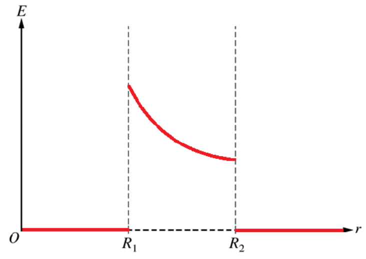


Figure 2

B For a multistep derivation that includes the equation $C = \dfrac{Q}{\Delta V}$ — **Point B1**

For indicating that C with the dielectric material inserted is C without the dielectric material inserted multiplied by κ (e.g., $C = \kappa\dfrac{Q}{\Delta V}$) — **Point B2**

For substitutions of both the total charge Q and the potential difference ΔV that are consistent with part A — **Point B3**

Example Response

Without the dielectric material

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\sigma_1(2\pi L R_1)}{\left(\dfrac{\sigma_1 R_1}{\varepsilon_0}\ln\left(\dfrac{R_2}{R_1}\right)\right)}$$

$$C = \frac{2\pi L\varepsilon_0}{\ln\left(\dfrac{R_2}{R_1}\right)}$$

With the dielectric material

$$C = \kappa\frac{Q}{\Delta V}$$

$$C = \frac{2\pi L\kappa\varepsilon_0}{\ln\left(\dfrac{R_2}{R_1}\right)}$$

Question 1: Free-Response Question — 15 points

(a) For using a correct equation for electric flux — **1 point**

Example Response

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

For the correct numerical answer — **1 point**

Scoring Note: Correct units are not required to earn this point.

Example Response

$$\Phi_E = 113\,\frac{\text{N}\cdot\text{m}^2}{\text{C}}$$

Example Solution

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

$$\Phi_E = \frac{1.0\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}}$$

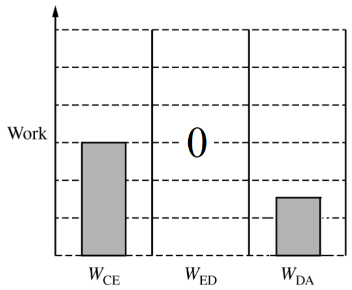
$$\Phi_E = 113\,\frac{\text{N}\cdot\text{m}^2}{\text{C}}$$

Total for part (a) — 2 points

(b)(i) For indicating that $W_{\text{ED}} = 0$ — **1 point**

For drawing a bar representing W_{DA} that has a height of approximately one and a half units — **1 point**

Example Response



AP Physics C: **Electricity and Magnetism: 2024 Set 2**

(b)(ii) For using an equation that relates the electric field to potential difference — **1 point**

Scoring Note: This point can be earned if the response begins with a correct relationship between electric field and potential difference in which numerical values are already substituted.

Example Responses

$$E_y = -\frac{dV}{dy} \quad \textbf{OR} \quad \left|E_y\right| = \left|-\frac{dV}{dy}\right| \quad \textbf{OR} \quad \left|E_y\right| = \left|-\frac{\Delta V}{\Delta y}\right| \quad \textbf{OR} \quad \Delta V = -\int \vec{E}\cdot d\vec{r}$$

For correct substitutions of values of electric potential and the distance between equipotential lines that can be used to calculate the approximate magnitude of the electric field at Position B — **1 point**

Example Response

$$\left|E_y\right| = \left|-\frac{-20.0\ \text{V} - (-10.0\ \text{V})}{1.4\ \text{m}}\right|$$

Example Solution

$$E_y = -\frac{dV}{dy}$$

$$\left|E_y\right| = \left|-\frac{dV}{dy}\right|$$

$$\left|E_y\right| = \left|-\frac{\Delta V}{\Delta y}\right|$$

$$\left|E_y\right| = \left|-\frac{-20.0\ \text{V} - (-10.0\ \text{V})}{1.4\ \text{m}}\right|$$

$$\left|E_y\right| = 7.1\,\frac{\text{V}}{\text{m}}$$

Total for part (b) — 4 points

(c) For selecting only $-y$ with an attempt at a relevant justification — **1 point**

For indicating that the direction of the electric field vector is perpendicular to a line that is tangent to the equipotential line at Position C — **1 point**

For indicating one of the following: — **1 point**

- The test charge moves from a higher electric potential to a lower electric potential.
- The test charge and the rod have charges of the opposite sign.
- The test charge and the sphere have charges of the same sign.

Example Response

$-y$. The direction of an electric field vector is perpendicular to an equipotential line. Because the test charge has a positive charge, the test charge would move from a position of higher electric potential to a position of lower electric potential when an electric force is exerted on the test charge. Therefore, at Position C, the electric force is downward because that is the direction that is perpendicular to the equipotential line and in the direction of decreasing electric potential.

Total for part (c) — 3 points

(d)(i)	For using an appropriate equation for determining the electric potential from a line of uniform charge	**1 point**

Example Responses

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i} \quad \textbf{OR} \quad V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r} \quad \textbf{OR} \quad \Delta V = -\int\vec{E}\cdot d\vec{r}$$

For a correct determination of r, the distance between Point P and a point on the line of uniform charge — **1 point**

Example Responses

$$V_P = k\sum\frac{Q}{y_P - y} \quad \textbf{OR} \quad V_P = k\int\left(\frac{1}{y_P - y}\right)dq$$

For a correct integral with $\lambda\, dy$ substituted for dq — **1 point**

Example Response

$$V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$$

For the correct limits of integration — **1 point**

Example Response

$$V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy$$

Example Solutions

$$V = \frac{1}{4\pi\varepsilon_0}\sum\frac{q_i}{r_i}$$

$$V_P = k\int\left(\frac{1}{y_P - y}\right)dq$$

$$dq = -\lambda\, dy$$

$$V_P = -k\lambda\int\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = -k\lambda\int_0^{2L}\left(\frac{1}{y_P - y}\right)dy$$

$$V_P = k\lambda\ln(y_P - y)\,\Big|_0^{2L} = -k\lambda\ln(y_P - y)\,\Big|_{2L}^{0}$$

$$V_P = -k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

OR

$$\Delta V = -\int\vec{E}\cdot d\vec{r}$$

$$E(y) = \int\frac{k}{r^2}dq$$

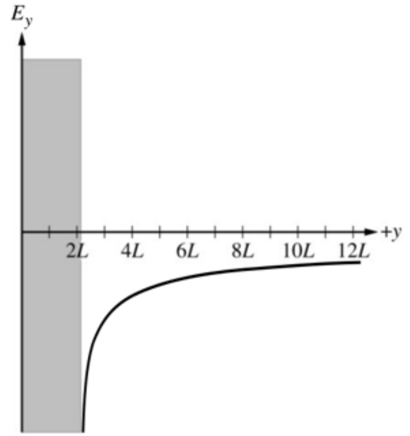
$$E(y) = \int_0^{2L}\frac{k\lambda}{(y - y')^2}dy' = k\lambda\left(\frac{1}{y - 2L} - \frac{1}{y}\right)$$

$$V_P = -\int_\infty^{y_P}E(y)\,dy = -k\lambda\int_\infty^{y_P}\left(\frac{1}{y - 2L} - \frac{1}{y}\right)dy$$

$$V_P = k\lambda\ln\left(\frac{y_P}{y_P - 2L}\right)$$

(d)(ii)	For sketching a curve or line that continually approaches the horizontal axis as position increases	**1 point**
	For sketching a concave down curve that is always negative	**1 point**

Example Response



Total for part (d) 6 points

Total for question 1 15 points

Question 1: Free-Response Question — 15 points

(a) For correctly determining the charge on the outer surface of the shell — **1 point**

Example Response

$$q_{net} = q_{inner} + q_{outer}$$
$$q_{outer} = q_{net} - q_{inner}$$
$$q_{outer} = +3Q - (+Q)$$
$$q_{outer} = +2Q$$

Scoring Note: A correct response may earn a point even if no work is shown.

Total for part (a) 1 points

(b) For using Gauss's law by substituting for either the area or the enclosed charge — **1 point**

Example Response

$$\frac{q_{enc}}{\varepsilon_0} = \oint E \bullet dA = E\left(4\pi r^2\right)$$

For using a correct expression for q_{enc} as a function of r — **1 point**

Example Response

$$q_{enc} = \rho V = \left(\frac{-Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = -Q\frac{r^3}{R^3}$$

Scoring Note: The response may earn this point regardless of the sign of q_{enc}.

For using the correct area — **1 point**

Example Response

$$E = q_{enc}\frac{1}{\varepsilon_0\left(4\pi r^2\right)} = \left(-Q\frac{r^3}{R^3}\right)\frac{1}{\varepsilon_0\left(4\pi r^2\right)}$$
$$E = -Q\frac{r}{4\pi\varepsilon_0 R^3}$$

Scoring Note: The response may earn this point regardless of the sign of E.

Total for part (b) 3 points

(c) For recognizing the electric field varies as an inverse square of the separation distance from the center of the nonconducting sphere — **1 point**

Example Response

$$E \propto \frac{1}{r^2}$$

For the correct magnitude, including units, of the electric field at $2R$ — **1 point**

Example Response

$$E_{new} = \frac{E_{old}}{4} = \frac{8\text{ N/C}}{4}$$
$$E_{new} = 2\text{ N/C}$$

Total for part (c) 2 points

(d) For using the equation relating potential difference to the electric field with an attempt at either integration limits or evaluating the integral — **1 point**

Example Response

$$V_f - V_i = -\int_{s_i}^{s_f} E\,dr$$
$$\Delta V = -\int_{r=R}^{r=4R} E\,dr$$

For substituting the correct expression for the electric field or an expression for E that is consistent with the explicit functional dependence of $E(r)$ from part (c) — **1 point**

Example Response

$$\Delta V = -\int_{r=R}^{r=4R} -\frac{Q}{4\pi\varepsilon_0 r^2}dr$$

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\int_{r=R}^{r=4R} \frac{1}{r^2}dr$$

For correctly integrating the electric field expression that was substituted in the previous point with correct limits of integration — **1 point**

Example Response

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r=R}^{r=4R} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{r=4R}^{r=R} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{4R}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{4-1}{4R}\right)$$
$$\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$$

Alternate Solution

For using a difference in potentials treating the inner sphere as a point charge by symmetry — **1 Point**

$$\Delta V = \frac{Q}{4\pi\varepsilon_0 r_f} - \frac{Q}{4\pi\varepsilon_0 r_i}$$

For using the correct charge on both of the terms; Q not Q and $3Q$ — **1 Point**

For using the correct values for r — **1 Point**

$$\Delta V = \frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 4R}$$
$$\Delta V = \frac{3Q}{16\pi\varepsilon_0 R}$$

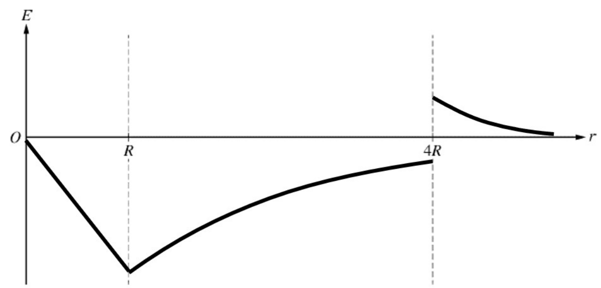
Total for part (d) 3 points

(e)(i) For indicating that E is negative and the magnitude of E increases linearly from 0 to R — **1 point**

For a curve that is continuous at R and asymptotically approaches zero in the region $R < r < 4R$ — **1 point**

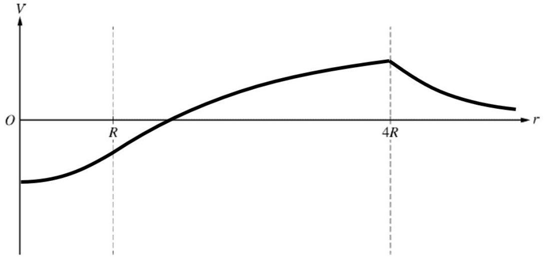
For a positive, concave-up, and decreasing curve for $r > 4R$ — **1 point**

Example Response



(e)(ii)	For a potential curve that is always increasing from 0 to $4R$	**1 point**
	For a potential curve that is decreasing for $r > 4R$	**1 point**
	For a continuous graph across all three regions	**1 point**

Example Response



Scoring Note: The intercept of the curve on the vertical axis is irrelevant. The intercept of the curve on the horizontal axis is irrelevant. The curve can, hence, cross the horizontal axis at any location or even be entirely on the negative side of the horizontal axis as long as the other criteria are met.

Total for part (e)	**6 points**
Total for question 1	**15 points**