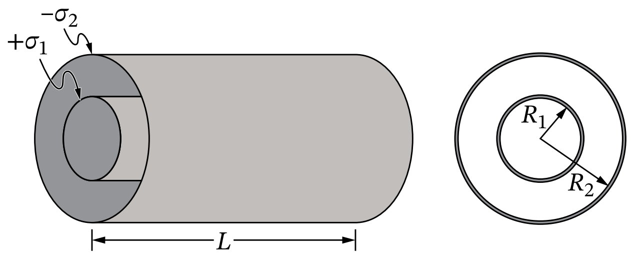
Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L. The inner shell has radius R_1 and the outer shell has radius R_2, as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Side View

Cross-Sectional View

Note: Figures not drawn to scale.

Figure 1

A.

i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1, σ_1, r, and physical constants, as appropriate.

ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1, R_2, σ_1, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

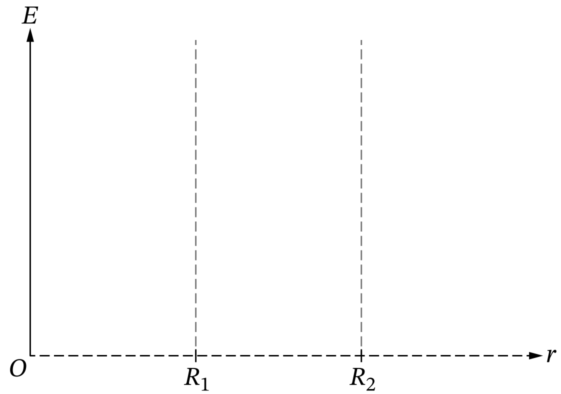
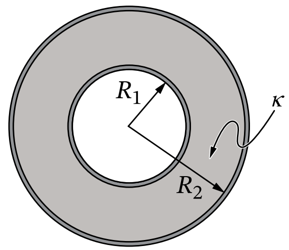


Figure 2

B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L, R_1, R_2, κ, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

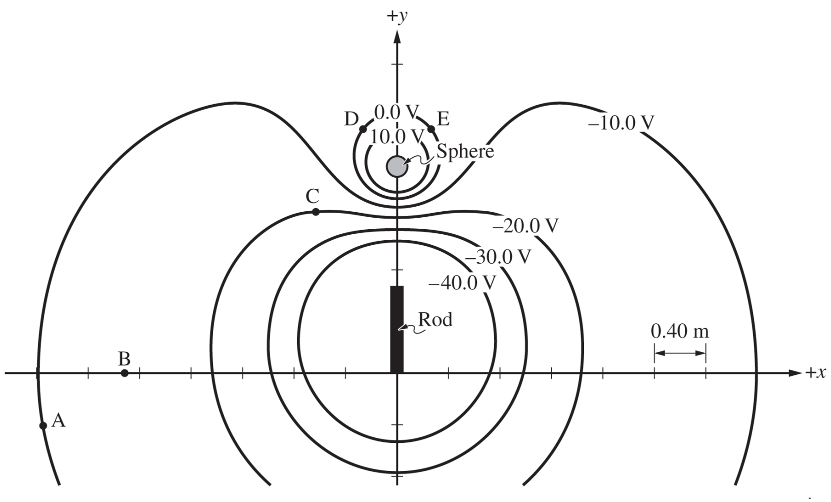


Figure 1

1. A nonconducting rod of uniform negative linear charge density is near a sphere with charge $+1.0\ \text{nC}$. The rod and sphere are held at rest on the y-axis, as shown in Figure 1. Equipotential lines and positions A, B, C, D, and E are labeled. Adjacent tick marks on the x-axis and on the y-axis are $0.40\ \text{m}$ apart.

   (a) **Calculate** the absolute value of the electric flux through the Gaussian surface whose cross section is the $0.0\ \text{V}$ equipotential line.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

A positive test charge (not shown) is placed and held at rest at Position C. An external force is applied to the test charge to move the test charge to different positions in the order of C→E→D→A. The test charge is held momentarily at rest at each position.

(b) The bar shown in Figure 2 represents the absolute value of the work W_{CE} done by the external force on the test charge to move the test charge from Position C to Position E.

i. Complete the following tasks on Figure 2.

- **Draw** a bar to represent the relative absolute value of the work W_{ED} done by the external force on the test charge to move the test charge from Position E to Position D.
- **Draw** a bar to represent the relative absolute value of the work W_{DA} done by the external force on the test charge to move the test charge from Position D to Position A.
- The height of each bar should be proportional to the value of W_{CE}. If $W_{ED} = 0$ and/or $W_{DA} = 0$, write a “0” in the corresponding columns, as appropriate.

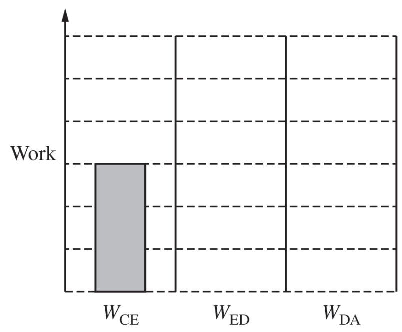


Figure 2

ii. **Calculate** the approximate magnitude of the x-component of the electric field at Position B.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

The positive test charge is placed at Position C. The test charge is then released from rest.

(c) **Indicate** the direction (not components) of the net electric force exerted on the test charge immediately after the test charge is released from rest.

_____ $+x$ _____ $+y$ _____ Directly away from the sphere

_____ $-x$ _____ $-y$ _____ Directly toward the sphere

Without using equations, **justify** your answer using physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

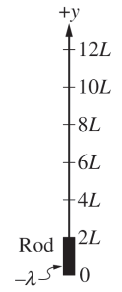


Figure 3

The sphere and the test charge are removed. The rod has length $2L$ and uniform negative linear charge density $-\lambda$. The rod is held at rest on the y-axis in the orientation shown in Figure 3. Position P (not shown) is located on the y-axis a distance y_P from the origin, where $y_P > 2L$.

(d) The electric potential V_P at y_P is $V_P = -k\lambda \ln\left(\dfrac{y_P}{y_P - 2L}\right)$.

i. Using integral calculus, **derive** the expression for V_P provided.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

ii. On Figure 4, **sketch** a graph of the y-component E_y of the electric field resulting from the rod as a function of y in the region $2L < y < 12L$.

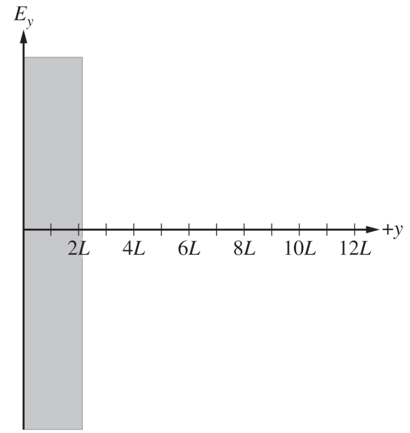


Figure 4

GO ON TO THE NEXT PAGE.

Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

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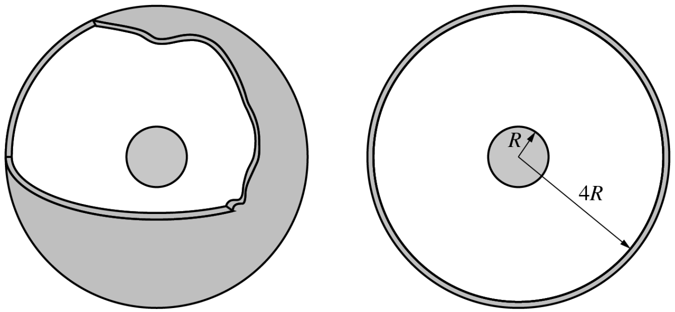


Cutout View   Cross-Section View

Note: Figures not drawn to scale.

1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of $-Q$ and the shell has a charge of $+3Q$. The radii of the inner sphere and spherical shell are R and $4R$, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

(c) The magnitude of the electric field at $r = R$ is $8\,\mathrm{N/C}$. Calculate the value of the electric field at $r = 2R$.

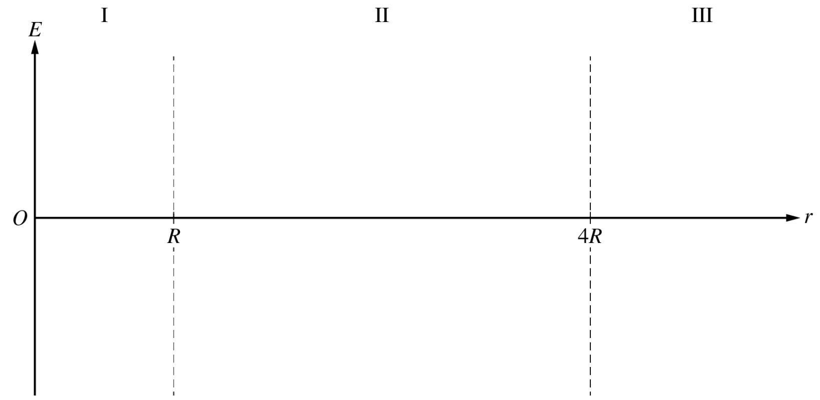
(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

GO ON TO THE NEXT PAGE.

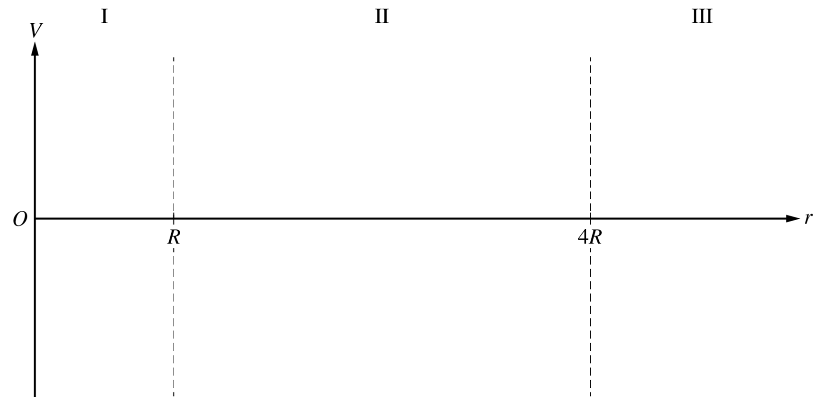
Continue your response to **QUESTION 1** on this page.

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.



ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.



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