Question 2: Translation Between Representations (TBR) — 12 points

A	For drawing a bar at $\frac{1}{4}T$	**Point A1**
	For drawing a bar at $\frac{1}{4}T$ that has a height of 3 units	**Point A2**
	For indicating that $\lvert\mathcal{E}\rvert$ is zero at times $t = 0$ and $\frac{1}{2}T$	**Point A3**

Example Response

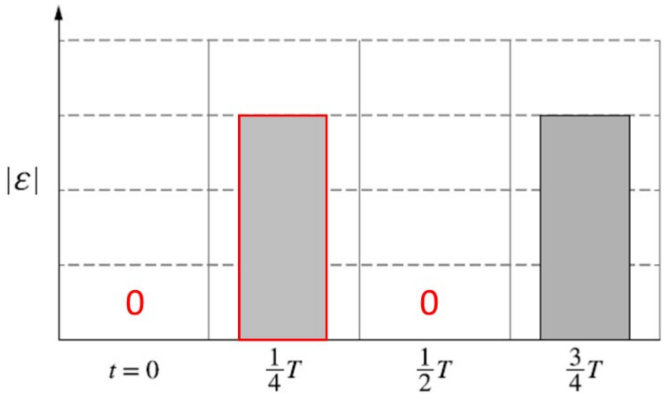


Figure 2

B	For a multistep derivation that includes the equation $\mathcal{E} = -\frac{d\Phi_B}{dt}$ **Scoring Note:** The negative sign does not have to be present in the expression for this point to be earned.	**Point B1**
	For a correct expression for the induced emf (e.g., $\mathcal{E} = BA\omega \sin(\omega t)$)	**Point B2**
	For using Ohm's law to relate current and emf (e.g., $I = \frac{\mathcal{E}}{R}$)	**Point B3**
	For a correct expression for the absolute value of the maximum induced current (e.g., $I = \frac{BA\omega}{R}$)	**Point B4**

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\left[BA\cos\left(\omega t\right)\right] = BA\omega\sin\left(\omega t\right)$$

$$\mathcal{E}_{\text{max}} = BA\omega$$

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$

$$I_{\text{max}} = \frac{\mathcal{E}_{\text{max}}}{R} = \frac{BA\omega}{R}$$

C	For a curve that is approximately sinusoidal	**Point C1**
	For a curve that shows exactly two cycles	**Point C2**
	For a curve that starts at the origin and has equal maximum values and equal minimum values **Scoring Note:** A curve that starts and ends at $P = 0$ and has a single maximum can earn this point.	**Point C3**

Example Response

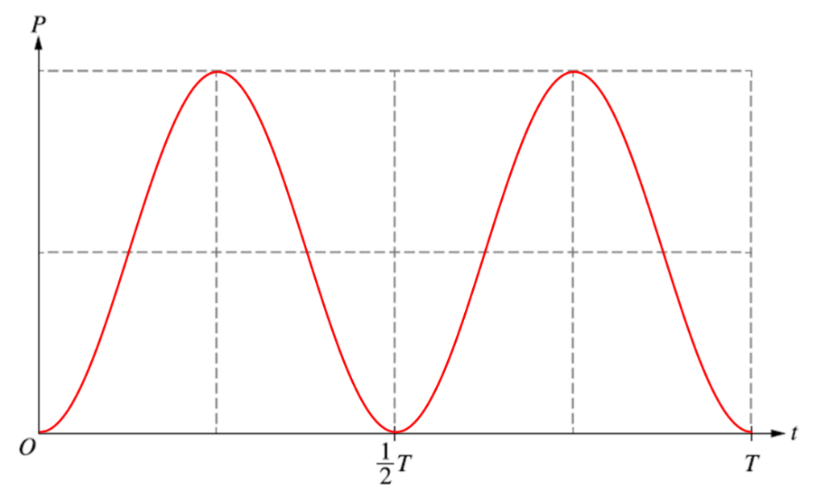


Figure 3

D	For correctly indicating whether the representations are consistent in parts A and C	**Point D1**
	For a correct justification that indicates that the maxima and/or minima of the graph in part C align with the bars drawn in part A because P is proportional to $\mathcal{E}^2$	**Point D2**

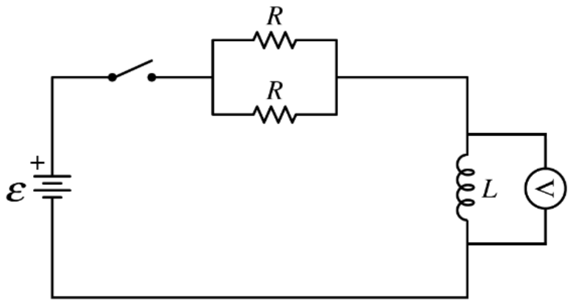
Example Response

Yes, part C is consistent with part A. P is proportional to $\mathcal{E}^2$. *When* $|\mathcal{E}|$ *is maximum, P is maximum.*

Question 2: Free-Response Question — 15 points

(a)(i)	For correctly placing the voltmeter in parallel with the inductor	**1 point**

Example Response



(a)(ii)	For a procedure that indicates that the voltmeter should be used to measure the potential difference for at least one time	**1 point**
	For measuring the potential difference from immediately after the switch is closed to when steady-state conditions have been established or during a time interval that would allow the time constant to be determined	**1 point**

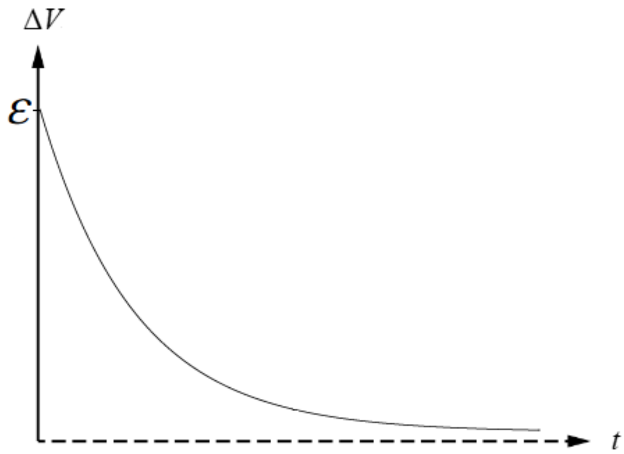
Example Response

Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.

Total for part (a) — 3 points

(b)(i)	For correctly labeling potential difference on the vertical axis and time on the horizontal axis	**1 point**
	For a concave-up and decreasing curve	**1 point**
	For a curve that asymptotically approaches zero, or a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i)	**1 point**
	For including a vertical intercept or a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i) that is correctly labeled as $\mathcal{E}$	**1 point**

Example Response



(b)(ii) For indicating that a curve fit to the graph is that of an exponential function or for correctly relating the area under the curve to the current in the circuit **1 point**

Alternate Solution

For indicating the time on the graph where the potential difference is approximately $0.37\mathcal{E}$, or $0.63\mathcal{E}$ for a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i)

For indicating that the coefficient in front of the t of the curve-fit equation is equal to $\frac{R}{2L}$ or for correctly relating R to L and the current **1 point**

Alternate Solution

For indicating that the time constant is equal to $\frac{2L}{R}$

Example Response

The data in the graph should be fit with an exponential function for the equation $V_L = \mathcal{E}\left(e^{-t\frac{R}{2L}}\right)$. Because $\mathcal{E}$ and L are known, R can be calculated.

Alternate Example Response

The potential difference at $0.37\mathcal{E}$ along the vertical axis corresponds to the time constant along the horizontal axis. Because the time constant is equal to $\frac{2L}{R}$, and L is known, R can be calculated.

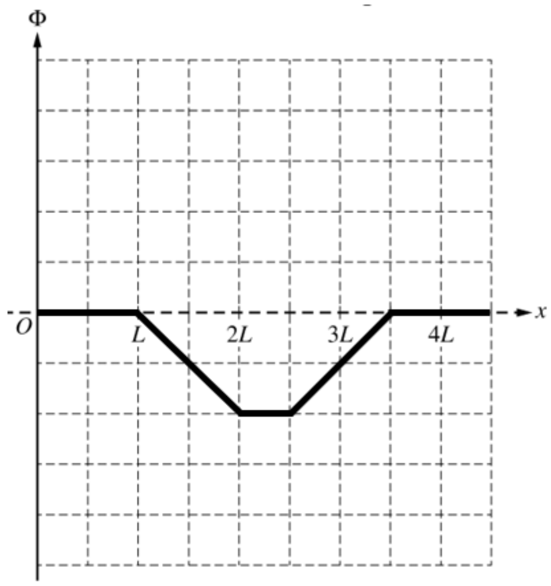
Total for part (b) 6 points

(c)	For a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule	**1 point**
	Example Response $\mathcal{E} - \Delta V_R - \Delta V_L = 0$	
	For indicating that the potential difference across the parallel combination of resistors is $I\left(\frac{R}{2}\right)$	**1 point**
	For indicating that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$	**1 point**
	Example Solution $\mathcal{E} - \Delta V_R - \Delta V_L = 0$ $\mathcal{E} - I\left(\frac{R}{2}\right) - L\frac{dI}{dt} = 0$ $\frac{\mathcal{E}}{L} - \frac{IR}{2L} = \frac{dI}{dt}$ $-\frac{R}{L}\left(\frac{I}{2} - \frac{\mathcal{E}}{R}\right) = \frac{dI}{dt}$	
	Total for part (c)	**3 points**
(d)	For selecting $\lvert\Delta V_2\rvert > \lvert\Delta V_1\rvert$ with an attempt at a relevant justification	**1 point**
	For indicating that the potential difference across the original inductor is zero when steady-state conditions are established	**1 point**
	For indicating that the potential difference across the inductor of nonnegligible resistance is nonzero when steady-state conditions are established	**1 point**
	Example Response $\lvert\Delta V_2\rvert > \lvert\Delta V_1\rvert$. *The potential difference* ΔV_1 *across the original, ideal inductor is zero at steady state. The potential difference across the new inductor would be nonzero due to the inductor's nonnegligible resistance. The value of* $\lvert\Delta V_2\rvert$ *would be the product of the steady-state current and the inductor's resistance.*	
	Total for part (d)	**3 points**
	Total for question 2	**15 points**

Question 3: Free-Response Question — 15 points

(a)	For a sketch that indicates that the magnetic flux is zero from $x = 0$ to $x = L$ and for the region $x > 3.5L$	**1 point**
	For a sketch that indicates that the absolute value of the magnetic flux increases with a constant slope in the region $L < x < 2L$	**1 point**
	For a sketch that indicates that the magnetic flux is constant and nonzero in the region $2L < x < 2.5L$	**1 point**
	For a sketch that indicates that the absolute value of the magnetic flux decreases from a nonzero value at $x = 2.5L$ to zero at $x = 3.5L$	**1 point**

Example Response



Scoring Note: A sketch that is reflected across the horizontal axis can earn the four points.
Scoring Note: The absolute values of the slopes in regions $L < x < 2D$ and $2.5L < x < 3.5L$ are not considered for earning points in the response in part (a).

Total for part (a) 4 points

(b)(i)	For selecting Counterclockwise with an attempt at a relevant justification	**1 point**
	For indicating the magnetic flux through the loop is increasing in the $-z$ -direction **OR** For indicating the magnetic field due to the induced current will be directed in the $+z$ -direction	**1 point**

Example Response

Counterclockwise. The magnetic flux through the loop is increasing in the $-z$ -direction. Therefore, a magnetic field is induced by the current in the loop to oppose the increasing magnetic flux. To establish this field, the current must be counterclockwise.

(b)(ii) For a multistep derivation that includes Faraday's law **1 point**

Scoring Note: The point can be earned if a negative sign is not included.

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA)$$

For indicating $\frac{dA}{dt}$ is $2Lv$ **1 point**

Scoring Note: The point can be earned if a negative sign is included.

Example Response

$$\mathcal{E} = B\frac{dA}{dt} = 2BLv$$

For using Ohm's law, resulting in an expression for I_R that is consistent with the expression determined for the emf $\mathcal{E}$ **1 point**

Scoring Note: The point can be earned if a negative sign is included.

Example Response

$$I_\mathrm{R} = \frac{2BLv}{R}$$

Example Solution

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\left(\int \vec{B} \cdot d\vec{A}\right)$$

$$\mathcal{E} = -\frac{d}{dt}(BA) = -\frac{d}{dt}(-B(2L)x)$$

$$\mathcal{E} = 2BL\frac{dx}{dt}$$

$$\mathcal{E} = 2BLv$$

$$I = \frac{\Delta V}{R}$$

$$I_\mathrm{R} = \frac{\mathcal{E}}{R}$$

$$I_\mathrm{R} = \frac{2BLv}{R}$$

(b)(iii) For using a correct general expression for P **1 point**

Example Responses

$P = I^2 R$ **OR** $P = \frac{(\Delta V)^2}{R}$ **OR** $P = I\Delta V$

For an expression for P that is consistent with the response in part (b)(ii) that is in terms of the provided quantities only **1 point**

Example Response

$$P = \frac{4B^2L^2v^2}{R}$$

Example Solution

$$P = I^2 R$$
$$P = \frac{(2BLv)^2}{R}$$
$$P = \frac{4B^2L^2v^2}{R}$$

Total for part (b) 7 points

(c) For selecting $E_{\text{new}} < E_{\text{original}}$ with an attempt at a relevant justification **1 point**

For indicating one of the following: **1 point**

- The change in magnetic flux is greater for the original scenario.
- The induced current occurs for a greater amount of time in the original scenario.
- The induced emf occurs for a greater amount of time in the original scenario.

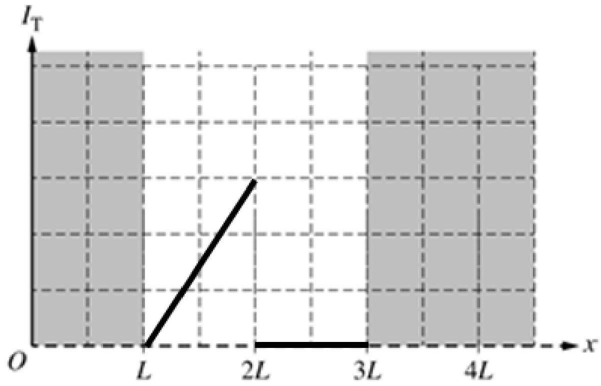
Example Response

The change in magnetic flux is greater in the original scenario, which produces an emf *and current for a longer time. Therefore,* $E_{\text{new}} < E_{\text{original}}$.

Total for part (c) 2 points

(d)	For a sketch that has an absolute value that only increases from $x = L$ to $x = 2L$	**1 point**
	For a sketch that is zero from $2L$ to $3L$	**1 point**

Example Response



Scoring Note: A response that is reflected across the horizontal axis earns both points.
Scoring Note: Any portion of the graph before $x = L$ and after $x = 3L$ are not scored.

Total for part (d) 2 points

Total for question 3 15 points