**Question 2: Version J**

**2.** A rotating, circular, conducting loop of area $A$ and resistance $R$ is in an external uniform magnetic field of magnitude $B$ that is directed in the $-z$-direction. At time $t = 0$, the magnetic field is perpendicular to the plane of the loop, as shown in Figure 1. The loop is rotating with constant angular speed $\omega$ and period $T$ about the dashed line that is along the diameter of the loop. The value of the magnetic flux through the loop as a function of time $t$ is $\Phi = BA\cos(\omega t)$.

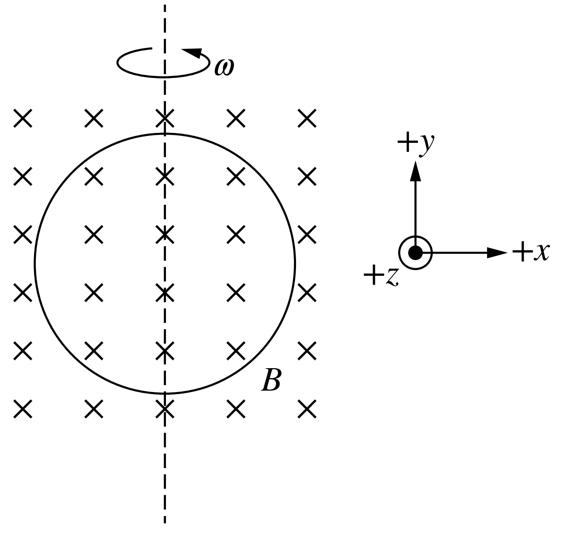


Figure 1

**A.** The absolute value of the induced emf in the loop is $|\varepsilon|$. The partially completed bar chart in Figure 2 shows a bar that represents $|\varepsilon|$ at $t = \frac{3}{4}T$. In Figure 2, **draw** bars to represent $|\varepsilon|$ at times $t = 0$, $\frac{1}{4}T$, and $\frac{1}{2}T$ relative to $|\varepsilon|$ shown at $\frac{3}{4}T$. If $|\varepsilon| = 0$, **write** a “0” in that column.

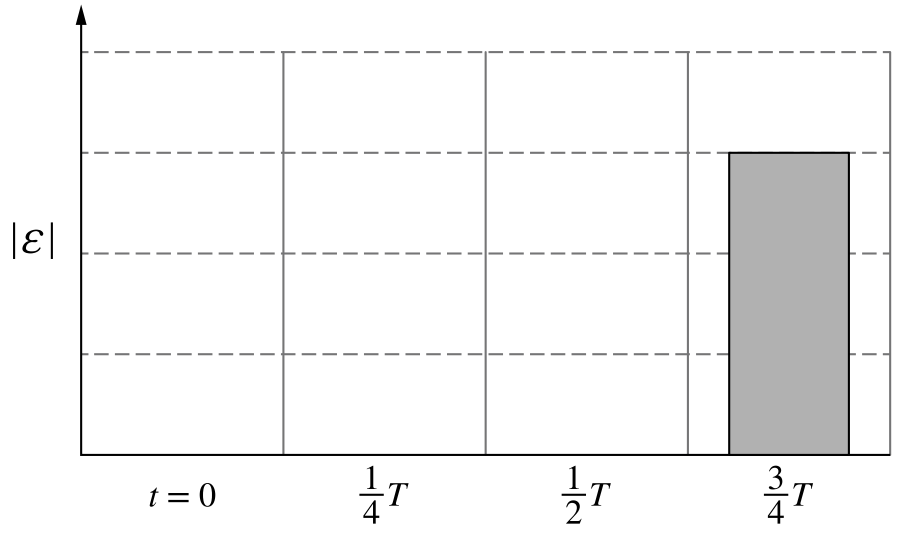


Figure 2

**B. Derive** an expression for the maximum induced current in the loop in terms of $A$, $R$, $B$, $\omega$, and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

**C.** On the axes shown in Figure 3, **sketch** a graph of the instantaneous power $P$ dissipated by the loop as a function of $t$ during the time interval $0 \le t \le T$.

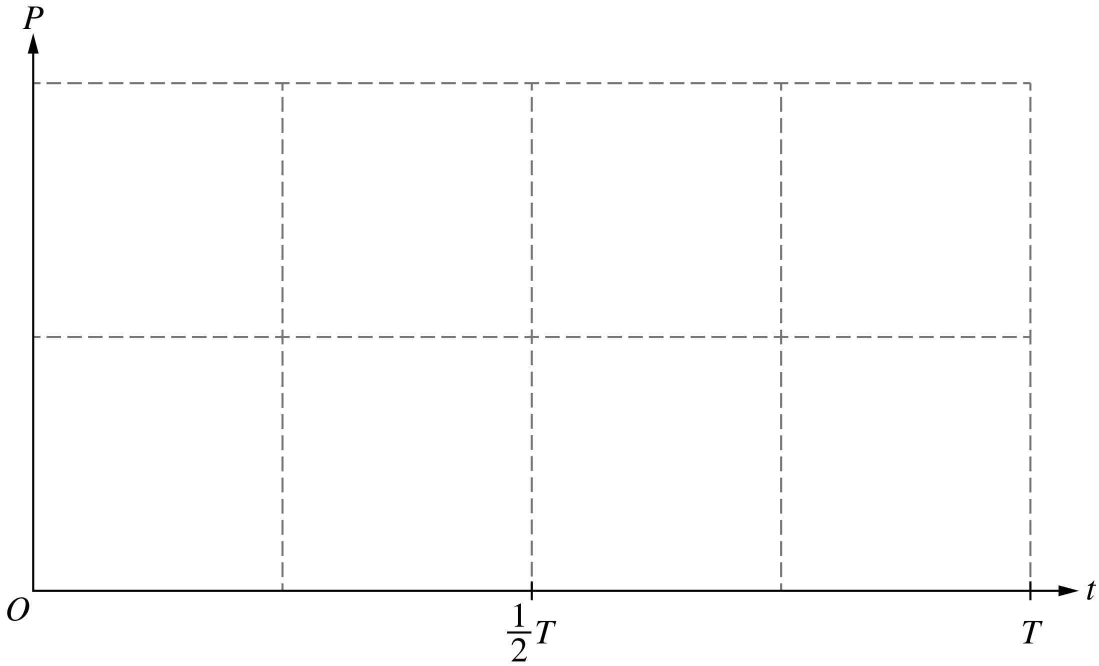


Figure 3

**D. Indicate** whether the sketch you drew in part C is or is not consistent with the bars that you drew in part A. Briefly **justify** your answer by referencing the functional dependence between $P$ and $|\varepsilon|$.

Begin your response to **QUESTION 2** on this page.

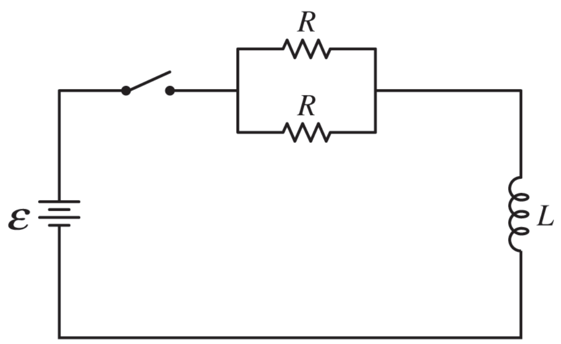


Figure 1

2. Students are asked to determine the resistance $R$ of two identical resistors. The resistors are in parallel with each other and are connected in series to a battery of known emf $\mathcal{E}$, an inductor of known inductance $L$, and a switch, as shown in Figure 1. The students have access to a voltmeter that can measure potential difference as a function of time. The students are required to measure a quantity that decreases with time to determine $R$.

(a)

i. On the circuit diagram shown in Figure 1, **draw** the voltmeter, using the following symbol, with connections that would allow the students to correctly measure a potential difference that decreases with time.


Voltmeter Symbol

ii. **Describe** a procedure for collecting data that would allow the students to graphically determine the experimental value for $R$ using the measured quantity that decreases with time. Provide enough detail so that another student could replicate the experiment.

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Continue your response to **QUESTION 2** on this page.

(b)

i. On the axes shown in Figure 2, produce a graph that represents the expected trend of the data by completing the following tasks.

- **Label** the quantities graphed on the vertical and horizontal axes.
- **Sketch** a line or curve that represents the expected trend of the collected data.
- **Label** any appropriate intercepts and/or asymptotes in terms of the quantities provided.

Figure 2

ii. **Describe** how the information from the graph in part (b)(i) would be used to determine the experimental value for $R$.

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Continue your response to **QUESTION 2** on this page.

(c) Starting with an appropriate application of Kirchhoff's loop rule, **derive**, but do NOT solve, a differential equation that can be used to determine the current $I$ in the inductor at time $t$ after the switch is closed. Express your answer in terms of $R$, $\mathcal{E}$, $L$, $t$, and physical constants, as appropriate.

After reaching steady state, the absolute value of the potential difference across the inductor is $|\Delta V_1|$. The students replace the original inductor with a new inductor that has nonnegligible resistance. The experiment is repeated. After a long time, the absolute value of the potential difference across the new inductor is $|\Delta V_2|$.

(d) **Indicate** whether $|\Delta V_2|$ is greater than, less than, or equal to $|\Delta V_1|$.

____ $|\Delta V_2| > |\Delta V_1|$ ____ $|\Delta V_2| < |\Delta V_1|$ ____ $|\Delta V_2| = |\Delta V_1|$

**Justify** your answer.

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Begin your response to **QUESTION 3** on this page.

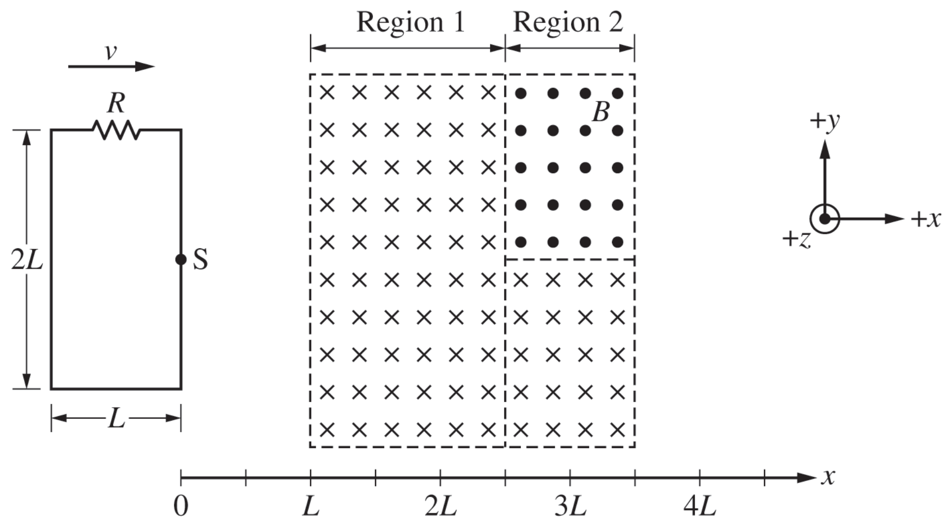


Figure 1

3. A wire is connected to a resistor of resistance $R$ to form a rigid rectangular loop of width $L$ and height $2L$. An external force is exerted on the loop so that the loop always moves with constant speed $v$ in the $+x$-direction, as shown in Figure 1. The loop then enters Region 1 of external uniform magnetic field of magnitude $B$ that is directed in the $-z$-direction. Region 1 has boundaries $x = L$ and $x = 2.5L$. The loop later enters Region 2 with two external, uniform magnetic fields, each of magnitude $B$, that are parallel but are directed in opposite $z$-directions. Region 2 has boundaries $x = 2.5L$ and $x = 3.5L$. Point S is the midpoint of the leading edge of the loop and is aligned with the horizontal boundary in Region 2 that separates the two magnetic fields.

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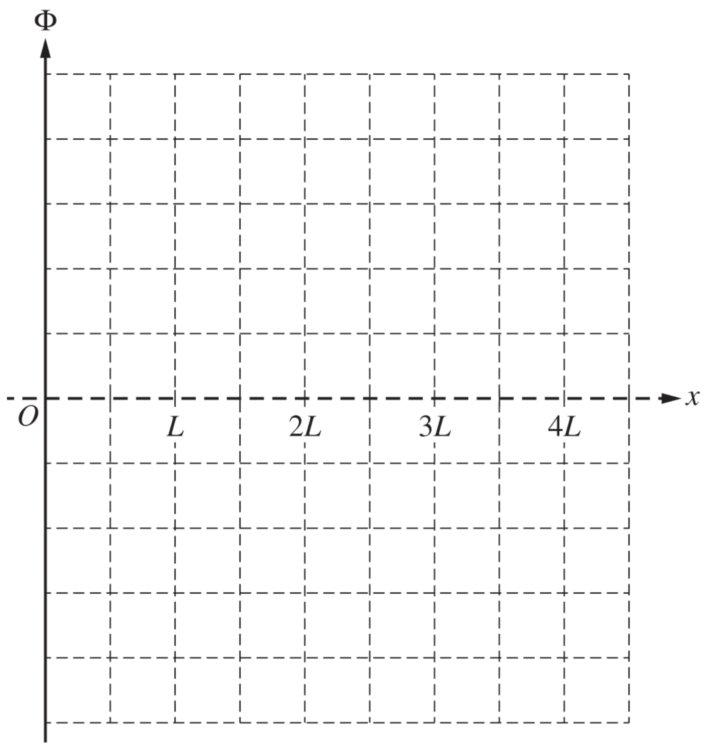
Continue your response to **QUESTION 3** on this page.

(a) On the following axes, **sketch** a graph of the magnetic flux $\Phi$ through the rectangular loop as a function of the position $x$ of Point S from $x = 0$ to $x = 4.5L$. The $+z$-direction indicated in Figure 1 corresponds to $+\Phi$.




(b) Consider the instant when Point S reaches $x = 1.5L$.

i. **Indicate** whether the current $I_R$ that is induced in the rectangular loop when Point S reaches $x = 1.5L$ is clockwise, counterclockwise, or zero.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

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Continue your response to **QUESTION 3** on this page.

ii. **Derive** an expression for $I_R$ when Point S reaches $x = 1.5L$. If $I_R = 0$, indicate how the derived expression shows that $I_R = 0$. Express your answer in terms of $R$, $L$, $v$, $B$, and physical constants, as appropriate.

iii. **Derive** an expression for the power $P$ dissipated by the resistor when Point S reaches $x = 1.5L$. Express your answer in terms of $R$, $L$, $v$, $B$, and physical constants, as appropriate.

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The total energy dissipated by the resistor in the rectangular loop as Point S moves from $x = 0$ to $x = 4.5L$ is $E_{original}$.

The vertical boundary between regions 1 and 2 is now shifted to $x = 1.5L$. After the boundary is shifted, the rectangular loop again moves with speed $v$ in the $+x$-direction, as shown in Figure 2. The total energy dissipated by the resistor as Point S moves from $x = 0$ to $x = 4.5L$ is $E_{new}$.

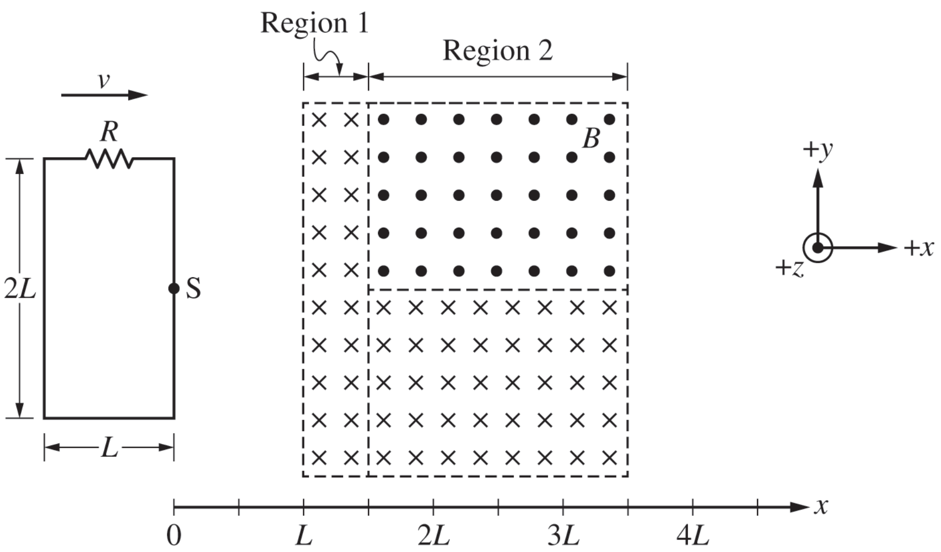


Figure 2

(c) **Indicate** whether $E_{new}$ is greater than, less than, or equal to $E_{original}$.

______ $E_{new} > E_{original}$ ______ $E_{new} < E_{original}$ ______ $E_{new} = E_{original}$

Briefly **justify** your answer.

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Continue your response to **QUESTION 3** on this page.

The original magnetic fields are modified so that the region $L < x < 3.5L$ contains an external uniform magnetic field of magnitude $B$ that is directed in the $-z$-direction.

A new wire is connected to a resistor of resistance $R$ to form a rigid triangular loop with base length $L$ and height $2L$. An external force is exerted on the loop so that the loop always moves with speed $v$ in the $+x$-direction, as shown in Figure 3. Point S represents the lower-leading corner of the loop.

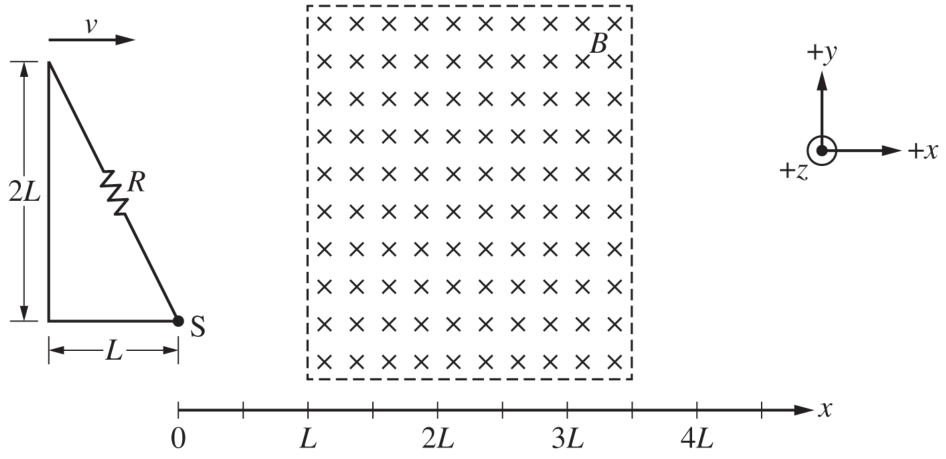
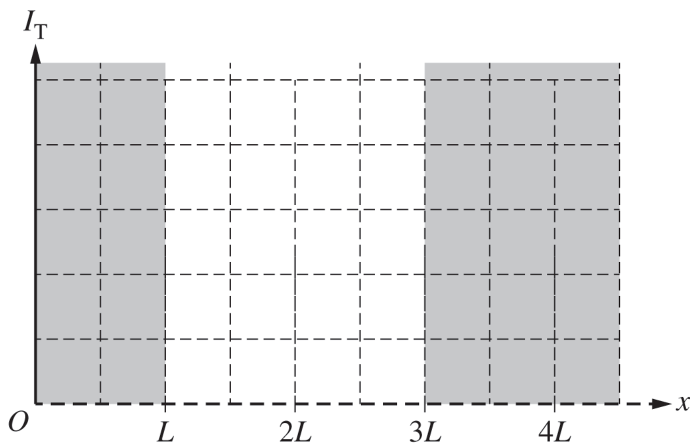



Figure 3

(d) On the following axes, **sketch** a graph of the induced current $I_T$ in the triangular loop as Point S moves from $x = L$ to $x = 3L$.

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**END OF EXAM**