Question 2: Translation Between Representations (TBR) — 12 points

A

For drawing a bar at $\frac{1}{4}T$ — **Point A1**

For drawing a bar at $\frac{1}{4}T$ that has a height of 3 units — **Point A2**

For indicating that $|\mathcal{E}|$ is zero at times $t = 0$ and $\frac{1}{2}T$ — **Point A3**

Example Response

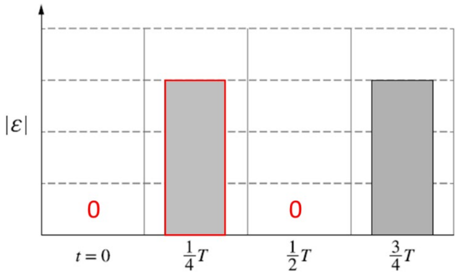


Figure 2

B

For a multistep derivation that includes the equation $\mathcal{E} = -\frac{d\Phi_B}{dt}$ — **Point B1**

Scoring Note: The negative sign does not have to be present in the expression for this point to be earned.

For a correct expression for the induced emf (e.g., $\mathcal{E} = BA\omega\sin(\omega t)$) — **Point B2**

For using Ohm's law to relate current and emf (e.g., $I = \frac{\mathcal{E}}{R}$) — **Point B3**

For a correct expression for the absolute value of the maximum induced current (e.g., $I = \frac{BA\omega}{R}$) — **Point B4**

Example Response

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -\frac{d}{dt}\left[BA\cos(\omega t)\right] = BA\omega\sin(\omega t)$$

$$\mathcal{E}_{\text{max}} = BA\omega$$

$$I = \frac{\Delta V}{R} = \frac{\mathcal{E}}{R}$$

$$I_{\text{max}} = \frac{\mathcal{E}_{\text{max}}}{R} = \frac{BA\omega}{R}$$

C

For a curve that is approximately sinusoidal — **Point C1**

For a curve that shows exactly two cycles — **Point C2**

For a curve that starts at the origin and has equal maximum values and equal minimum values — **Point C3**

Scoring Note: A curve that starts and ends at $P = 0$ and has a single maximum can earn this point.

Example Response

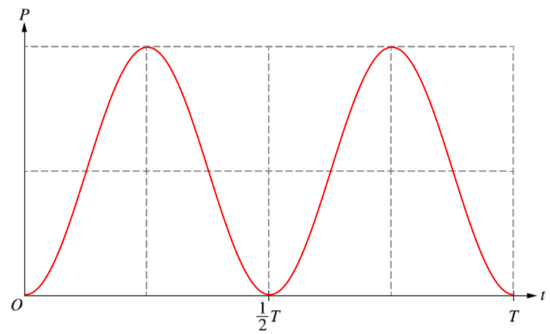


Figure 3

D

For correctly indicating whether the representations are consistent in parts A and C — **Point D1**

For a correct justification that indicates that the maxima and/or minima of the graph in part C align with the bars drawn in part A because P is proportional to $\mathcal{E}^2$ — **Point D2**

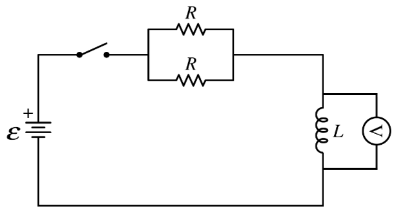
Example Response

Yes, part C is consistent with part A. P is proportional to $\mathcal{E}^2$. *When* $|\mathcal{E}|$ *is maximum, P is maximum.*

Question 2: Free-Response Question — 15 points

(a)(i) For correctly placing the voltmeter in parallel with the inductor — **1 point**

Example Response



(a)(ii) For a procedure that indicates that the voltmeter should be used to measure the potential difference for at least one time — **1 point**

For measuring the potential difference from immediately after the switch is closed to when steady-state conditions have been established or during a time interval that would allow the time constant to be determined — **1 point**

Example Response

Close the switch. Using the voltmeter, record the potential difference as a function of time until steady-state conditions are established.

Total for part (a) 3 points

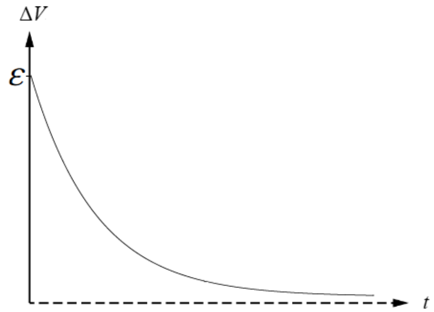
(b)(i) For correctly labeling potential difference on the vertical axis and time on the horizontal axis — **1 point**

For a concave-up and decreasing curve — **1 point**

For a curve that asymptotically approaches zero, or a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i) — **1 point**

For including a vertical intercept or a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i) that is correctly labeled as $\mathcal{E}$ — **1 point**

Example Response



(b)(ii) For indicating that a curve fit to the graph is that of an exponential function or for correctly relating the area under the curve to the current in the circuit — **1 point**

Alternate Solution

For indicating the time on the graph where the potential difference is approximately $0.37\mathcal{E}$*, or* $0.63\mathcal{E}$ *for a curve that starts at the origin and approaches a horizontal asymptote that is consistent with the placement of the voltmeter in the response in part (a)(i)*

For indicating that the coefficient in front of the t of the curve-fit equation is equal to $\frac{R}{2L}$ or for correctly relating R to L and the current — **1 point**

Alternate Solution

For indicating that the time constant is equal to $\frac{2L}{R}$

Example Response

The data in the graph should be fit with an exponential function for the equation $V_L = \mathcal{E}\left(e^{-t\frac{R}{2L}}\right)$. *Because* $\mathcal{E}$ *and* L *are known,* R *can be calculated.*

Alternate Example Response

The potential difference at $0.37\mathcal{E}$ *along the vertical axis corresponds to the time constant along the horizontal axis. Because the time constant is equal to* $\frac{2L}{R}$*, and* L *is known,* R *can be calculated.*

Total for part (b) 6 points

(c)	For a multi-step derivation that begins with an attempt at using Kirchhoff's loop rule	**1 point**

Example Response

$\mathcal{E} - \Delta V_R - \Delta V_L = 0$

For indicating that the potential difference across the parallel combination of resistors is $I\left(\frac{R}{2}\right)$	**1 point**
For indicating that the absolute value of the potential difference across the inductor is $L\frac{dI}{dt}$	**1 point**

Example Solution

$$\mathcal{E} - \Delta V_R - \Delta V_L = 0$$
$$\mathcal{E} - I\left(\frac{R}{2}\right) - L\frac{dI}{dt} = 0$$
$$\frac{\mathcal{E}}{L} - \frac{IR}{2L} = \frac{dI}{dt}$$
$$-\frac{R}{L}\left(\frac{I}{2} - \frac{\mathcal{E}}{R}\right) = \frac{dI}{dt}$$

Total for part (c) 3 points

(d)	For selecting $\lvert\Delta V_2\rvert > \lvert\Delta V_1\rvert$ with an attempt at a relevant justification	**1 point**
	For indicating that the potential difference across the original inductor is zero when steady-state conditions are established	**1 point**
	For indicating that the potential difference across the inductor of nonnegligible resistance is nonzero when steady-state conditions are established	**1 point**

Example Response

$\lvert\Delta V_2\rvert > \lvert\Delta V_1\rvert$. *The potential difference* ΔV_1 *across the original, ideal inductor is zero at steady state. The potential difference across the new inductor would be nonzero due to the inductor's nonnegligible resistance. The value of* $\lvert\Delta V_2\rvert$ *would be the product of the steady-state current and the inductor's resistance.*

Total for part (d) 3 points

Total for question 2 15 points

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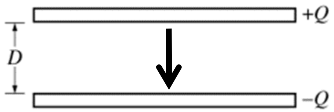
Question 1

15 points total **Distribution of points**

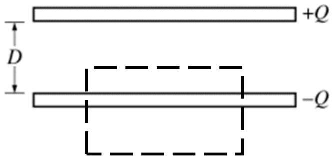
(a)

i. 1 point



For at least one arrow between the plates pointing downward from the positive plates toward the negative plate and no extraneous arrows pointing in any other direction — 1 point

ii. 1 point



For drawing an appropriate Gaussian surface (enclosing at least the inner edge of one of the plates) that can be used to determine the electric field between the plates — 1 point

iii. 3 points

For using a correct statement of Gauss's law — 1 point

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

For applying Gauss's law, using an enclosed charge and surface area consistent with the surface drawn in part (a)-ii — 1 point

$E(A_{GS}) = \dfrac{q_{enc}}{\varepsilon_0}$ (A_{GS} is the area of the end of the Gaussian surface between the plates)

$$E = \frac{q_{enc}}{\varepsilon_0 A_{GS}} \quad \left(\text{using } \sigma = \frac{q_{enc}}{A_{GS}}\right)$$

$$E = \frac{\sigma}{\varepsilon_0} \quad \left(\text{using } \sigma = \frac{Q}{A}\right)$$

For a correct answer with work shown — 1 point

$$E = \frac{Q}{\varepsilon_0 A}$$

Question 1 (continued)

Distribution of points

(b) 1 point

Comparing the equation for electric field between parallel plates to the given equation:

$$E = \frac{Q}{\kappa \varepsilon_0 A} = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$$

For an answer consistent with part (a)-iii — 1 point

$$\kappa = \kappa_0 e^{-x/D}$$

(c)

i. 1 point

Using the equation relating the electric field to potential difference

$$E = -\frac{dV}{dx}$$

For a correct differential equation — 1 point

$$\frac{dV}{dx} = -\left(-\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}\right)$$

$$\frac{dV}{dx} = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$$

Alternate Solution: — *Alternate Point*

Using the equation relating the electric field to potential difference:

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

For a correct differential equation — *1 point*

$$\Delta V = -\int -\frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A} dx$$

$$\Delta V = \int \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A} dx$$

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Question 1 (continued)

Distribution of points

(c) (continued)

ii. 4 points

Separating the variables in the differential equation from part (c)(i):

$$\frac{dV}{dx} = \frac{Q}{\varepsilon_0 \kappa_0 e^{-x/D} A}$$

$$dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) e^{x/D} dx$$

For using the correct limits of integration in attempting to integrate the equation above — 1 point

$$\int_{V_0}^{V_D} dV = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right) \int_0^D e^{x/D} dx$$

For correctly integrating the equation — 1 point

$$[V]_{V_0}^{V_D} = \left(\frac{Q}{\varepsilon_0 \kappa_0 A}\right)\left[De^{x/D}\right]_0^D$$

$$(V_D - V_0) = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)\left(e^{D/D} - e^0\right)$$

For an expression that gives the correct absolute value of the potential difference between the plates — 1 point

For having the potential difference be positive — 1 point

$$\Delta V = \left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e - 1)$$

$$\Delta V = \frac{1.72QD}{\varepsilon_0 \kappa_0 A}$$

(d) 1 point

Using the equation for capacitance:

$$C = \frac{Q}{\Delta V} = \frac{Q}{\left(\frac{QD}{\varepsilon_0 \kappa_0 A}\right)(e-1)}$$

For an answer consistent with part (c)-ii — 1 point

$$C = \frac{\varepsilon_0 \kappa_0 A}{D(e-1)}$$

$$C = \frac{\varepsilon_0 \kappa_0 A}{1.72D}$$

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Question 1 (continued)

Distribution of points

(e) 3 points

For selecting $U_V > U_C$ — 1 point

For correctly comparing the capacitance or the potential difference with the varying dielectric constant to the capacitance or the potential difference with the uniform dielectric constant — 1 point

For correctly comparing the two stored energies consistent with the comparison of the capacitances or potential differences — 1 point

Example: According to the equation from part (d), $C_C > C_V$. Since $U = \frac{Q^2}{2C}$, if the charge stored on the two capacitors is the same, then $U_V > U_C$.

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Question 2

15 points total **Distribution of points**

(a)

i. 2 points

Using Ohm's law:

$V = IR$

For a correct application of Kirchhoff's loop rule — 1 point

$\mathcal{E} = Ir + IR$

$I = \dfrac{\mathcal{E}}{(r+R)}$

For a correct expression for the measured voltage across the variable resistor — 1 point

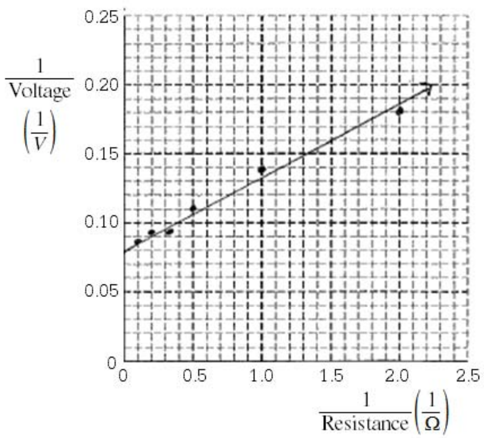
$V = \dfrac{\mathcal{E}}{(r+R)}R$

ii. 1 point

For an expression of $1/V$ as a function of $1/R$ consistent with answer from part (a)(i) — 1 point

$\dfrac{1}{V} = \left(\dfrac{r}{\mathcal{E}}\right)\dfrac{1}{R} + \dfrac{1}{\mathcal{E}}$

(b) 4 points



For correctly labeling both axes with variables and units — 1 point

For correctly scaling both axes with an acceptable and appropriate scale — 1 point

For correctly plotting the data points — 1 point

For correctly drawing a straight line that best represents the data — 1 point

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Question 2 (continued)

Distribution of points

(c)

i. 2 points

For using a value for the y-intercept consistent with the straight line drawn in part (b) — 1 point

$y = mx + b$

$b = 0.080\ 1/\text{V}$

For a correct substitution into the equation from part (a)(ii) — 1 point

$\dfrac{1}{V} = \dfrac{(r+R)}{\mathcal{E}}\dfrac{1}{R}$

$\dfrac{1}{V} = \dfrac{r}{\mathcal{E}}\dfrac{1}{R} + \dfrac{1}{\mathcal{E}}$

$b = 1/\mathcal{E}$

$\mathcal{E} = 1/(0.080\ 1/\text{V})$

$\mathcal{E} = 12.5\ \text{V}$

ii. 2 points

For calculating the slope using the straight line from part (b) and not data points — 1 point

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{(0.151 - 0.100)}{(1.50 - 0.40)} = 0.0463\ \Omega/\text{V}$

For a correct substitution into equation from part (a)(ii) — 1 point

$m = \dfrac{r}{\mathcal{E}}$

$r = m\mathcal{E}$

$r = (0.0463\ \Omega/\text{V})(12.5\ \text{V})$

$r = 0.58\ \Omega$

(d) 2 points

For using Ohm's law — 1 point

$\mathcal{E} = Ir$

For a correct substitution of values from part (c): — 1 point

$(12.5\ \text{V}) = I(0.58\ \Omega)$

$I = 21.6\ \text{A}$

(e) 2 points

For selecting "The voltmeter with high resistance" — 1 point

For a correct justification — 1 point

Example: The voltmeter acts like a resistor in a circuit with the battery. It will measure the potential difference across its own internal resistance. The higher its internal resistance, the closer its potential difference will be to the emf of the battery.

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Question 3

15 points total — **Distribution of points**

(a) 3 points

For properly using a correct equation to calculate the magnetic flux — 1 point

$\Phi_B = \int \vec{B} \bullet d\vec{A}$

For correct substitution of the area and trigonometric function — 1 point

$\Phi_B = BA(\cos\theta) = B\pi r^2(\cos\theta)$

For correct substitution into the above equation — 1 point

$\Phi_B = 4(1 - 0.2t)\pi(0.10\text{ m})^2(\cos 60°)$

$\Phi_B = 0.063 - 0.013t$ (or in terms of a and b, $\Phi_B = a(1 - bt)(0.016)$)

(b) 2 points

For using a correct equation to solve for the emf — 1 point

$\mathcal{E} = -\dfrac{d\Phi_B}{dt}$

Substituting the expression from part (a):

$\mathcal{E} = \dfrac{d}{dt}(0.063 - 0.013t)$

For an answer consistent with part (a) — 1 point

$\mathcal{E} = 0.013\text{ V}$

Note: Any sign on the answer is ignored.

(c)

i. 1 point

For a substitution into Ohm's law consistent with the answer from part (b) — 1 point

$V = IR$

$I = \dfrac{(0.013\text{ V})}{(50\ \Omega)}$

$I = 2.6 \times 10^{-4}\text{ A}$

ii. 2 points

The correct choice is "Counterclockwise".

For a justification that incorporates that the original magnetic field is changing — 1 point

For a justification that correctly relates the induced current to the direction of a new magnetic field created by that current — 1 point

Example: Looking down at the loop from *P*, the vertical component of the magnetic field of the loop is upward and decreasing. To oppose this change, the current in the loop must create a magnetic field that is directed upward at point *P*. This requires a counterclockwise current in the loop.

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Question 3 (continued)

Distribution of points

(d) 2 points

For using a correct equation to calculate the energy dissipated — 1 point

$E = Pt = I^2Rt$ or $E = Pt = \left(\mathcal{E}^2/R\right)t$

For a substitution into either of the above equations with the answer from part (c)(i) or part (b) — 1 point

$E = \left(2.6 \times 10^{-4}\text{ A}\right)^2(50\ \Omega)(4.0\text{ s})$ or $E = \dfrac{(0.013\text{ V})^2}{(50\ \Omega)}(4.0\text{ s})$

$E = 1.4 \times 10^{-5}\text{ J}$

(e) 4 points

For selecting both "Zero" for the net magnetic force and "Nonzero" for the Net magnetic torque — 1 point

For indicating the forces on directly opposite sides of the loop are in opposite directions — 1 point

For concluding that the forces cancel & net force is zero — 1 point

For indicating that the torques add & net torque is nonzero — 1 point

Example: Since the current changes direction relative to the magnetic field as you go around the loop, the magnetic field will exert force of equal magnitude but opposite direction on opposite sides of the loop. These forces all cancel out resulting in zero net force. However, since this force is up on one side of the loop and down on the other side of the loop, this will create torques that rotate the loop in the same direction. Therefore, the net torque is not zero.

Units 1 point

For correct units on at least two parts with a calculated numerical answer and no incorrect units — 1 point