Question 1: Mathematical Routines (MR) — 10 points

A **(i)** For a multistep derivation that includes the equation $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$ — **Point A1**

Scoring Note: Vector notation is not required for this point to be earned.

For a correct substitution of the area of an appropriate Gaussian surface with nonzero flux for the region $R_1 < r < R_2$ (e.g., $2\pi r\ell$) — **Point A2**

For a correct expression for the enclosed charge (e.g., $\sigma_1(2\pi R_1\ell)$) — **Point A3**

Example Response

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(2\pi r\ell) = \frac{\sigma_1(2\pi R_1\ell)}{\varepsilon_0}$$

$$E = \frac{\sigma_1 R_1}{\varepsilon_0 r}$$

(ii) For substituting the expression for E from part A (i) into $\Delta V = -\int_a^b \vec{E} \cdot d\vec{r}$ — **Point A4**

Scoring Notes:
- Vector notation is not required for this point to be earned.
- The sign of ΔV is not considered for this point to be earned.

For an attempt to solve the integral for $|\Delta V|$ that includes correct limits (e.g., $|\Delta V| = \frac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r} dr$) — **Point A5**

Scoring Note: This point may be earned regardless of the order of the limits of integration; for example, $\frac{\sigma_1 R_1}{\varepsilon_0}\int_{R_2}^{R_1} \frac{1}{r} dr$.

Example Response

$$|\Delta V| = \left| -\int_a^b \vec{E} \cdot d\vec{r} \right|$$

$$|\Delta V| = \int_{R_1}^{R_2} \frac{\sigma_1 R_1}{r\varepsilon_0} dr = \frac{\sigma_1 R_1}{\varepsilon_0}\int_{R_1}^{R_2} \frac{1}{r} dr$$

$$|\Delta V| = \frac{\sigma_1 R_1}{\varepsilon_0} \ln(r)\Big|_{R_1}^{R_2} = \frac{\sigma_1 R_1}{\varepsilon_0} \ln\left(\frac{R_2}{R_1}\right)$$

(iii) For sketching a graph that is zero for both $0 < r < R_1$ and $r > R_2$ — **Point A6**

For sketching a graph that is decreasing and concave up for $R_1 < r < R_2$ — **Point A7**

Scoring Note: The curve does not have to intersect the vertical dashed lines for this point to be earned.

Example Response

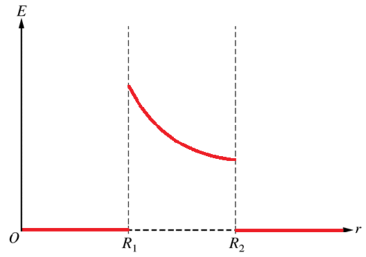


Figure 2

B For a multistep derivation that includes the equation $C = \frac{Q}{\Delta V}$ — **Point B1**

For indicating that C with the dielectric material inserted is C without the dielectric material inserted multiplied by κ (e.g., $C = \kappa\frac{Q}{\Delta V}$) — **Point B2**

For substitutions of both the total charge Q and the potential difference ΔV that are consistent with part A — **Point B3**

Example Response

Without the dielectric material

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{\sigma_1(2\pi L R_1)}{\left(\frac{\sigma_1 R_1}{\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)\right)}$$

$$C = \frac{2\pi L\varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}$$

With the dielectric material

$$C = \kappa\frac{Q}{\Delta V}$$

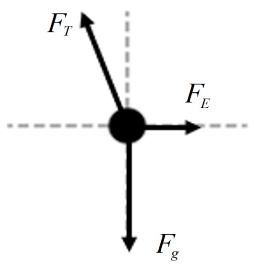
$$C = \frac{2\pi L\kappa\varepsilon_0}{\ln\left(\frac{R_2}{R_1}\right)}$$

Question 1: Free-Response Question — 15 points

(a)	For correctly drawing and labeling the electrostatic force directed to the right	**1 point**
	For drawing the force of tension up and to the left and the gravitational force in the downward direction	**1 point**

Scoring Note: A maximum of 1 point may be earned if extraneous forces are included.

Example Response



Total for part (a) — 2 points

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(b)	For equating the horizontal component of tension to the electrostatic force	**1 point**

Example Response

$$F_E = F_T \sin(\theta)$$

For equating the vertical component of tension to the gravitational force — **1 point**

Example Response

$$F_g = F_{Ty}$$
$$Mg = F_T \cos\theta$$

For an attempt to simultaneously solve the equations — **1 point**

Example Response

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$$

Example Solution

$$\Sigma F_y = 0$$
$$F_{Ty} = F_g$$
$$F_T \cos\theta = Mg$$
$$F_{Tx} = F_E$$
$$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}\frac{1}{\sin\theta}\cos\theta = Mg$$
$$d^2 = \frac{Qq\cos\theta}{4\pi\varepsilon_0 Mg\sin\theta}$$
$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$$

Total for part (b) — 3 points

(c)	For applying Coulomb's law to determine tension	**1 point**

Scoring Note: This point may be earned if the student used the vertical component of tension.

Example Response

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T \sin(\theta)$$

For correct substitution into an expression for tension consistent with part (b) or a correct expression for tension	**1 point**

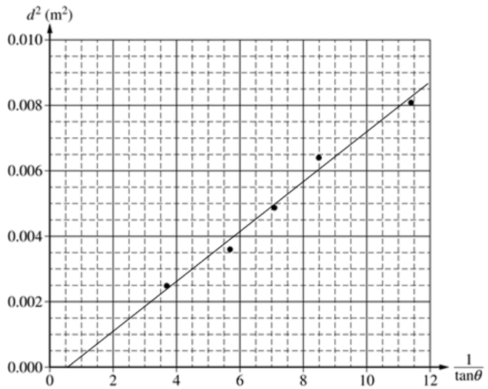
Example Solution

$$\Sigma F = 0$$
$$F_E - F_{Tx} = 0$$
$$F_E = F_{Tx}$$
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2} = F_T \sin(\theta)$$
$$F_T = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2 \sin(\theta)}$$
$$F_T = \frac{1}{4\pi\left(8.85\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N{\bullet}m}^2}\right)}\frac{\left(6.0\times10^{-8}\ \mathrm{C}\right)^2}{(0.057\ \mathrm{m})^2\sin\left(12^\circ\right)}$$
$$F_T = 0.048\ \mathrm{N}$$

Total for part (c) 2 points

(d)(i)	For a line that approximates the trend of the data	**1 point**

Example Response



(d)(ii)	For using two points from the trend line drawn by the student to calculate the slope	**1 point**

Scoring Note: Points of data may be used only if points of data are located directly on the line.

Example Response

$$\text{Slope} = \frac{\Delta y}{\Delta x}$$
$$\text{Slope} = \frac{\Delta(d^2)}{\Delta\left(\frac{1}{\tan(\theta)}\right)}$$
$$\text{Slope} = \frac{\left(0.0075\ \mathrm{m}^2 - 0.001\ \mathrm{m}^2\right)}{(10.5-2)}$$
$$\text{Slope} = 7.647\times10^{-4}\ \mathrm{m}^2$$

For correctly relating the slope of the graph to the equation $d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$	**1 point**

Example Response

$$d = \sqrt{\frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}}$$
$$d^2 = \frac{Qq}{4\pi\varepsilon_0 Mg\tan\theta}$$
$$d^2 = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)\frac{1}{\tan\theta}$$
$$\text{slope} = \left(\frac{Qq}{4\pi\varepsilon_0 Mg}\right)$$

For substituting the value of the slope of the graph into the equation $\varepsilon_0 = \frac{Qq}{4\pi Mg(\text{slope})}$ to calculate an experimental value of ε_0	**1 point**

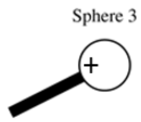
Example Solution

$$\text{slope} = \frac{Qq}{4\pi\varepsilon_0 Mg}$$
$$\varepsilon_0 = \frac{Qq}{4\pi Mg(\text{slope})}$$
$$\varepsilon_0 = \frac{\left(6.0\times10^{-8}\ \mathrm{C}\right)^2}{4\pi(0.005\ \mathrm{kg})\left(9.8\ \mathrm{m/s}^2\right)\left(7.647\times10^{-4}\ \mathrm{m}^2\right)}$$
$$\varepsilon_0 = 7.6\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N{\bullet}m}^2}$$

Total for part (d) 4 points

(e)(i)	For a "+" drawn on the left side of the sphere	**1 point**

Example Response



(e)(ii)	For a statement that indicates correct charge rearrangement on Sphere 3 due to the electric forces from the charges on Sphere 2	**1 point**

Example Response

The negative charges on Sphere 3 move to the right due to the attractive forces from the positive charges on Sphere 2, leaving a net positive charge on the left side of Sphere 3.

(e)(iii)	For selecting "$\theta_2 < \theta_1$" with an attempt at a relevant justification	**1 point**
	For statement that indicates **one** of the following: • The average distance between the repulsive charges is greater. • The electrostatic or repulsive force is less.	**1 point**

Scoring Note: Points 1 and 2 of part (e)(iii) can be earned with an answer that is consistent with the location of the excess positive charges drawn in part (e)(i).

Example Response

Excess charges on Sphere 3 are now free to move, so excess like charges will be concentrated on the far ends of Sphere 3 when the spheres are in static equilibrium. The excess like charges, located on opposite sides of Sphere 3, repel with less force than if the excess charges were located at the centers of Sphere 3. Thus, the downward force due to gravity on Sphere 2 causes the center of Sphere 2 to hang closer to the center of Sphere 3.

Total for part (e) **4 points**

Total for question 1 **15 points**

Question 3: Free-Response Question — 15 points

(a)	For a loop rule expression that includes terms for the equivalent resistance $\frac{R}{2}$ and the potential difference across the battery	**1 point**
	For an expression that includes charge Q in the term relating the potential difference across the capacitor and includes charge per unit time $\frac{dQ}{dt}$ in the term relating the potential difference across the pair of resistors	**1 point**

Example Response

$$\mathcal{E} - \Delta V_C - \Delta V_{R,eq} = 0$$

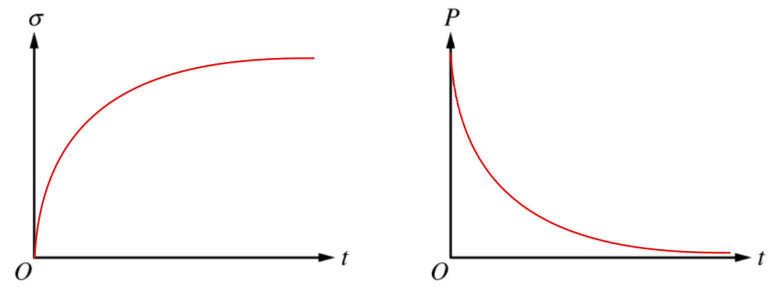
$$\mathcal{E} - \frac{Q}{C} - \frac{R}{2}\frac{dQ}{dt} = 0$$

Total for part (a) **2 points**

(b)	For sketching a concave down and increasing curve on the graph σ as a function of t	**1 point**
	For sketching a curve that is concave up and decreasing on the graph of P as a function of t	**1 point**
	For sketching both curves that approach a slope of zero as time increases	**1 point**

Scoring Note: The third point can be earned even if the first two points are not earned.

Example Response



Total for part (b) **3 points**

(c)(i) For a correct justification that could include one of the following: **1 point**

- An indication that the current is to the right with a justification that includes a statement that indicates that positive charge has accumulated on the top plate of Capacitor 1 and/or negative charge has accumulated on the bottom plate of Capacitor 1 when the switch was closed to Position A
- An indication that the current is to the right with a justification that includes a statement that indicates that the value of the electric potential of the top plate of Capacitor 1 is larger than the electric potential of the bottom plate of Capacitor 1 when the switch was closed to Position A

Example Responses

The current is directed towards the right because the top plate of Capacitor 1 is positively charged, meaning conventional current will flow clockwise.

OR

Toward the right. Current flows from high to low potential so it will flow from the top plate up and right through the switch.

(c)(ii) For indicating that the total charge on the positive plate of Capacitor 2 is $\frac{2}{3}Q_0$ **1 point**

Scoring Note: This point can be earned without supporting calculations.

Example Response

The potential difference across Capacitor 1 is equal to the potential difference across Capacitor 2. Capacitor 2 has twice the capacitance of Capacitor 1. Therefore, Capacitor 2 stores twice the charge that is stored on Capacitor 1. Due to conservation of charge, Capacitor 2 stores an amount of charge equal to $\frac{2}{3}Q_0$.

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(c)(iii) For an indication that the total energy dissipated by the resistors is the difference between an initial electric potential energy stored in one or both capacitors at time $t = t_1$ and a final electric potential energy stored on one or both capacitors after the new steady state conditions have been reached **1 point**

Example Response

$$E_R = U_C - U_{0C}$$

For indicating that only Capacitor 1 stores nonzero electric potential energy initially and both capacitors store nonzero electric potential energy after the new steady state conditions have been reached, or alternative consistent with part (c)(ii) **1 point**

Example Response

$$U_{0C} = U_{01}$$
$$U_C = U_1 + U_2$$

For correct substitutions for the charges stored on the capacitors after the new steady state conditions have been reached consistent with part (c)(ii) **1 point**

Example Response

$$\Delta E_R = U_C - U_{0C}$$
$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2C}\right)\left(\frac{2Q_0}{3}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$

Example Solution

$$\Delta E_R = U_C - U_{0C}$$
$$\Delta E_R = \left(\frac{1}{2}\left(\frac{1}{C}\right)\left(\frac{Q_0}{3}\right)^2 + \frac{1}{2}\left(\frac{1}{2C}\right)\left(\frac{2Q_0}{3}\right)^2\right) - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = \frac{Q_0^2}{6C} - \frac{1}{2}\frac{Q_0^2}{C}$$
$$\Delta E_R = -\frac{Q_0^2}{3C}$$

Total for part (c) 5 points

(d)	For indicating that the potential difference across each capacitor is the same or that the charge stored on each capacitor in steady state is the same	**1 point**

Example Responses

$\Delta V_1 = \Delta V_2$

OR

$Q_1 = Q_2$

For recognizing that the capacitance of each capacitor is now the same in the new configuration — **1 point**

Example Response

After steady state conditions are reached, both capacitors have the same potential difference. The new capacitance of Capacitor 2 *is equal to the capacitance of Capacitor* 1 *because the capacitance of a capacitor is inversely related to the distance between the plates of a capacitor. Therefore, since* $U_C = \frac{1}{2}C(\Delta V)^2$, $\frac{U_2}{U_1} = 1$.

Total for part (d) — 2 points

(e)(i)	For a loop rule that includes the terms for the emf of the battery, the potential difference across the pair of resistors, and the potential difference across Capacitor 1	**1 point**

Example Response

$\mathcal{E} - \Delta V_R - \Delta V_C = 0$

For a correct answer — **1 point**

Example Response

$I = \frac{Q_0}{RC}$

Example Solution

$$\mathcal{E} - \Delta V_R - \Delta V_C = 0$$
$$\mathcal{E} - I\left(\frac{R}{2}\right) - \left(\frac{Q_0}{2C}\right) = 0$$
$$\frac{Q_0}{C} - I\left(\frac{R}{2}\right) - \left(\frac{Q_0}{2C}\right) = 0$$
$$\frac{Q_0}{2C} - I\left(\frac{R}{2}\right) = 0$$
$$\frac{Q_0}{2C} = I\left(\frac{R}{2}\right)$$
$$\frac{Q_0}{C} = IR$$
$$I = \frac{Q_0}{RC}$$

(e)(ii)	For indicating that the current is zero	**1 point**

Total for part (e) — 3 points

Total for question 3 — 15 points