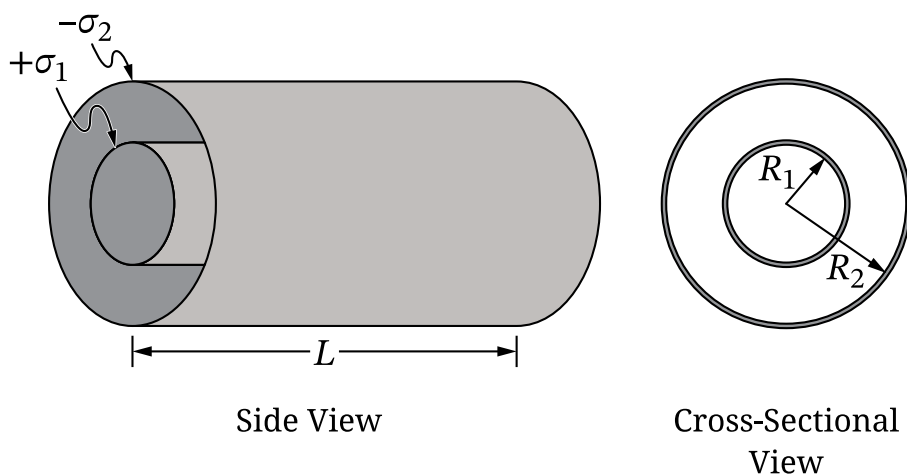


Question 1: Version J

1. An isolated, air-filled, charged capacitor consists of two conducting, coaxial, cylindrical shells that each have length L . The inner shell has radius R_1 and the outer shell has radius R_2 , as shown in Figure 1, where $R_1 < R_2 \ll L$. The surface charge densities (amounts of charge per unit area) of the inner and outer shells are $+\sigma_1$ and $-\sigma_2$, respectively. The absolute values of the total charges on the shells are equal.



Note: Figures not drawn to scale.

Figure 1

A.

- i. Using Gauss's law, **derive** an expression for the magnitude E of the electric field as a function of the radial distance r from the center of the capacitor for the region $R_1 < r < R_2$. Express your answer in terms of R_1 , σ_1 , r , and physical constants, as appropriate.
- ii. **Derive** an expression for the absolute value $|\Delta V|$ of the potential difference between the outer and inner shells in terms of R_1 , R_2 , σ_1 , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

- iii. On the axes shown in Figure 2, **sketch** a graph of E as a function of r from $r = 0$ to a position that is outside the outer shell.

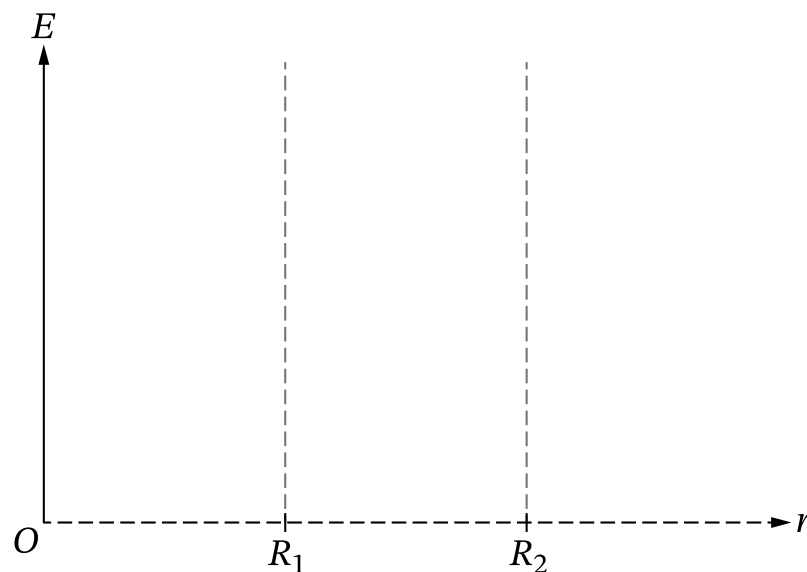
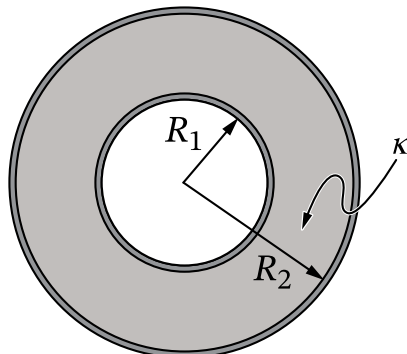


Figure 2

- B. A material of dielectric constant κ is inserted into the isolated, charged capacitor such that the material fills the region $R_1 < r < R_2$, as shown in Figure 3.



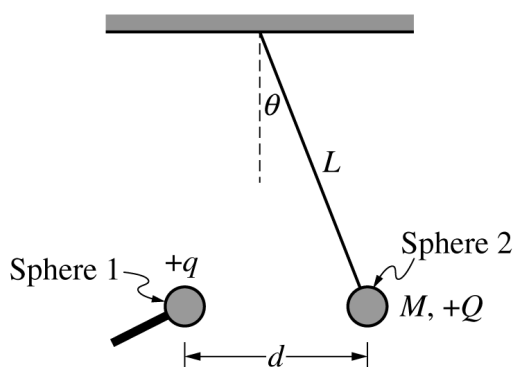
Cross-Sectional View

Figure 3

Derive an expression for the capacitance C of the capacitor with the material inserted in terms of L , R_1 , R_2 , κ , and physical constants, as appropriate. Begin your derivation by writing a fundamental physics principle or an equation from the reference information.

PHYSICS C: ELECTRICITY AND MAGNETISM**SECTION II****Time—45 minutes****3 Questions**

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

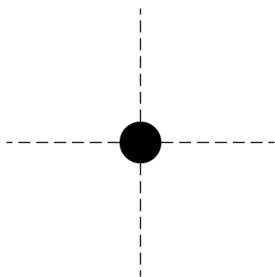


Students perform an experiment to determine the value of vacuum permittivity ϵ_0 . Sphere 1 is nonconducting with charge $+q$ and is attached to an insulating rod. Sphere 2 is nonconducting with charge $+Q$ and has mass M . Sphere 2 is hung from a string of negligible mass and length L . Sphere 1 is brought near, without touching, Sphere 2, as shown. Equilibrium is established when the centers of the two spheres have the same vertical position, are a horizontal distance d apart, and the string is at an angle θ from the vertical.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 1** on this page.

- (a) On the following dot that represents Sphere 2 at the position shown in the previous figure, draw and label the forces (not components) that act on Sphere 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.

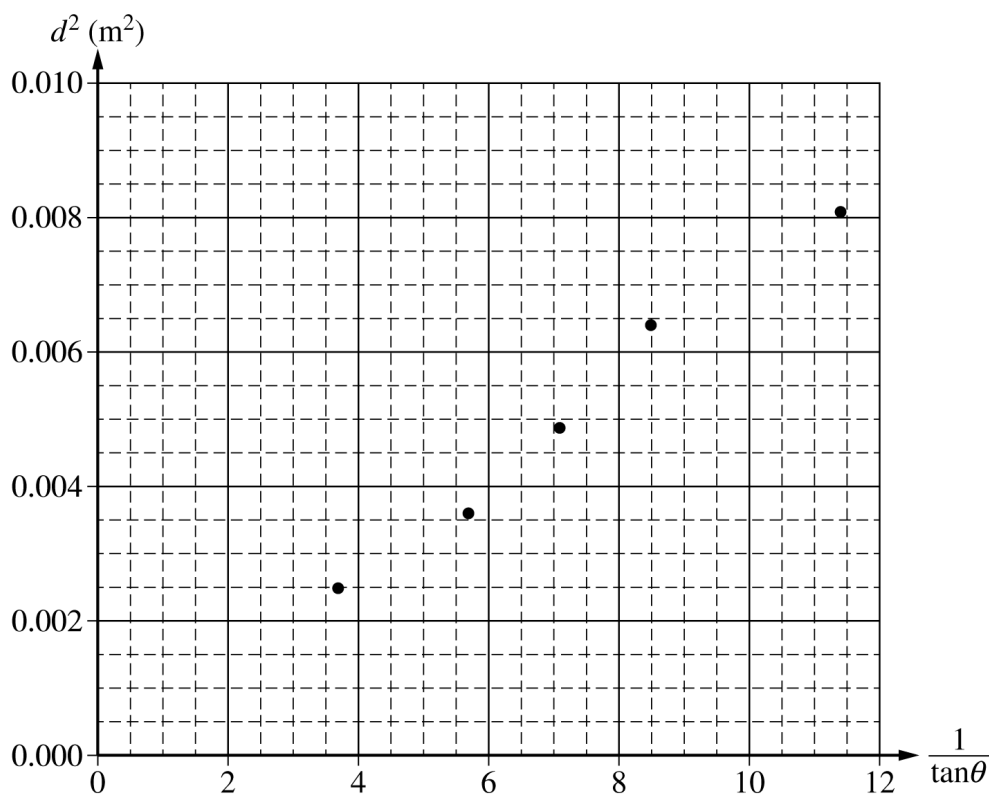


- (b) Derive the relationship between the distance d and the angle θ to show that $d = \sqrt{\frac{Qq}{4\pi\epsilon_0 Mg \tan\theta}}$.

- (c) These values are collected in one trial: $Q = q = 6.0 \times 10^{-8} \text{ C}$, $\theta = 12^\circ$, and $d = 0.057 \text{ m}$. Calculate the expected force of tension exerted on Sphere 2 by the string.

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- (d) The students vary d and measure θ after equilibrium is reached. The students use the collected data to plot the following graph of d^2 vs. $\frac{1}{\tan\theta}$.

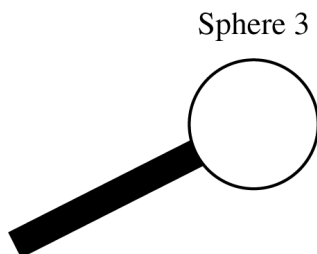


- Draw the best-fit line for the data.
- Using the best-fit line, calculate an experimental value for the vacuum permittivity ϵ_0 when $M = 0.0050$ kg and $Q = q = 6.0 \times 10^{-8}$ C.

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(e) The students modify the experiment by replacing Sphere 1 with a conducting Sphere 3 that has the same size and charge $+q$. The experiment is repeated.

i. The circle in the following figure represents Sphere 3 when spheres 2 and 3 are at equilibrium. On the circle, draw a single “+” sign to represent the location of highest concentration of the excess positive charges.



ii. Briefly explain your reasoning for the sketch drawn in part (e)(i).

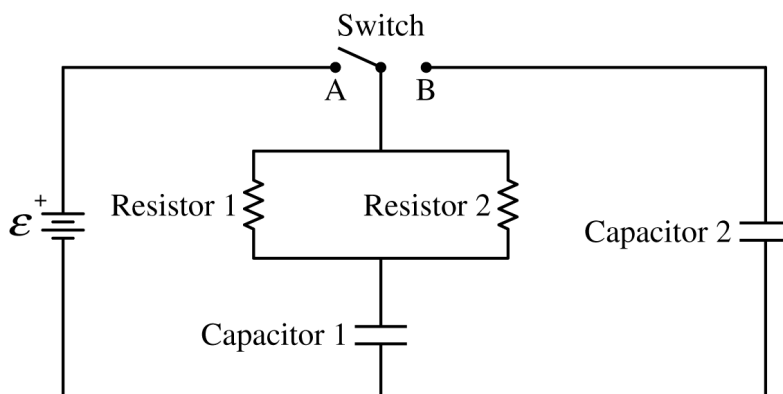
iii. In the original experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_1 from the vertical. In the modified experiment, when the centers of the two spheres are a horizontal distance d_1 apart, the string makes an angle θ_2 from the vertical.

Is θ_2 greater than, less than, or equal to θ_1 ?

_____ $\theta_2 > \theta_1$ _____ $\theta_2 < \theta_1$ _____ $\theta_2 = \theta_1$

Briefly justify your answer.

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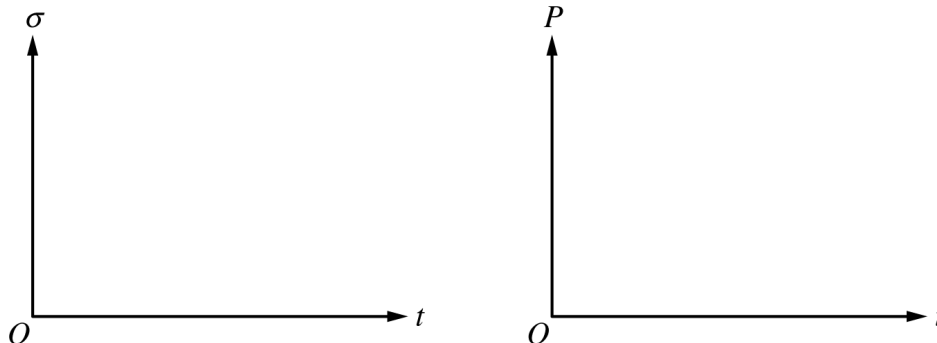
The circuit shown consists of a battery of emf $\mathcal{E}$, resistors 1 and 2 each with resistance R , capacitors 1 and 2 with capacitances C and $2C$, respectively, and a switch. The switch is initially open and both capacitors are uncharged.

At time $t = 0$, the switch is closed to Position A.

- (a) Write, but do NOT solve, a differential equation that can be used to determine the charge Q on the positive plate of Capacitor 1 as a function of time t after the switch is closed to Position A. Express your answer in terms of $\mathcal{E}$, R , C , Q , t , and fundamental constants, as appropriate.

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- (b) On the axes shown, sketch graphs of the surface charge density σ on the positive plate of Capacitor 1 and the total power P dissipated by the resistors as functions of time t from time $t = 0$ until steady-state conditions are nearly reached.



A long time after the switch is closed to Position A, the charge on the positive plate of Capacitor 1 is Q_0 and Capacitor 2 is uncharged.

- (c) At time t_1 , the switch is closed to Position B.

i. Immediately after time t_1 , is the direction of the current in the switch directed toward the left, directed toward the right, or is there no current? Briefly justify your answer.

ii. Determine an expression for the total charge on the positive plate of Capacitor 2 a long time after t_1 . Express your answer in terms of Q_0 and fundamental constants, as appropriate.

GO ON TO THE NEXT PAGE.

iii. Derive an expression for the total energy E_R dissipated by resistors 1 and 2 from immediately after time t_1 until new steady-state conditions have been reached. Express your answer in terms of C , Q_0 , and fundamental constants, as appropriate.

With the switch still closed to Position B, the parallel plates of Capacitor 2 are moved so that the separation distance increases by a factor of 2.

(d) Determine the ratio $\frac{U_2}{U_1}$ of the energy U_2 stored in Capacitor 2 to the energy U_1 stored in Capacitor 1 a long time after the plates of Capacitor 2 have been moved. Briefly justify your answer.

GO ON TO THE NEXT PAGE.

With the capacitors still charged as in part (d), the switch is now closed to Position A.

(e) Express your answers to part (e)(i) and part (e)(ii) in terms of R , C , Q_0 , and fundamental constants, as appropriate.

i. Derive an expression for the current I_0 from the battery immediately after the switch is closed to Position A.

ii. Determine the current I_∞ from the battery a long time after the switch is closed to Position A.

GO ON TO THE NEXT PAGE.

STOP

END OF EXAM