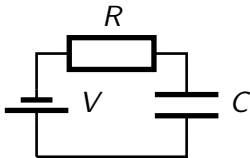


Electric Circuits

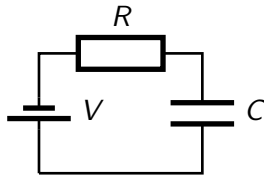
AP Physics C: E&M ■ Unit 11

AP Physics C: E&M



What we will cover

- 1 Electric current
- 2 Ohm's law and power
- 3 Compound circuits and internal resistance
- 4 Kirchhoff's rules
- 5 RC circuits



1

Current

Electric current and Ohm's law

Rate of charge flow

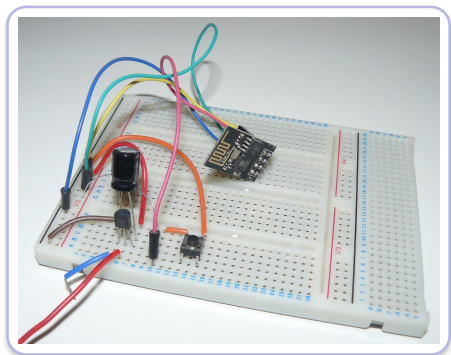
$$I = \frac{dQ}{dt}$$

Conventional current 常规电流 is opposite to the electron **drift** 漂移速度.

Ohm's law

$$V = IR, \quad R = \frac{\rho L}{A}$$

Resistivity 电阻率 ρ is a material property; R is the object's.



A breadboard circuit for testing $V = IR$

What a circuit is, and what Ohm's law claims

a circuit 电路

closed loops of wires, batteries, resistors, bulbs, capacitors, inductors, switches and meters – charge flows **only** around a closed loop

steady state 稳态

current the same everywhere along one branch; nothing accumulates

Ohm's law 欧姆定律

$I = \frac{\Delta V}{R}$ for an **ohmic** 欧姆性 element – one whose R does not change with I

resistance

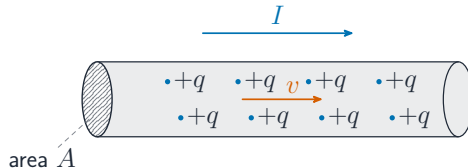
$R = \frac{\rho \ell}{A}$: material through ρ , and **geometry** through ℓ and A

ideal battery 理想电池

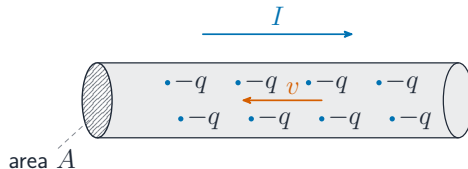
maintains a fixed emf with **zero** internal resistance; a real one has r in series

Current is a **scalar** 标量 with a chosen direction, not a vector. Conventional current runs the way **positive** charge would move.

Electric Current



(a) positive carriers drift with I



(b) electrons drift against I

Charge carriers drift slowly through a conductor to make a current

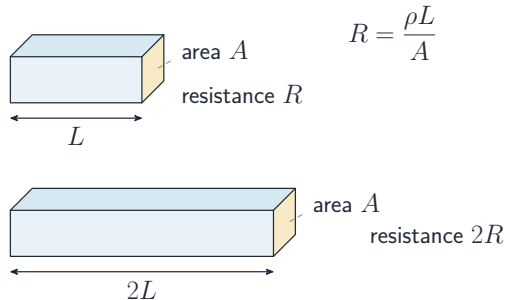
Worked example – how slowly electrons actually drift

A copper wire of area $1.0 \times 10^{-6} \text{ m}^2$ with $n = 8.5 \times 10^{28} \text{ m}^{-3}$ carries 1.7 A. Find the drift velocity 漂移速度.

$$\begin{aligned}
 \blacksquare \quad v_d &= \frac{I}{nqA} \\
 \blacksquare &= \frac{1.7}{(8.5 \times 10^{28})(1.6 \times 10^{-19})(1.0 \times 10^{-6})} \\
 \blacksquare &= 1.3 \times 10^{-4} \text{ m/s} - \text{about 0.5 m per hour}
 \end{aligned}$$

The *signal* travels near light speed; the electrons crawl. **Current density** 电流密度 $\vec{J} = nq\vec{v}_d$ is the same idea per unit **cross-sectional area** 横截面积, tied to the field by $\vec{E} = \rho\vec{J}$, with $I = \int \vec{J} \cdot d\vec{A}$ in general.

Resistance, Resistivity, and Ohm's Law



twice the length $\Rightarrow$ twice the resistance

A longer conductor has more resistance; a wider one has less

Worked example – stretch a wire to double its length

**A wire is stretched to twice its length.
What happens to its resistance?**

- the volume is fixed, so doubling ℓ halves A
- $R' = \frac{\rho(2\ell)}{A/2} = 4 \frac{\rho\ell}{A}$
- $= 4R$

Both geometric factors move, and they move the *same* way. Changing only ℓ would give $2R$ – the missing half of the answer is the area.

2

Internal resistance

Compound circuits and internal resistance

Reduce series/parallel resistors to one **equivalent resistance** 等效电阻. A real battery is an ideal **emf** 电动势 plus a small **internal resistance** 内阻 r :

Terminal voltage

$$V_{term} = \varepsilon - Ir$$

Worked example

$\varepsilon = 12\text{ V}$, $r = 1\ \Omega$, load $5\ \Omega$:

$$I = \frac{12}{6} = 2\text{ A}$$

$$V_{term} = 12 - 2(1) = 10\text{ V}$$

2 V lost inside the battery.

Power, and how to rank bulb brightness

where it comes from

a charge q falling through ΔV gives up $q\Delta V$ of energy, so the rate is $P = I\Delta V$

three forms

$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$ – use the one whose two quantities you actually know

in a resistor

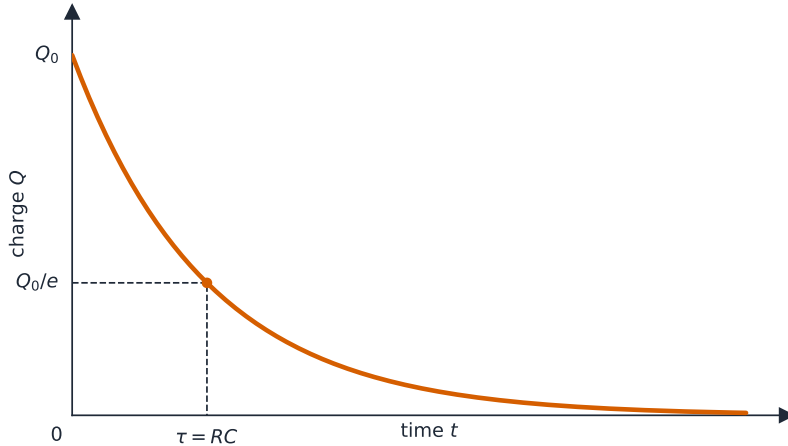
all of it becomes **heat**

brightness

rank bulbs by **power**, never by current or by voltage alone – a bulb can carry more current and still be dimmer

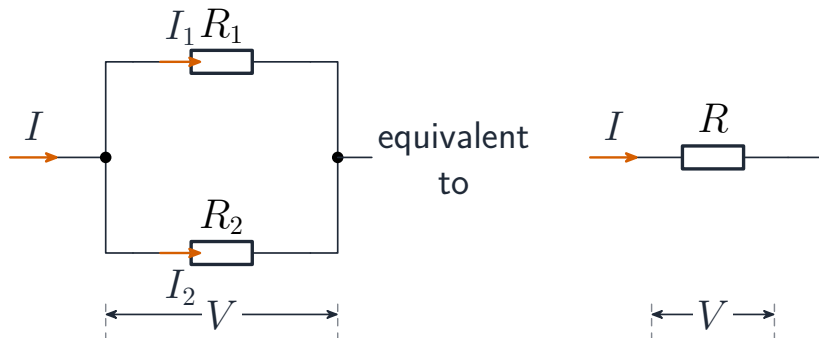
In **series** the current is shared, so $P = I^2 R$ ranks them: bigger R is brighter. In **parallel** the voltage is shared, so $P = (\Delta V)^2 / R$ ranks them: smaller R is brighter. The ranking reverses.

Resistor-Capacitor (RC) Circuits



The charge on a capacitor decays exponentially as it discharges

Compound Direct Current Circuits



Resistors in parallel combine to a smaller equivalent resistance

Where each meter goes, and why

ammeter 电流表

in series at the point whose current you want – ideal $R = 0$, so it changes nothing

voltmeter 电压表

in parallel across the element – ideal $R = \infty$, so it draws no current

get it backwards

an ammeter in parallel is a short circuit; a voltmeter in series stops the current

circuit diagram 电路图

redraw before you calculate – series and parallel are far easier to see in a tidy diagram than in the given one

The ideal resistances are chosen so the meter does not disturb what it measures. Every real meter fails that a little, which is what internal-resistance questions probe.

3

Kirchhoff

Kirchhoff's rules

Loop rule (energy)

$$\sum \Delta V = 0 \quad \text{around a loop}$$

A rise across an emf, a drop $-IR$ across a resistor.

Junction rule (charge)

$$\sum I_{in} = \sum I_{out}$$

An oscilloscope: voltage against time reveals how current and charge evolve in a circuit

Both Kirchhoff rules are conservation laws

junction rule

charge is **conserved**: current in equals current out at any node

loop rule

energy is conserved: charges exchange $\Delta U_E = q \Delta V$, so $\sum \Delta V = 0$ around any closed path

signs

walk the loop one way; a drop across a resistor with the current is $-IR$, a rise through a battery from $-$ to $+$ is $+\mathcal{E}$

how many equations

one per independent loop plus one per independent junction – enough for the unknowns, never more

If your loop equation does not close, you have a sign error, not an arithmetic one. Redo the walk and check each element's polarity.

4

RC

RC circuits

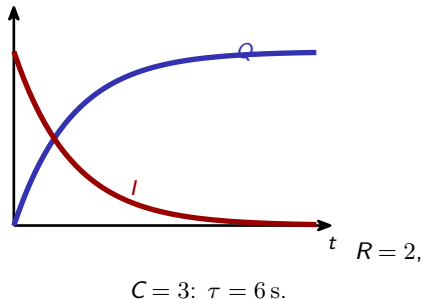
A capacitor charges through a resistor with **time constant** 时间常数 $\tau = RC$:

Charging

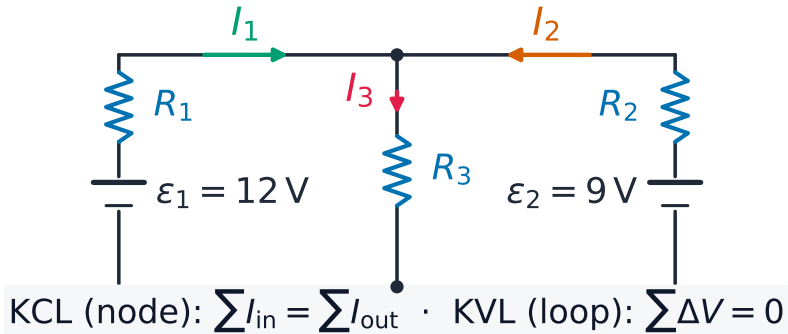
$$Q(t) = C\varepsilon \left(1 - e^{-t/\tau}\right)$$

$$I(t) = \frac{\varepsilon}{R} e^{-t/\tau}$$

Empty capacitor = a wire at first; full = blocks steady current.

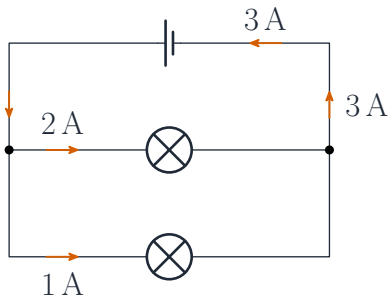


Kirchhoff's Junction Rule



Two loop equations and one junction equation solve this two-battery circuit

Current divides at a junction



at each junction: $3\text{ A} = 2\text{ A} + 1\text{ A}$

Current divides at a junction: what flows in equals what flows out

Exam tips

- Relate current to charge flow $I = \frac{dQ}{dt}$ and use $J = \sigma E$, $R = \frac{\rho L}{A}$ for resistance.
- Apply **Kirchhoff's laws** (junction: charge; loop: energy) with consistent sign conventions.
- Analyse **RC circuits** with calculus: charging/discharging give exponentials with time constant $\tau = RC$.
- Combine resistors (series add, parallel reciprocal) and track power $P = IV = I^2 R$.
- At $t = 0$ a capacitor acts like a wire; after a long time ($t \rightarrow \infty$) it acts like an open branch.

5

Recap

Unit 11 – key takeaways

1

Current & Ohm

$$I = dQ/dt; V = IR; P = IV = I^2 R = V^2/R$$

2

Real battery

$$V_{term} = \varepsilon - Ir, \text{ small internal resistance}$$

3

Kirchhoff

$$\text{loop } \sum \Delta V = 0; \text{ junction } \sum I_{in} = \sum I_{out}$$

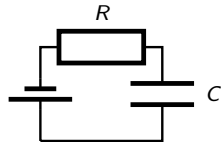
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RC

$$\tau = RC; \text{ charge and current decay exponentially}$$

Check yourself

- 1 Write current as a derivative of charge.
- 2 What conservation law is Kirchhoff's loop rule?
- 3 At $t = 0$, how does an empty capacitor behave?
- 4 After a long time, how much current flows through a charged capacitor?



Answers 1. $I = dQ/dt$ 2. conservation of energy 3. like a plain wire 4. zero